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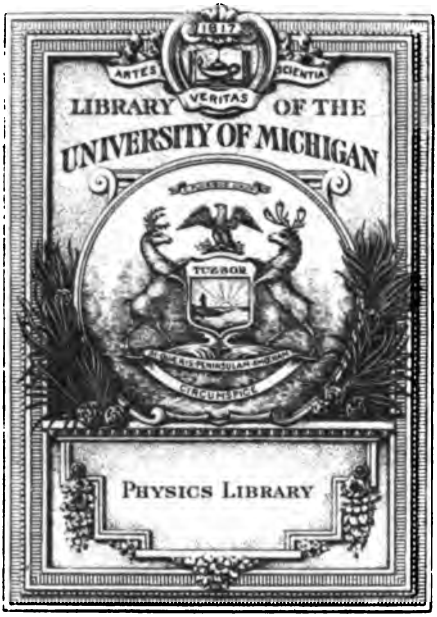

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GENERAL PHYSICS

AN ELEMENTARY TEXT-BOOK FOR COLLEGES

BY

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FAYERWEATHER PROFESSOR OF PHYSICS
IN NORTHWESTERN UNIVERSITY

THIRD EDITION

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PREFACE

In these days of numerous text-books the author who dares still further to increase the number owes to the public, at least, an explanation in which he shall set forth his purpose, however far short of accomplishment he may have fallen.

Accordingly, in this volume, the author's hopes have been: —

(i) To prepare a text-book adapted to the needs of first-year students in physics at Northwestern University. One fear in this connection is that the transfer of a course of lectures from flexible manuscript to rigid type metal may correspond to that stage of development which, in the life history of an animal, is known as *rigor mortis*. This difficulty, it is hoped, may be avoided by varying, from year to year, the illustrative phenomena employed in the lecture demonstrations.

(ii) To keep the treatment elementary, and yet include all the fundamental principles of physics; and at the same time to bind them together with "connective tissue" in such a way as to make clear to the student the essential unity of the subject.

(iii) Not merely, or even mainly, to impart information, but to set before the student a large and compact body of truth obtained by a method which shall remain for him, throughout life, a pattern and norm of clear and correct thinking.

To succeed completely in an undertaking so ambitious as that suggested by the threefold purpose just stated is more than the author dares to hope.

To my friend, Mr. John Mackenzie of Minneapolis, I am indebted for those paragraphs in Chapter III which describe Professor Osborne Reynolds' remarkable theory of gravitation.

The imperfections of the present text would have been many more except for the clever revision of the proof by my colleague, Professor R. R. Tatnall, and by my friend, Professor

A. A. Knowlton of the Armour Institute. Special thanks are due Mr. J. H. Spencer of the United States Weather Bureau, to the *Youth's Companion*, and to the De Laval Steam Turbine Company, for the use of cuts. Mr. John A. Fleming of the Carnegie Institution was good enough to revise those paragraphs dealing with the secular variation of the earth's magnetic field. To my sister, Caroline L. Crew, I am indebted for the excision of many awkward English constructions.

H. C.

EVANSTON, ILLINOIS,
April 8, 1908.

PREFACE TO SECOND EDITION

THE author begs to express his indebtedness to the following friends for the elimination of many errors of statement and of typography in the first edition: Professor F. E. Austin, Professor James Barnes, Mr. N. E. Bennett, Professor A. W. Duff, Dr. L. R. Ingersoll, Professor N. A. Kent, Professor G. E. Ripley, Professor L. P. Sieg, Professor N. F. Smith, Professor O. M. Stewart, Professor R. R. Tatnall, and Dr. Samuel Willard.

H. C.

EVANSTON, ILLINOIS,
June 1, 1909.

PREFACE TO THIRD EDITION

The changes involved in this revision are mainly the following: —

(i) A more concrete and historical treatment of Dynamics; more prominence to Statics.

(ii) A simpler and more unified treatment of Electricity and Magnetism, secured by use of the electron theory.

(iii) A new chapter on Electromagnetism, in which the entire discussion is based upon the two great discoveries of Oersted and Faraday.

Besides there are a number of minor additions, including sections on Liquid Air, Diamagnetism, Atmospheric Electricity, and Photometry.

For all these and many other suggestions, the author is deeply indebted to Professor F. E. Kester of Kansas, A. Zeleny and P. E. Klopsteg of Minnesota, Gordon S. Fulcher of Wisconsin, and G. F. Hull of Dartmouth.

The author wishes to record his especial obligation to Professor D. A. Kreider of Yale, who with penetrating judgment and perfect candor criticized nearly every chapter in the earlier edition.

Dr. R. B. Owens of the Franklin Institute has courteously granted permission for the use of certain electrotypes; so also have Messrs. Leeds and Northrup, and the Weston Electrical Instrument Company. My colleagues, Professors George V. McCauley and K. K. Smith, have generously performed the distasteful task of going over the proof.

H. C.

Evanston, Illinois,
June 21, 1916.

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GENERAL PHYSICS

GENERAL PHYSICS

Introductory; the Place of Physics Among the Sciences

The Greeks used the same word (κόσμος) to denote both *nature* and *order*. In this fact is discovered an admirable attitude of mind in which to approach the study of physics, a science which has for one of its principal objects the discovery of order in nature.

Physics, in the present sense of the word, is not an ancient science, but a creation of the last three hundred years. Compared with astronomy and medicine, it is a mere upstart. When, however, one considers the work of the Egyptians in handling tremendous blocks of stone, such as those used in the construction of the pyramids, or the work of the Greeks in devising powerful catapults, or the work of the Romans in distributing water through well-constructed aqueducts, or any one of their numerous clever devices, such as the steelyards, locks, and surgical instruments which have been recovered at Pompeii, it becomes evident that the ancients were familiar with a wide range of physical facts. That there were among them keen observers is patent to every one who has considered the Hermes of Praxiteles or the columns of the Parthenon. The skill of the ancients in hunting and the accuracy with which they observed the motions of the planets are proverbial.

Nor were they wanting in the desire for a unitary view of the world about them. This search for unity is evinced in the idea of Heraclitus that igneous vapor (fire) was the one great principle, in the idea of Thales that aqueous vapor was the key to the situation, in the idea of Anaximenes that air was the essential principle, and in the notion of Democritus that the origin of all things was to be sought in the atom. The failure of these various views gave rise to the general opinion that four " elements " — earth, air, fire, and water — constituted an irreducible minimum for the physical universe. The Ptolemaic system of crystalline spheres, cycles, and epicycles is another product of their eagerness for unity. Well, then, in what respect does modern physical science differ from the ancient? Why is the beginning of this science supposed to lie only some three hundred years back of the present?

The answer to this question, like that to most others which begin with "why," is not easy; and all that can be done here is to point out briefly *how* ancient differs from modern science rather than to explain *why* this difference exists.

During the fifteenth century there seemed to come over the minds of thinking men a change, marked especially by an increased freedom from the trammels of authority. Such freedom had already, even as early as the fourteenth century, made itself evident in painting, architecture, and music. A new literature had found expression in Dante, Petrarch, and Boccaccio, whose works were now being disseminated by the newly invented art of printing. An increased boldness was shown by navigators, which culminated in the discovery of America. This same century was marked by the birth of Copernicus, with Paracelsus and Leonardo da Vinci for contemporaries. Without shedding any new light upon the " cause " of this changed attitude of mind, it may perhaps be best described as " the spirit of the Renaissance." Previous to this time, those great generalizations which we now call " physical laws " seem to have been stumbled upon, so to speak; after this period, the search for law, for the constant elements in the behavior of nature, seems to have been more a matter of deliberate and definite plan, while the results of the search have been tremendously prolific of other important results.

It was upon such a background as this, and into such a changed world as this, that on the 15th of February, 1564, at Pisa, in Italy, the founder of modern physics was born. But, while all science is the creation of man, it is equally true that no science is the creation of any single man. In what sense, therefore, can Galileo be spoken of as the founder of modern physics? Just this: he it was who first discovered the fundamental principles of modern dynamics, the one branch of learning upon which the entire superstructure of the physical sciences has been erected. His formulation of these principles was, as we shall later see, imperfect; but his suggestion that bodies change their motion when, and only when, acted upon by some external force — or, if you please, the substitution of the conception " force " in place of " cause " of motion — has proved to be the most fertile principle, as well as the first fundamental and perfectly general principle, of modern dynamics, a principle which finds its later and more perfect expression in Newton's Three Laws of Motion. Among the ancients, Archimedes (287–221 B.C.) appears to be the one man who approached the modern method in physics. He established a firm foundation for the science of statics — the conditions of equilibrium for bodies at rest; but this is a very special case of dynamics, and the achievements of Archimedes, which early attracted the attention of Galileo, — remarkable and valuable as they impress us to-day, — bore little fruit until the idea of force as the condition for change of motion was introduced by the great Italian. From Galileo's time on, motions cease to be classified as " natural " and " violent." *All* motions are henceforth " natural," and are proper subjects for special investigation. The faith and skill in experiment exhibited by Galileo set a new pace, one which had possibly never before been reached, and one which has certainly seldom since been surpassed.

Very striking is the manner in which experiment continues to grow in favor. During the last half century astronomers have made many experiments in order to interpret their observations. Still more recently have biologists taken to experiment and with such earnestness that a leading American naturalist (*Science*, vol. 41, p. 4, 1915) considers it the " most salient feature of the new zoölogy."

The experimental method once established, new facts and new generalizations accumulated with marvelous rapidity; and the subject of natural philosophy, enriched by the labors of Hooke, Boyle, and Newton in England, Descartes and Mersenne in France, Huygens in Holland, Torricelli and other pupils of Galileo in Italy, became so bulky that certain natural cleavage lines began to appear.

Certain phenomena were soon recognized as peculiar to living bodies — bodies which possess the ability to assimilate food, the power of reproduction, sensation, etc., and these came to be studied under the head of natural history and medicine. This entire group, including, as it now does, botany, zoölogy, physiology, anatomy, paleontology (as distinguished from geology), bacteriology, etc., has, by wide consent, been given the name of the **natural sciences.** All matter, other than that of living bodies, is said to be inanimate; and it is with this class of bodies especially that the **physical sciences** have to do.

In the early days, nearly all the phenomena of the physical sciences were studied under the head of **Natural Philosophy,** a term which was employed in contradistinction to **Natural History,** and one which included such diverse subjects as the motions of the solar system, engineering structures, and meteorology. But Physics, or Natural Philosophy, in its narrow and proper sense, does not deal with *all* the phenomena of dead matter. There are, for instance, certain bodies which are so distant from us that they can be studied only by specially designed instruments such as the telescope. Accordingly, the study of the fixed stars, the planets, comets, etc., is pursued under a special department of physical science called **Astronomy.** There is another exceedingly large and important branch of physical science which deals especially with those changes which occur in homogeneous bodies — mainly changes of composition. The systematic study of these changes is carried on under the head of **Chemistry.**

Those properties of inanimate bodies which are not primarily connected with a change in their chemical composition are then the especial subjects of study pursued under the head of **Physics.**

Besides these there are other important, but more specialized, branches of physical science which deal with particular groups of bodies. Thus the comprehensive science which takes up the study of the earth's crust from a physical and chemical standpoint is called **Geology**; while **Meteorology,** a science of great significance and of increasing practical value, deals exclusively with the physical phenomena of the earth's atmosphere.

Built upon the physical and natural sciences, is still another group of studies distinguished from the preceding "pure" sciences mainly by the purpose and attitude of mind on the part of the student. Such are the various branches of engineering, which are largely branches of applied physics; such also is the study of modern medicine, a science which has become immediately dependent upon Biology, Chemistry, Physics, Physiology, and Anatomy.

In attempting roughly to locate Physics among the various sciences, we have so far considered only those groups of phenomena which are associated with matter, and have left to one side the phenomena of mind, also the historical sciences which deal with the purposes and the controlling wills of individuals and nations. To one side also have been left those sciences which deal with the consensus of sane opinion and with the expression of the will of the entire race, sciences which Münsterberg calls "Normative" and which include Logic and Mathematics.

The place of Physics in the family of the sciences is then a very modest one, for it deals only with a particular group of phenomena, associated with inanimate matter, and limited essentially to those changes in which the composition of the various substances involved remains unaltered. Within this limited domain the purpose of the physicist is to discover and to describe the universal and constant modes in the behavior of nature. Very often these results can be given a simple and accurate mathematical formulation. With what splendid success this work has been pushed forward will appear when we come to study in more detail the work of the men who have followed the great Galileo, men such as Newton, Faraday, and Maxwell in England; Fresnel and Ampère in France; Gauss, Weber, Kirchhoff, Helmholtz, and Hertz in Germany; Volta and Melloni in Italy; Franklin, Henry, and Rowland in America.

Vast and bewildering as this great body of discovered truth appears, we have only to look beyond the frontiers of knowledge to realize how contemptible, in comparison with the unknown, are the few scientific results which go to make up the present science of Physics.

"The Book of Nature is a fine large tapestry rolled up which we are not able to see all at once."
ROBERT BOYLE, *Works*, VI, p. 796.

CHAPTER I

KINEMATICS. THE SCIENCE OF MOTION

POSITION

1. Kirchhoff (1824–1886), a distinguished German physicist, first clearly pointed out that the aim of the physicist is to describe the motions of bodies completely and in the simplest possible manner. But since a body is a limited portion of matter, which we may consider as made up of a number of small particles, it will be simpler to consider first the motion of a *particle* and then pass to the consideration of *bodies* in motion. The word "particle" is used always to denote a body so small that its dimensions may be neglected in comparison with other distances involved. Thus a baseball may be considered a particle when one is thinking only of its path through the air; but in the pitcher's hands it becomes a body of very appreciable size, a body which he sets into rotation about a definite axis in order to give it any desired "curve." However, since motion is merely change of position, it will be convenient first to learn how the position of a particle is defined.

POSITION OF A PARTICLE IN A PLANE

2. When geographers describe the position of a town, they assume that the reader already knows the position of some point on the earth, which may be called a point of reference. Such a point of reference for the surface of the earth is the intersection of the meridian through Greenwich with the equator. This fixed, the location of a town is definitely described when two things are known about it; namely, its *latitude* and its *longitude*. Thus the island of St. Helena is in 15° 55′ south latitude and 5° 43′ west longitude.

A similar method is frequently used in physics; *e.g.* the position of a point P (Fig. 1), with reference to another point O, *in the same plane,* is described when two mutually rectangular straight lines are drawn through the point O, and the perpendicular distances of the point P from these two lines respectively are given. **The point of reference O is generally called the origin. The horizontal line of reference $\overline{OX}$ (Fig. 1) is generally called the axis of X; and the vertical line of reference $\overline{OY}$, the axis of Y. The longitude x of the point P is called its abscissa; while the latitude y is known as its ordinate.** Abscissas are positive when drawn to the right from the axis of Y; negative when drawn to the left. Ordinates are positive when drawn upward from the axis of X; negative when drawn downward. These quantities x and y are known as the **rectangular coördinates** of the point P; they are a special kind of Cartesian coördinates, so called after Descartes, who first employed an algebra of this kind.

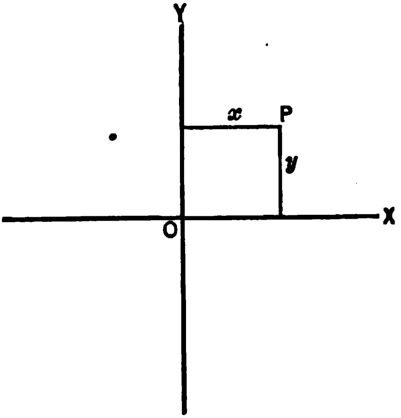

FIG. 1.—Illustrating rectangular coördinates.

In the laboratory we have frequent occasions to draw curves, and this is generally done by locating points on coördinate paper in the manner just described.

Navigators and surveyors generally employ a different method of locating one point with reference to another in the same plane. **They give the straight-line distance from the first point to the second, then they give the direction of the line joining the points. This method also is frequently used in physics and is known as the method of polar coördinates.** The idea will be clear from Fig. 2, where O is the point of reference and $\overline{OX}$ any known direction. There is no doubt about the exact position of a particle at P as soon as we know the distance r between the points O and

P, and the angle θ between the lines *OP* and *OX*. **The angular distance θ we shall always consider positive when measured in a direction contrary to that in which the hands of a clock move, a direction which is called counterclockwise.** In like manner, θ is considered negative when measured in a clockwise sense. The line $\overline{OX}$ we may call the line of reference.

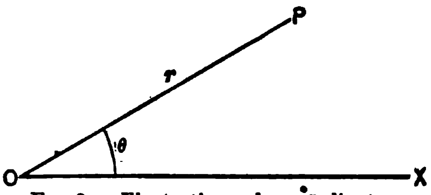

FIG. 2. — Illustrating polar coördinates.

POSITION OF A PARTICLE IN SPACE

3. If a point on the earth's surface is to be described still more completely, we may choose the sea level as a plane of reference, and give the altitude of the point. So in mathematics and physics one frequently describes a point by locating it first in a certain plane as described in the preceding sections, and then stating the position of this plane with respect to some other plane of reference. It will be observed that, in general, *three specifications are needed to completely describe the position of one point with reference to another:* and when the numerical values of these three coördinates are once known, the position of the point is given without ambiguity. Thus if the rectangular coördinates of a particle in space have the values

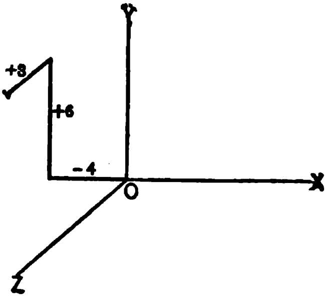

FIG. 3.

$$\begin{cases} x = -4, \\ y = +6, \\ z = +3, \end{cases}$$

one can proceed to locate it as indicated in Fig. 3.

POSITION VECTORS

4. But a third and simpler way to think of the position of a point is to consider it as completely specified by a single straight line running from the point of reference to the point under consideration. If in Fig. 4 we know, not only the length, but also the direction (including the sense) of the straight line $\overline{OP}$, we know all that can possibly be known about the position of the particle at P.

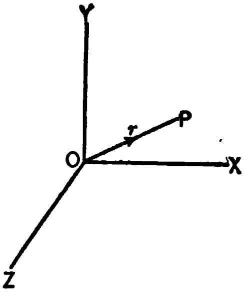

FIG. 4.

To any such quantity as $\overline{OP}$, possessing direction, sense, and amount, the name "vector" has been given. This class of quantities includes many of the most important studied in physics.

A vector quantity is simply one which *resembles a limited straight line* in three particulars; namely, *in the possession of direction, sense, and numerical magnitude.* A vector which locates one point with reference to another, as in Fig. 4, is called a position vector, and is a type of vector quantities in general, of which we shall meet many examples in later pages. The usefulness of the vector, in representing the position of a point, lies in its clearness and simplicity; for purposes of numerical calculation it has small value.

DIGRESSION ON SCALAR QUANTITIES

5. The mere length of a straight line is the same in whatever direction the line may lie. The *length* of a straight line is, therefore, *not* a vector quantity. It is independent of direction. **Quantities which do not have direction are called scalars.**

Philadelphia is a city 90 miles from New York. This distance is a scalar quantity. The information contained in this statement tells me only that Philadelphia lies somewhere on a circle of 90 miles radius described with New York as a center. But "90 miles southwest of New York" gives me a vector quantity, which locates the position of Philadelphia without ambiguity. A pound of sugar, three yards of calico, two gallons of water, a volume of anything, any amount of matter, are illustrations of scalars.

SUMMARY

6. Summarizing, then, the position of a particle is nearly always described by one or the other of the three following methods: —

(i) *Rectangular coördinates*, in which the position of a point is defined by giving its distance from each of three mutually perpendicular planes. These distances are called the rectangular coördinates of the particle and are usually denoted by x, y, z. The three mutually perpendicular reference planes are indicated in Fig. 3 by YOZ, ZOX, and XOY.

(ii) *Polar coördinates*, in which the position of a particle is described by its distance r from the origin, and by its latitude and longitude on a sphere whose center is at the origin and whose radius is the distance r just described. These three quantities are called the polar coördinates of the particle and are usually denoted by the letters r, θ, and ϕ respectively.

(iii) *Vectors*, a graphical method in which the position of a particle is located as the terminal point of a single straight line drawn in a definite sense and in a definite direction from the origin.

POSITION OF A BODY

7. Having thus far considered the position of a *particle* only, we now take up the position of a *body* which has already been defined as a limited portion of matter. For the present let us deal only with perfectly rigid solids, that is, with bodies whose size and shape are each constant. If we consider a body merely as a system of particles, it is evident that its position is completely defined only when the position of *each* particle in the system is known. But since the body is rigid, this is equivalent to saying that its position is completely determined when we know the positions of any three points in the body, provided of course that these points do not lie in the same straight line. In other words, if a straight line and one point outside this line be located in a rigid body, the position of the entire body is completely defined. It is clear, therefore, that a body may have many different positions in which two points in the body are fixed; for under this condition it may be rotated through any angle about the straight

line joining these two points. As we shall see later, it is often quite as necessary to consider the angular as well as the linear position of a body; while for a single particle angular position is a term which has no meaning.

MEASUREMENT OF LENGTHS

8. The actual determination of the position of any point in the world about us leads at once to the measurement of a distance, by which is meant simply the comparison of the distance in question with a given distance which we agree to take as unity. As later experience will show, comparison is the essential feature of all measurement. Ten yards of cloth is simply a piece of cloth which is ten times as long as a certain piece of wood called a yardstick.

UNITS OF LENGTH. CENTIMETER. FOOT

9. **Almost the entire scientific world has agreed to take as the standard length the distance between two marks on a certain platinum-iridium bar preserved in the International Metric Bureau at Sèvres, near Paris. This distance is called a meter. The one hundredth part of the distance between these two marks is taken as the unit and is called a centimeter.** For many laboratory measurements this smaller unit will be found more convenient than the meter.

It will be observed that this definition is purely arbitrary, a mere matter of convention. The fact that the meter is so chosen as to be nearly equal to one ten-millionth of the distance from the earth's equator to its north pole is interesting; but it does not enter into the definition of the unit of length. As a matter of fact, the length of the earth's quadrant has been found to be more nearly 10,000,880 meters.

English and American engineers generally employ the **yard** as the standard of length and the **foot**, which is one third of a yard, as the unit of length.

In Great Britain the yard is, by definition, the distance between two rulings on a certain bronze bar preserved in the Standards Office at London. In the United States the yard is defined as 36/39.37 of the standard meter at Sèvres.

From the definition of the yard one sees that

One inch = 1/36 yard = 2.54001 centimeters

and One mile = 1760 yards = 1609.35 meters.

MEASUREMENT OF AREAS. UNIT OF AREA

10. Having settled upon the centimeter as a unit of length, the simplest possible unit of area is the area of one square centimeter. In consideration of this fact, the unit of area almost universally employed in science is the **square centimeter.** Very seldom, indeed, are areas measured by direct comparison with a unit of area. More frequently it will be found convenient to measure the *linear* dimensions of the surface in question, and, from these, to *compute* by geometry the area of the surface. Engineers, of course, employ the square foot as the unit of area.

MEASUREMENT OF VOLUMES. UNIT OF VOLUME

11. Standards of volume, such as the bushel, the quart, the cubic foot, are already familiar. **In scientific work the cubic centimeter is adopted as the unit of volume,** and for the same reasons that led to the adoption of the square centimeter as the unit of area. An acquaintance with the metric system makes the wisdom of this choice at once apparent.

In the case of volumes whose linear dimensions can be measured, *e.g.* a parallelepiped or a sphere, it is frequently more convenient to compute the volume than to measure it directly.

ANGULAR POSITION. UNIT ANGLE

12. When two straight lines do not coincide in direction, they are said to have different angular positions. The difference of their directions is the angular displacement of one line with respect to the other. This angular displacement is expressed by surveyors and astronomers in degrees. But navigators use "points" of the compass, or 11¼°, as units. In physics and mathematics the unit of angle mostly employed in the description and discussion of phenomena is different from either of these. It is defined as follows: If two straight lines lying in the same plane differ in direction, they will intersect. About the point of intersection as center, and in the plane of these two lines, draw a circle. **These lines are said to include a unit angle when they intercept an arc of this circle equal in length to its radius. See Fig. 5. This unit angle is called the radian.**

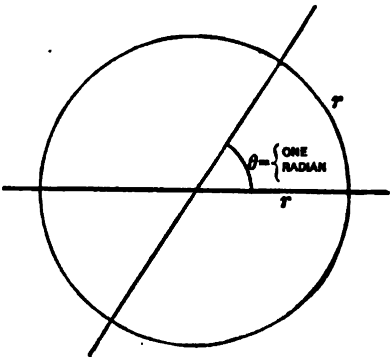

FIG. 5. — Unit angle.

It is evident, since the circumference of a circle is a length equal to 2π times the radius, that in one right angle there will be one quarter of 2π radians, *i.e.* $\frac{\pi}{2}$ radians. Compute, and on the margin of this page record, the number of degrees in one radian.

Problems

1. Explain just how it is that any limited straight line is a vector quantity, while the length of such a line is a scalar quantity.

2. What light does the etymology of the word "vector" shed upon its present meaning?

3. How many radians in the angle subtended by a semicircle?

4. The volume of a cube is 921 c.c. Find the area of one face of the cube.

5. Draw a pair of axes at right angles to each other. With reference to these axes, locate the points A, B, and C which have abscissas and ordinates as follows: —

POINT	ABSCISSA	ORDINATE
A	$+2$	$+7$
B	$+5$	-10
C	-8	-4

6. How many cubic centimeters in the volume of a hemisphere of 8 cm. radius?

7. Draw three mutually rectangular axes of coördinates and locate the two points whose coördinates are (− 2, + 3, + 4) and (+ 1, + 5, + 2).

8. Find the length of the vector joining the two points whose positions are given in the preceding problem.

9. Which of the two following points is farther from the origin, (+ 5, − 9) or (+ 7, + 7)?

10. The two mutually perpendicular sides of a right-angled triangle are 9 and 20 centimeters, respectively, in length. Find the area of the triangle in square centimeters, and the length of the hypothenuse in centimeters.

11. Compute the number of seconds of arc in one radian.

12. Locate on the plane of the blackboard the following position vectors:—

$$\begin{Bmatrix} r = 30, & \theta = 15^\circ \\ r = 30, & \theta = 345^\circ \\ r = 30, & \theta = 195^\circ \end{Bmatrix}.$$

13. Considered as a unit of volume, what advantage has the cubic centimeter over the cubic inch?

14. How many cubic centimeters are there in a cubic foot?

15. How many threads to the inch must a screw have in order that its nut may be advanced one half millimeter for each rotation?

CHANGE OF POSITION

I. *Case of a Particle*

13. Imagine a bird sitting on a tree at the point *P* in the diagram (Fig. 6); and suppose yourself an observer standing at the point *O*. The line $\overline{OP}$ will represent the position of the bird. But if while you remain at *O* the bird flies to another tree at *Q*, it is evident that the position of the bird will now be represented by the straight line *OQ*, which differs from *OP* in both direction and length. How shall

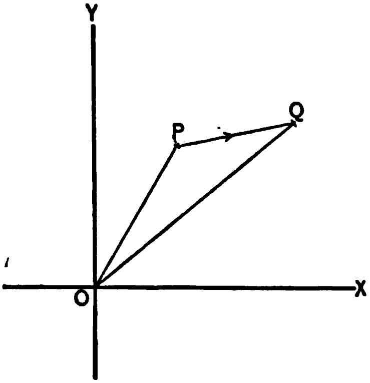

FIG. 6.—Change of position, a quantity having direction.

one most clearly describe the *change* in the bird's position? Evidently the simplest way is to draw a straight line from P to Q and let the line $\overline{PQ}$ represent the change in position. For the new position of the bird may be defined either by giving the two straight lines $\overline{OP}$ and $\overline{PQ}$, or by simply giving the one straight line, $\overline{OQ}$; by either process we definitely locate the new position, Q.

If the point Q lies upon the line $\overline{OP}$, or upon $\overline{OP}$ extended, the change of position involves a change of distance only; if however $\overline{OP}$ and $\overline{OQ}$ have each the same length, then the change is one of direction only.

The upshot of the whole matter is that change of position is described in exactly the same way as is position itself; namely, by stating the length, sense, and direction of a straight line.

Digression on the Addition and Subtraction of Vectors

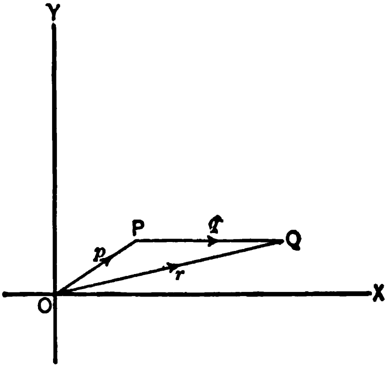

Fig. 7.—Vector addition.

14. Referring to Fig. 7, let us denote the vector $\overline{OP}$ by $\bar{p}$, the vector $\overline{PQ}$ by $\bar{q}$, and the vector $\overline{OQ}$ by $\bar{r}$. Then it will be clear that the new position Q of the particle may be located in either one of two different but strictly equivalent methods. By one method, the particle at Q is located as the terminal point of the vector $\bar{r}$. By the other method, it is the terminal point of two vectors $\bar{p}$ and $\bar{q}$, placed end to end with their arrows in the same sense. **And since $\bar{r}$ locates the point Q at the same distance and in the same direction from the origin, $\bar{r}$ is said to be the resultant or vector sum of $\bar{p}$ and $\bar{q}$; while $\bar{p}$ and $\bar{q}$ are said to be the components of $\bar{r}$.** In the algebra of vectors this fact is expressed by writing

$$\bar{p} + \bar{q} = \bar{r}, \qquad \text{Eq. 1}$$

an equation which contains the whole story of vector addition.

In ordinary algebra, which deals only with scalar quantities, we should have

$$p + q > r,$$

but this inequation refers merely to lengths and not at all to directions.

The **resultant** of any number of vectors is obtained by precisely the same process, namely, **placing these vectors end to end, in any order, but all in the same sense, and then joining the initial point of the first vector to the terminal point of the last vector.**

This graphical method gives rise to the vector polygon, as illustrated in Fig. 8, where $\bar{r}$ is the resultant of the vectors $\bar{p}$, $\bar{q}$, and $\bar{s}$. From what precedes it will be seen that vector addition means nothing more than what is implied in the statement that, so far as mere transportation is concerned, it is all the same whether one travels by train directly from Chicago to Pittsburg, or goes first from Chicago to Toledo, thence to Cleveland, and thence to Pittsburg.

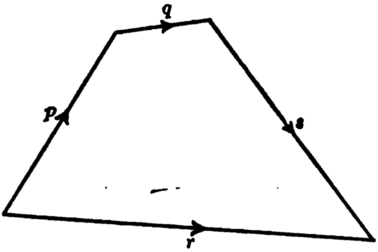

FIG. 8.—Polygon of vectors.

Since the sign of a vector determines its sense, one sees that the subtraction of any given vector from another is precisely the same as reversing the sense of the given vector and then adding it to the other.

When a particle receives a displacement which is represented by any given vector, say $\overline{OP}$, Fig. 9, the question often arises as to what the amount of the displacement is in some other direction, say $\overline{OH}$.

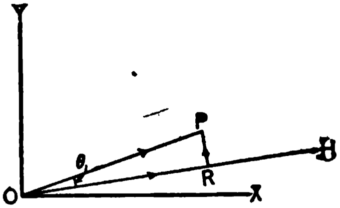

FIG. 9. — Illustrating the resolution of a vector.

This is found by letting fall from the terminal point P a perpendicular upon the line $\overline{OH}$. If R denotes the foot of this perpendicular, we may consider the vector $\overline{OP}$ as broken up into two components $\overline{OR}$ and $\overline{RP}$, of which the former lies entirely along

$\overline{OH}$; the latter has no component along $\overline{OH}$. From the definition of a cosine it is evident that

$$OR = OP \cos \theta. \qquad \text{Eq. 2}$$

It is customary to speak of OR as the **projection** of $\overline{OP}$ upon the straight line OH, or as the **component** of $\overline{OP}$ along OH.

The correct method of adding and subtracting vectors seems to have been first set forth clearly by Newton and Varignon in 1687. For in this same year Newton published his *Principia* and Varignon his *Plan for a New Mechanics.*

II. *Case of a Rigid Body*

15. As has been observed (§ 7), the position of a rigid body is fixed when any three points in the body, not lying in the same straight line, are fixed.

Accordingly, one might expect the position of a rigid body to change in a great variety of ways. But as a matter of fact there are only three different methods in which a rigid body can shift its position.

(i) The first of these is the motion one sees in the flywheel of a sewing machine or in a grindstone, or in any wheel and axle, such as that indicated in Fig. 10, where the rotation is about a fixed axis. **This motion of a rigid body about a fixed axis is called a pure rotation.**

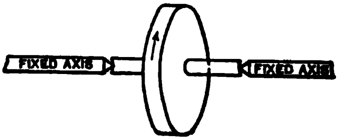

FIG. 10.—A case of pure rotation.

(ii) The second of these motions is one which occurs in the piston rod of an ordinary steam engine, and in a sled which slides to and fro in its own track. Here no point in the body is fixed. The motion is such that **each point in the body describes a path equal and similar to the path of every other point.** Such a motion is called a **pure translation.**

From the above definition, it is evident that, if one places a compass excentrically on a horizontal glass plate which is slowly rotating about a fixed vertical axis, the circular motion of the compass needle will be one of pure translation.

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(iii) The only other motion which a rigid body can have is a combination of the two preceding, that is, a body may be rotated and translated at the same time. Such a motion is called, in technical language, a **twist**. The first example of a twist which will occur to the reader is doubtless the motion of a nut with reference to its screw, or of a ring sliding over a rod, or of a baseball pitched in such a way as to acquire a " curve."

The student who pursues the subject further will find that any change of position, such for instance as that which occurs when one takes his watch from his pocket and lays it upon the table, may be accomplished by a single twist.

The fact is then that every motion of a rigid body turns out to be either a pure translation or a pure rotation or a twist.

As we have already seen, linear displacements are represented by vectors; so also are angular displacements. For imagine the rough outline in Fig. 11 to represent any rigid body having one point O fixed. Take this point as origin. Imagine the body to be turned through any angle θ about an axis $\overline{OP}$. Such an angular displacement is what we have just called a pure rotation. It requires for its description three specifications, namely, (i) the direction of the axis of rotation, (ii) the amount of rotation, *i.e.* the angle through which the body has turned, and (iii) the sense of rotation. In short, this angular displacement requires a vector to represent it.*

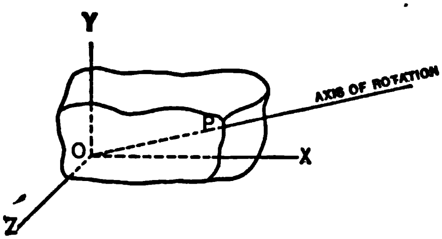

Fig. 11. — Angular displacement a vector.

Thus, if a grindstone be rotated through 90°, its angular displacement would be represented by a vector drawn in the direction of the axis of the grindstone, and of a length $\frac{\pi}{2}$, the sense being such that if one looks in the direction of the arrow on the vector, the stone will appear to rotate in a clockwise direction.

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* The student's attention is here especially directed to the fact that in adding vectors which represent angular displacement, the resultant is not independent of the order in which the vectors are added, except when the angular displacements are infinitesimal.

Such a relation between the direction of the vector and the direction of the rotation is exactly that which exists between the direction of advance and the direction of rotation in an ordinary woodscrew. It is hence called the right-handed screw relation. It is important to observe that the rule for the addition of vectors applies to angular displacements only when these are very small.

In the later parts of physics, we shall meet many pairs of quantities related in the same way as are these two displacements, linear and angular, which we have just been considering.

DEGREES OF FREEDOM

I. *Case of a Particle*

16. It is interesting to observe that when a particle is free to change its position by moving in any direction whatever, it can move in three, and only three, directions, such that its motion along any one of these directions has no component along either of the other two. And it is evident that these three directions are mutually perpendicular; for if they were inclined at any angle other than 90°, say θ, a displacement r along one axis would have a component $r \cos \theta$ along the other. These three directions are represented, therefore, by any three rectangular axes. In view of these facts, a particle is said to have three **degrees of freedom.** In other words, a particle has three possibilities in the way of translational motion. It cannot rotate because, by definition, it has no appreciable dimensions. When the motion is confined to a single plane, it has, of course, only *two* degrees of freedom; and when its motion is limited to a line, a particle is said to have only *one* degree of freedom.

II. *Case of a Rigid Body*

17. The case is quite different with an extended body. Consider a coin, a metal cylinder, or any rigid body whatever; if it is perfectly free, it can be translated parallel to any of the three rectangular axes. Like the particle, it has three degrees of translational freedom. Now imagine this body supported on a single pivot, in the same manner as a spinning top. Here one point is fixed so that the body has no freedom of translation; yet it is clearly possible for the body to rotate about any one of three mutually perpendicular axes passing through this fixed point. A rotation about any one of these axes will have no component of rotation about either of the other axes. A rigid body not constrained in any way is said, therefore, to have three degrees of freedom of rotation, making in all six degrees of freedom for any unhampered body, namely, three of translation and three of rotation.

The student will find it both interesting and important to discover, in any particular case of constrained motion, just how many degrees of freedom a body has; for the advanced student will find later that the whole behavior of a body may be described by use of as few coördinates as the body has degrees of freedom

Interesting Special Case. Uniplanar Motion

18. When the motion is in a single plane, the rigid body can be transferred from any one position to any other position (in that plane, of course) by a pure rotation alone.

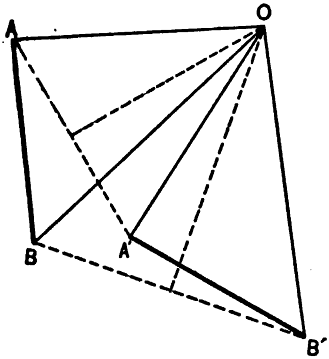

FIG. 12. — Illustrating the most general uniplanar motion.

To prove this theorem, consider any two positions of the body as represented by AB and $A'B'$ in Fig. 12. Bisect AA' and also BB'. Draw perpendiculars through these points of bisection. Denote by O the point where these perpendiculars intersect. Then the axis about which the body AB must be rotated to bring it into the position $A'B'$ is perpendicular to the plane, and passes through the point O. For evidently the triangle AOB may be rotated about this axis until it is exactly superposed upon the congruent triangle $A'OB'$.

Problems

16. Does the arm of a rocking chair in actual use rotate about one fixed axis or about a variable axis?

17. A marble free to roll on a floor, but not free to leave the floor, has how many degrees of freedom?

18. Design a rigid body having one degree of freedom of translation and one of rotation.

19. How many degrees of freedom has one of the balls on the governor of a steam engine?

20. Most motions that are employed in actual machinery are constrained, *i.e.* they have less than six degrees of freedom. What, in your estimation, is the special case which occurs most frequently in machines?

21. In the following motions of rigid bodies distinguish clearly which are cases of pure translation, which are cases of pure rotation, and which are cases involving both translation and rotation: —

i. The motion of a door on its hinges.
ii. The motion of a carriage wheel with reference to its axle.
iii. The motion of a carriage wheel with reference to the road on which it travels.
iv. The motion of a rocking-chair as one rocks in it.
v. The motion of the eyepiece as one focuses a pair of opera glasses.
vi. The motion of the blade, with reference to the handle, when one opens a pocketknife.

22. What vector will describe the final position of a man who walks 10 miles N.W., then turns and walks 10 miles S.W.?

23. Two boats sail from the same port; at the end of 5 hr. one has traveled 50 mi. due E., the other $\sqrt{5000}$ mi. S.E. What vector will then represent their relative positions? *Ans.* 50 mi. due S.

24. The coördinates of two points are (+ 2, + 3) and (− 1, + 4). Find the vector which joins this pair of points.

25. A particle moves in a circle starting from a given point which we may call the origin. Find the vector which describes the position of the particle when ⅔ of the circumference has been traversed.

26. Plot the following four points and show that they are the corners of a parallelogram: (+ 3, + 2); (+ 4, + 1); (+ 2, + 5); (+ 3, + 4).

27. What are the rectangular components of the vector which joins the following two points: (+ 7, + 3); (+ 10, + 5)?

RATE OF CHANGE OF POSITION. VELOCITY

19. It is to be carefully observed that *up to this point* we have been concerned not at all with the *history* of the body between the time of its leaving its old position and the time of its reaching its new position. We do not care how long a body may have been in changing its position, nor does it matter through what " cause " it may have changed its position. We have been concerned only with a method for describing, most simply and most completely, the change in position of the body. When, however, it is asked *how* this change occurred, we are led to the consideration of *Time* and *Velocity*.

In the world about us we find motion to be a state of matter which is quite as natural — perhaps even more natural — than rest. Displacement may be considered as the result of motion. But very often we shall be concerned rather with the rate of motion. Accordingly, we must now consider the method of describing a rate of motion, *i.e.* a velocity.

When a man buys a railroad ticket from New Orleans to Boston, he pays for " displacement," and, in general, the price of a railroad ticket varies directly as the amount of displacement. But one's choice of a train depends largely upon its speed, or rate of displacement. The difference in price between a good watch and a poor one depends principally upon the rate at which the angular position of the hands change. If this rate is very uniform, the watch is well made; if the hands do not move at a fairly constant rate, the watch is poorly made.

The word **rate** (which is merely the English form of the Latin word *ratio*) denotes the quotient of any one quantity by another. A **time rate** is a rate in which *time* appears in the denominator. Thus the rate of interest is the amount of money one pays for the use of a dollar, divided by the number of years he has used the dollar. The death rate is the total number of deaths among a thousand people during any period, divided by the number of years in that period. The rate with which we are here especially concerned, however, is the time rate at which a particle passes over distance.

But before we can measure time rates we must decide upon a standard and a unit of time. These are described in the following paragraph.

TIME. UNIT OF TIME

20. The mean (*i.e.* **the average) time occupied by the sun in one apparent revolution about the earth is known as the mean solar day.** This interval is universally employed as the standard of time. **The $\frac{1}{86400}$ part of a mean solar day is called a second and is used by the scientific world as the unit of time.**

Intervals of time can be compared with marvelous accuracy by means of watches, clocks, or chronographs.

DEFINITION OF SPEED

21. More than a century ago it was proved by Lavoisier, the great French chemist, that all matter has a continuous existence; from which fact it follows that, as a particle moves from one point to another, it must occupy every intermediate position along the route. This route is then a continuous geometrical curve. Accordingly the **path** of a particle is defined as the continuous locus of all the successive positions which it occupies. We are now able to define clearly what is meant by **speed**, namely, **the rate at which a particle moves along its path.** Direction of motion, it will be observed, does not enter into this definition. A man who, during fifteen minutes, walks a mile along a winding cowpath travels with precisely the same average speed as another man who walks a mile in fifteen minutes along a straight city pavement.

Let us denote by p the distance which a particle has moved along any path during the time t. Then for purposes of computation, the average speed of the particle is expressed as follows: —

$$\text{Average Speed} = S = \frac{p}{t}. \qquad \text{Eq. 3}$$

But if the speed of the particle is varying from point to point, as is usually the case in nature, then denote by p_1 the distance traversed by the particle at the time t_1; and by p_2 the distance traversed by the end of the time t_2. Now if t_1 and t_2 be so chosen that t_1 is very little earlier than t; and t_2 a very little later than t, the speed at any instant t will be the limit approached by the ratio $\frac{p_2 - p_1}{t_2 - t_1}$. Accordingly one writes

$$\text{Instantaneous Speed} = S_t = \left[\frac{p_2 - p_1}{t_2 - t_1}\right]_{t_2 \doteq t_1} \qquad \text{Eq. 4}$$

where the subscript $t_2 \doteq t_1$ indicates that t_2 approaches indefinitely near to t_1.

The necessity for some expression of this kind will appear when one asks such a question as, "What is the speed of a baseball thrown vertically into the air?" The question has no meaning unless one specifies some particular instant of time; for the ball has every speed from the initial speed with which it left the hand of the thrower to zero, which is its speed at the top of its path.

Speed is, therefore, really, a *limit*. The conception of a limit is something which each student should bring with him from a study of elementary geometry; or, not having acquired it there, he should now master it once for all, since the idea is one which is essential even in elementary physics.

Unit of Speed

22. Having adopted the centimeter as the unit of length and the second as the unit of time, we have no trouble in settling upon the proper scientific unit of speed; namely, a speed of one centimeter per second. The engineer, however, generally employs the foot per second for his unit of speed.

Problems

28. Find the average speed, in miles per hour, of a steamer which makes the passage from New York to Liverpool, 3032 mi., in 6 da.

29. A 24-hour train from New York to Chicago, 926 mi., must make what average speed?

30. A shot travels a distance of 4 km. in 10.5 sec. Find its average speed in meters per second.

31. A speed of 1 ft. per second is equivalent to a speed of how many centimeters per second?

32. A gun is fired at a distance of 1 km.; the report is heard 3.25 sec. later. From this observation, compute the speed of sound in air.

33. Light travels from the sun to the earth in about 8 min. The distance of the sun from the earth is about 95 million mi. Find the approximate speed of light in miles per second. What is the value of this speed in meters per second?

34. One man walks at the rate of 3 mi. per hour; another walks at the rate of 7 km. per hour. Which one travels faster?

35. Assuming that the circumference of the earth at the equator is 25,000 mi., find the speed of a particle on the equator rotating once in 24 hr. Express this speed in miles per second, also in meters per second.

Velocity of a Particle

23. It is evident from the preceding examples that speed alone is not sufficient to determine the rate at which a particle is changing *position*. Consider, for instance, a particle which moves through a small distance from P_2 to P_3 along the curved path indicated in Fig. 13, where P_2 is located by the vector $\bar{r}_2$ and P_3 by the vector $\bar{r}_3$. In order to describe the rate of change of *position*, it is necessary to know not only how far the particle moved in passing from P_2 to P_3, but also in what direction the line joining P_2 and P_3 lies.

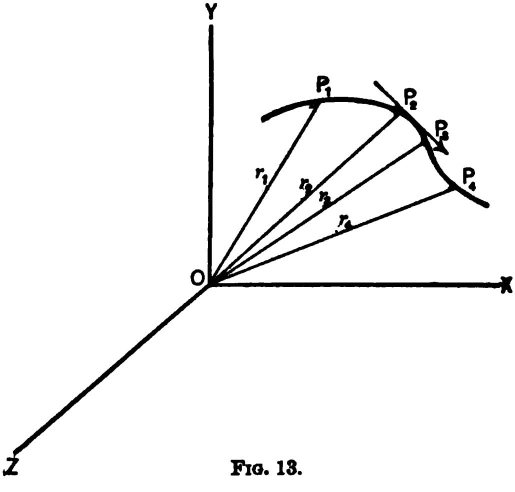

Fig. 13.

It is clear, of course, that when a particle moves, it always moves at any particular instant, in some one direction, *and* in some one sense. To completely describe the motion at any instant it is necessary, therefore, to know not only the speed but also the direction and the sense of the motion at that instant. This can be most conveniently done by use of **a vector called "velocity" which has the same direction and sense as that of the motion and which has a length equal to the speed of the motion.** Such a vector completely describes the velocity of the particle at the particular instant under consideration. Velocity is, therefore, a vector quantity, and speed a scalar. To illustrate: "A boat leaves her pier and steams in a straight line at a speed of 10 miles an hour for 3 hours." From this information, all that we know of the boat's position is that she now lies somewhere on a circle of 30 miles' radius, having the wharf for center. If, however, her velocity (instead of her speed) had been given by saying that "she steamed for 3 hours at a 10-mile rate in a northeasterly direction," her present position would be accurately known. **Velocity** is, accordingly, defined as the **time rate of change of position** of the particle. Or, in terms of algebra, if we denote velocities by V,

$$V = \left[\frac{\overline{P_1P_2}}{t_2 - t_1}\right]_{t_1 \doteq t_2} \qquad \text{Defining equation for velocity.} \qquad \text{Eq. 5}$$

Note carefully the difference between this equation, which defines velocity, and Eq. 4, which defines speed; the numerator in (5) is a vector, while the numerator in (4) is a scalar. From this follows the

DISTINCTION BETWEEN SPEED AND VELOCITY

24. Speed represents merely the rapidity of motion, while velocity represents rapidity, direction, and sense. Speed is defined by a single number, while velocity demands three specifications; namely, amount, direction, and sense. Imagine two trains traveling along the same piece of double track, the one going north 30 miles an hour, the other going south 30 miles an hour. Both trains have the same speed and the same direction; but their velocities are as different as they can be; for they are directly opposite in sign, *i.e.* in sense.

VELOCITY OF A BODY

25. We leave now the discussion of a particle and pass to the consideration of an extended body, *i.e.* of a body whose dimensions cannot be neglected in comparison with other quantities involved. And since we have found (§ 15) that any change in the position of a body can be defined by not less than two vectors, a translation and a rotation, it follows that the *rate* at which a body changes position will require for its description two velocities.

If the motion of the body is one of translation only, it is the same as that of a car along a straight piece of railroad; the linear velocity is the same for every particle in the car; and this velocity is completely described when the speed of *any* one point on the car and the direction of the track are given.

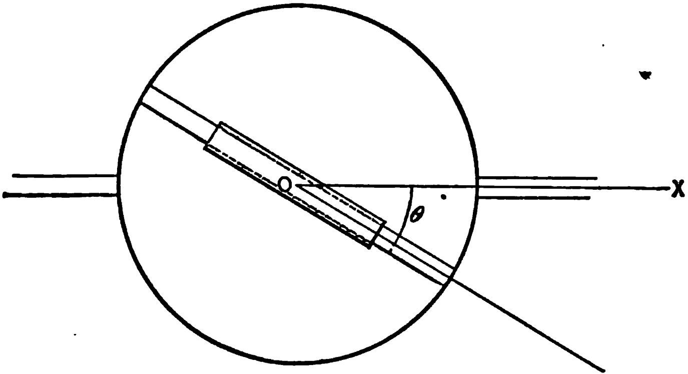

FIG. 14.

But suppose the car is placed on a turntable (Fig. 14), and the table then made to rotate; particles near the end of the car will move rapidly, those near the center will move slowly. In general, the speed of a particle will vary as the distance from the axis of the turntable. How shall we describe a motion which is apparently so complicated? This is very easily done when we recall that practically the only quantity changing is the angular position of the car, and that this change is the same for every part of the car.

Imagine this car to have turned through an angle of θ radians. Consider next any *horizontal* line drawn in the car, the back of a seat, or the edge of a window sill; each has been turned through the same angle θ. Indeed, the new position of the car is definitely fixed, when the motion is one of rotation only, by saying that *the axis of rotation is vertical, the amount of rotation, θ* The rate at which the angle θ varies with time is called the **angular speed** of the car (regardless of the axis about which the rotation takes place).

ANGULAR SPEED

26. And, in general, if θ_1 be the angular position of any line in a body at a time t_1, and θ_2 its angular position at time t_2, then its average angular speed Ω is defined and completely described by the following equation: —

$$\Omega = \frac{\theta_2 - \theta_1}{t_2 - t_1}. \qquad \text{Eq. 6}$$

Defining equation
for *average* angu-
lar speed.

Unit angular speed is evidently that with which a body must rotate in order to describe one radian per second.

The engineer generally employs a unit of angular speed which is only about one tenth as large as this, namely, *one revolution per minute;* for which he uses the symbol "R.P.M."

If, instead of the average angular speed, we wish to measure the actual angular speed at any particular instant, we must then take the average speed over an indefinitely small interval of time, including this particular instant. This operation is indicated in the following notation: —

$$\Omega = \left[\frac{\theta_2 - \theta_1}{t_2 - t_1}\right]_{t_1 \doteq t_2} \qquad \text{Eq. 7}$$

Defining equation
for angular speed
at any instant.

ANGULAR VELOCITY

27. The numerical value of Ω (in Eq. 7) together with the direction and sense of the axis about which the rotation occurs are the three specifications which describe any angular velocity. It is evident, then, that one specification is sufficient to define angular speed, but three specifications are required to define angular velocity.

The graphical representation of angular velocity is quite as simple as that of linear velocity; for we have only to lay off along the axis of rotation as many units of length (centimeters) as there are radians per second in the angular speed.

With this convention, the straight line $\overline{OX}$ (Fig. 15) **completely represents an angular velocity; for its length measures the angular speed and its direction is the direction of the axis of rotation, while the arrow indicates the sense of rotation.**

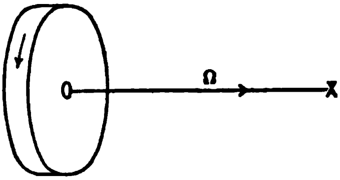

FIG. 15.—Angular velocity represented by a vector.

An angular velocity is said to be positive when, on looking along the positive direction of the axis OX, the body appears to rotate in the same sense that a right-handed screw appears to rotate when one looks along the direction of its advance. When the rotation is in the opposite sense, it is called negative.

Thus the angular velocity which one gives to the steering wheel of an automobile is represented by a vector drawn in the direction of the steering post. If the motion of the wheel is clockwise, the arrow on the vector must point down; if the motion is counterclockwise, the arrow points up.

The angular *velocity* of the earth is continually changing because its axis is continually varying in direction; but the angular *speed* of the earth is so uniform that no variation in it has ever been certainly detected. From Eqs. 4 and 6 it is clear that angular speed is a quantity which is strictly analogous to linear speed; in like manner linear and angular velocities are strictly analogous.

RELATION BETWEEN LINEAR AND ANGULAR SPEED

28. Consider any point P (Fig. 16) in a rigid body rotating about any fixed axis, with constant angular speed, a wheel on its axle, for instance. Let the perpendicular distance of this point from the axis of rotation be r. If the angular speed of the body is Ω, it will rotate once in $\frac{2\pi}{\Omega}$ seconds.

But while the body performs one rotation, the particle at P has moved over a distance of $2\pi r$.

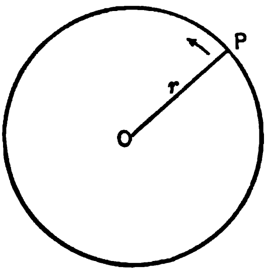

FIG. 16.—Relation between linear and angular speed.

Let S denote the average *linear* speed of the particle; then,

$$\frac{\text{Distance}}{\text{Time}} = \frac{2\pi r}{\left(\frac{2\pi}{\Omega}\right)} = r\Omega = S.$$

In the case of a fixed axis, the angular speed of the body, and the linear speed of any particle on it, are, therefore, related by the following equation:—

$$S = r\Omega \qquad \text{Eq. 8}$$

Problems

36. A boy attaches a small stone to the end of a string which is 80 centimeters long, and then swings it about his head at the rate of 3 turns per second. What is the linear speed of the stone in centimeters per second?

37. The belt of a driving pulley travels at a speed of 15 m. per second. The pulley rotates 10 times per second. Assuming that the belt does not slip on the face of the pulley, compute the diameter of the pulley. *Ans.* $\frac{150}{\pi}$ cm.

38. In the case of a watch, compute the angular speeds of the second hand and the hour hand, respectively. *Ans.* $\frac{\pi}{30}$ and $\frac{\pi}{21600}$.

39. What is the angular speed of a bicycle wheel making 30 revolutions per minute? *Ans.* π radians per second.

40. The front wheel of a bicycle is 28 in. in diameter. Find the angular speed of the wheel about its axle when it is ridden at the rate of 12 mi. per hour *Ans.* 15.09 radians per second.

41. How may the wheel of a car rounding a curve at 5 mi. an hour be used to illustrate the case of a body having a constant angular speed, but at the same instant a variable angular velocity?

42. Show that the fly wheel of an engine, when the steam is cut off, illustrates an angular velocity which is uniform in direction, but variable in speed.

43. What is the aim of the clockmaker, to produce an instrument which will give constant angular speed or constant angular velocity?

CHANGE OF VELOCITY. ADDITION AND SUBTRACTION OF VELOCITIES

29. Up to this point we have considered a particle which is urged only by one single velocity. It often happens, however, that a particle has impressed upon it two or more different velocities at the same time. For instance, if a man be sitting in a moving street car, his motion with respect to the earth is directly along the rails. If, however, he change his seat to one directly opposite, on the other side of the car, his motion with respect to the earth, as he walks across the car, is not along the rails, nor across the rails, but in a diagonal direction, $\overline{AC}$, as indicated in

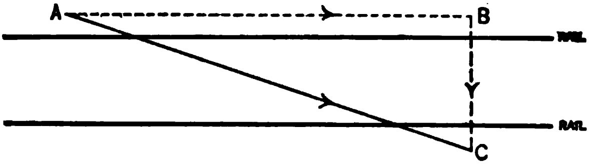

Fig. 17.—Addition of velocities.

Fig. 17. $\overline{AB}$ is a straight line which indicates the velocity of the car; $\overline{BC}$ is a straight line which represents the velocity of the man with respect to the car. These vectors are added as are all other vectors, and $\overline{AC}$ is said to be the *resultant velocity* of the *components* $\overline{AB}$ and $\overline{BC}$. $\overline{AC}$ is, therefore, the straight line which represents the man's motion with respect to the earth.

Since velocity is a vector quantity, it is evident that we may at once apply to it the general rules for vector addition and subtraction. For these see § 14 above. Thus having given a particle whose speed is v and whose direction is defined by θ, its velocity will be the vector $\overline{OV}$, in Fig. 18. Let us suppose the motion to take place in the $\overline{XY}$ plane. Then if we desire the velocity of the particle in a direction parallel to the axis of X, it is only necessary to project $\overline{OV}$ on the axis of $\overline{OX}$. We thus obtain $\overline{OH}$ as the required velocity. In like manner $\overline{HV}$ will be the velocity parallel to the axis of Y; or in terms of algebra

$$\left.\begin{aligned}\overline{OH} &= \overline{OV}\cos\theta,\\ \overline{HV} &= \overline{OV}\sin\theta,\\ \overline{OH}^2 + \overline{HV}^2 &= \overline{OV}^2.\end{aligned}\right\} \qquad \text{Eq. 9}$$

In like manner, if it is desired to find the component of the velocity $\overline{OV}$, resolved in any other direction, say, along the line $\overline{OR}$, which makes an angle ϕ with $\overline{OV}$, we have

$$\overline{OR} = \overline{OV} \cos \phi. \qquad \text{Eq. 10}$$

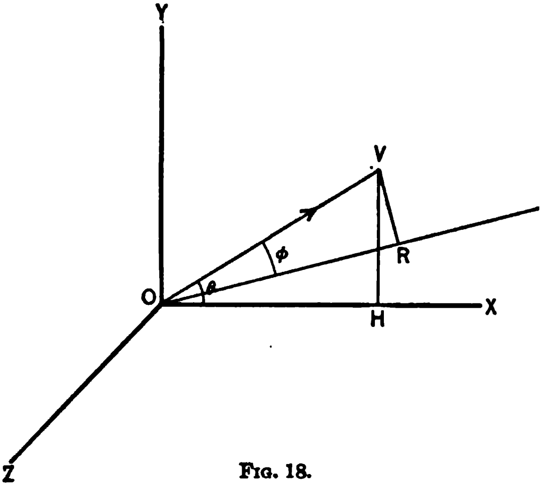

Fig. 18.

This same result may be expressed graphically by saying that the component of a velocity in any given direction is the projection of the vector which represents that velocity upon a straight line drawn in the given direction. Angular velocities are compounded and resolved in precisely the same manner as linear velocities.

All Motion Relative

30. In the case above considered, where a man changes his seat from one side of a car to the other, the question might have been asked: What is his motion *with respect to the car?* The answer would then have been that his velocity with respect to the car is completely represented by the straight line $\overline{BC}$, Fig. 17.

Whether, then, the man's velocity is represented by $\overline{AC}$ or $\overline{BC}$, depends entirely upon whether we take the earth or the car as our reference point.

And so it is with all velocities. Each one is a velocity with reference to some point which for the time being we consider as fixed. If a man be seated in a railway car which is traveling at the rate of 30 miles an hour, we may say that the man is at rest or in motion, according as we take for our reference point some part of the car or some part of the earth. In the case of an aëroplane, all the problems of equilibrium and stability are just the same for a machine which is flying through still air at the rate of 50 miles an hour as for one which is tethered at its center of gravity in the face of a gale which is blowing 50 miles an hour.

Problems

44. Represent by a diagram the velocity of a boat which is being rowed east, at the rate of 4 mi. an hour, while the current is carrying it south at the rate of 1 mi. an hour.

45. A wheelman is riding north at the rate of 8 mi. an hour against a head wind which is retarding him at the rate of 2 mi. an hour. What speed would he make if there were no wind?

46. What speed would this same wheelman make if he turned around and rode with the wind?

47. A canal boat is towed at the rate of 3 mi. an hour. How fast must a man walk its deck in order that he may remain at rest with respect to the towpath?

48. A wheel rider travels east at the rate of 6 mi. per hour while the wind is blowing from the north with a speed of 6 mi. per hour. From what direction will the wind appear to strike the rider?

49. A boat is rowed on the river so that its speed in still water would be 6 mi. an hour. Suppose the river flows at the rate of 4 mi. an hour. Make a diagram showing the direction in which the boat must head in order that its motion with reference to the bottom of the river may be at right angles to the current.

50. Two trains of the same length are running with the same speed on parallel tracks, but in opposite directions. Their combined length is 800 ft., and they pass each other in 6 sec. What is the velocity of the trains relative to the track? From Shearer's *Notes and Questions in Physics*, No. 49.

51. Find the resultant of the following three velocities, 10, 6, and $10\sqrt{2}$, which make the angles $+45°$, $0°$, and $-30°$ respectively with the horizontal. *Ans.* 25.3 in a horizontal direction.

52. Two velocities each of 16 cm. per second include an angle of 120° between them. Find the resultant.

RATE OF CHANGE OF VELOCITY. ACCELERATION

31. Having now considered three fundamental ideas (position, change of position, and velocity), we proceed to the study of a fourth fundamental quantity; namely, rate of change of velocity.

If all the motions we encounter in nature had velocities which were uniform both in direction and in speed, the vocabulary of physics would be even smaller than it now is. Although the behavior of a good clock and the rotation of the earth on its axis are marvelously close approximations to constant angular speeds, it is an interesting fact that no instance of perfectly constant speed has yet been discovered in nature or invented by man. The motion of a cutting tool on a shaper is fairly constant so far as direction is concerned; but its speed is by no means constant.

DEFINITION OF LINEAR ACCELERATION

32. If the velocity of a particle is not uniform, it must be because either the direction of the motion, or the speed of the motion, is changing, or both. For these are the only two continuously variable elements in velocity. When either or both of these changes occur, *i.e.* when the velocity of the particle is changing, the motion is said to be accelerated (*ad* + *celero*).

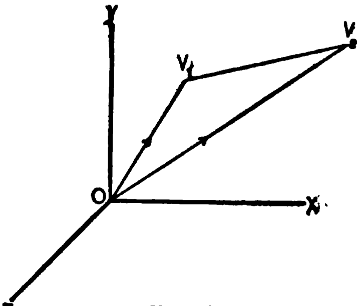

FIG. 19.

If the vector $\overline{OV_1}$ (Fig. 19) represents the velocity of the particle at a time t_1, and the vector $\overline{OV_2}$ represents it at a time t_2, we know from the properties of vectors that the total change in velocity during the interval $t_2 - t_1$ is represented by the vector $\overline{V_1V_2}$. In other words, the straight line joining $\overline{V_1}$ and $\overline{V_2}$ is as many centimeters long as there are units of speed in the *increment* of velocity $\overline{V_1V_2}$. Not only so, but the direction of this straight line is the same as that of the added velocity.

To the rate, $\dfrac{\overline{V_1V_2}}{t_2 - t_1}$, has been given the name **acceleration.** Let us denote this acceleration by a, the change in velocity by $v_2 - v_1$, then

$$a = \frac{\overline{V_1V_2}}{t_2 - t_1} = \frac{v_2 - v_1}{t_2 - t_1}.$$

Defining equation for *average* acceleration. Eq. 11

Acceleration, then, is simply the rate at which the particle is gaining or losing velocity. This rate is not always constant; consequently, the average acceleration will depend in such a case upon how long an interval, $t_2 - t_1$, is covered by the average. In order to get the actual acceleration at any particular point of time t, we must take the average over an indefinitely small interval of time, including t.

$$a = \left[\frac{\overline{V_1 V_2}}{t_2 - t_1}\right]_{t_2 \doteq t_1} = \left[\frac{v_2 - v_1}{t_2 - t_1}\right]_{t_2 \doteq t_1} \qquad \text{Defining equation for acceleration } \textit{at any instant t.} \qquad \text{Eq. 12}$$

Acceleration is accordingly the limit approached by the ratio between an increment of velocity and the corresponding increment in time. If, therefore, a particle is moving with uniform velocity, its acceleration is zero. Whether the particle is moving fast or slowly makes no difference. So long as its velocity is not changing, its acceleration is zero.

UNIT OF ACCELERATION

33. From this "defining equation," our unit of acceleration is obtained exactly as we obtained our unit of velocity, viz., we make the numerator and denominator of the right-hand member each unity; then a must be unity. We thus arrive at the following definition: **Unit acceleration is an acceleration in which unit change of velocity is produced in unit of time.**

Since, in pure science, the centimeter per second is the unit of velocity, the natural unit of acceleration is the *centimeter per second per second,* often written cm./sec.^{2}. Thus a coin dropped from one's hand gains during each second a velocity of approximately 981 centimeters per second toward the center of the earth. Accordingly the acceleration of a body falling freely under gravity is said to be 981 cm./sec.^{2}. In engineers' units this quantity would read approximately 32.2 ft./sec.^{2}.

ANGULAR ACCELERATION

34. Angular velocity, like linear, depends for its value upon two variables, namely, the direction of the axis of rotation, and rate of rotation. If, in any spinning body, either of these two variables changes, the body is said to have an angular acceleration. Thus the shaft of a turbine steamer may be rotated with a constant angular speed, but if the steamer is changing her course, the angular velocity of the shaft is changing, and the shaft is said to have an angular acceleration. If, at another time, the boat be moving in a perfectly straight line and the speed of the turbine be varying, the motion of the shaft is also said to have an angular acceleration.

In general, the velocity of a spinning body varies from instant to instant; as, for example, the fly wheel of a stationary engine; so that if we wish to obtain the angular velocity at a particular instant t, we much choose t_2 and t_1 in such a way that t_2 shall be a time a very little later than t, and t_1 a time a very little earlier than t. Then, if we denote by Ω_1 the angular velocity at the instant t_1, and by Ω_2 the angular velocity at the instant t_2, the angular acceleration at any instant t will be the limit approached by A as $t_2 - t_1$ approaches zero.

$$A = \left[\frac{\Omega_2 - \Omega_1}{t_2 - t_1}\right]_{t_2 \doteq t_1} \quad \text{Defining equation for angular acceleration.} \qquad \text{Eq. 13}$$

Since the numerator in the right-hand member is a vector quantity, while the denominator is a scalar, it follows that the angular acceleration A is also a vector quantity, and is, therefore, compounded and resolved as are other vector quantities.

UNIT OF ANGULAR ACCELERATION

35. When the rate at which a body is rotating changes by one radian per second during each second, the value of its angular acceleration is unity.

TWO IMPORTANT SPECIAL CASES

36. Having now considered in some detail the three fundamental conceptions of kinematics; namely, position, velocity, and acceleration, we proceed to the study of two highly interesting special cases of linear acceleration.

CASE I. *Motion of a Particle along a Straight Line: Direction of Velocity Constant; Speed Variable*

37. If, in Fig. 20, the velocity of a particle be changed from $\overline{OV_1}$ to $\overline{OV_2}$ (the points, O, $\overline{V_1}$, $\overline{V_2}$ all lying on the same straight line), then the acceleration results from a change of speed only. For the velocity of the particle is all the while along the straight line $\overline{OV_2}$. Here the acceleration is in the same direction as the motion of the particle.

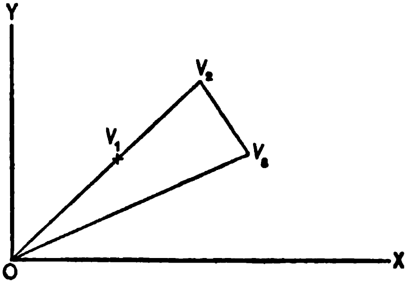

FIG. 20.

Let us imagine that the speed of a particle changes from OV_1 to OV_2 in any interval of time t, and that, throughout the motion, *the acceleration is constant.* Let us call the acceleration a; then the particle will gain a units of speed during each second. The *total gain of speed* during any time t is therefore given by the following expression: —

$$s = at. \qquad \text{Eq. 14}$$

If now, we happen to know the speed of the particle at the beginning of the interval and call it S_0, then the actual speed at the end of the interval will be the original speed plus the speed gained, viz.: —

$$S = S_0 + s,$$

or

$$S = S_0 + at. \qquad \text{Eq. 15}$$

This equation is very useful; for it not only describes the variable speed S at each instant of time, but it also enables us to compute the value of any one of the four quantities involved as soon as we know the other three.

In case a particle loses speed, we have a negative value for s, and hence a negative value for a in the equation, $s = at$. For in these problems, t is always positive.

Not infrequently we shall want to know something more about the motion of the particle than its mere speed. We often want to know what the position of the particle is after a certain interval during which its motion has been accelerated.

If the acceleration a is a constant, then the gain in speed as time goes on will be proportional to the time,

$$s' = at.$$

And the *mean* speed of the particle from the beginning to the end of the interval is the value of the speed at the middle of this interval, viz., the speed represented by the line *LM*, in Fig. 21, but this speed is, by definition, the mean of S_0 and $S_0 + at$, which is $S_0 + \frac{at}{2}$. Call this average speed S. We may now take this average value of the speed and substitute it for S in Eq. 3, § 21, thus obtaining

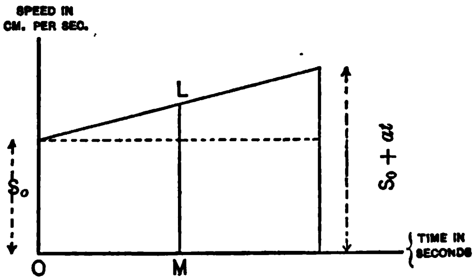

Fig. 21.

$$p = St = \left(S_0 + \frac{at}{2}\right)t,$$

or $$p = S_0t + \tfrac{1}{2}at^2, \qquad \text{Eq. 16}$$

where p is the length of path (*i.e.* simply the distance) covered by the moving particle.

For the special case which we are considering, namely, motion in a straight line, this equation, together with Eq. 15, $S = S_0 + at$, tells the whole story. One gives us the speed of the particle when we know the acceleration and the duration of its action; the other gives us the change in the position of the particle.

If between these two equations we eliminate t, we get another useful expression giving us the speed of a particle in terms of its acceleration and the distance traversed, namely: —

$$S^2 = S_0^2 + 2\,ap. \qquad \text{Eq. 17}$$

To solve any problem, then, under this special case which we have just been considering (motion in a straight line: acceleration constant), one has merely to observe which of the above five quantities (S, S_0, a, t, and p) are given, which required, and lastly which of these three equations contains the desired and the known quantities. He has then one equation to solve for the unknown quantity.

Problems

53. A bicycle and rider start from rest at the top of a hill. At the end of 16 sec. the foot of the hill is reached, with a speed of 640 cm. per second. Find the average acceleration of the bicycle during the descent of the hill. Here S, S_0, and t are given *Ans.* $a = 40$.

54. Having computed the acceleration, use this and the speed at the foot of the hill to compute the length of the hill. *Ans.* $x = 51.2$ m.

55. A train running at 36 km. per hour is stopped by a sudden application of the brakes. What acceleration must be produced by the brakes in order to stop the train in 8 sec.? Here, again, S, S_0, and t are given. *Ans.* $a = +125$.

56. When steam is turned on again, a constant acceleration of 50 units is produced. How long will it be before the train has again acquired its original speed of 36 km. per hour? *Ans.* 20 sec.

57. Over what distance will this train have traveled while coming to rest in problem 55? *Ans.* 40 m.

58. How far will the train travel while again acquiring its original speed in problem 56? *Ans.* 100 m.

59. A football player running north at the rate of 8 m. per second reverses his velocity and in an interval of 2 sec. is again running, this time south, with a speed of 8 m. per second. Find his mean acceleration during the interval. *Ans.* $a = -800$.

60. A particle is simultaneously urged to move with speeds of 40, 15, and 20 ft./sec. respectively. Can it remain at rest?

61. A train which is uniformly accelerated starts from rest, and at the end of 3 sec. has acquired a speed with which it would travel through 1 km. in the next 5 min. Find the acceleration.

62. Two bodies, whose respective velocities are accelerated in every second by 30 and 50 cm. per second, begin to move toward each other at the same instant. At first they are 2 km. apart, and have no initial velocity. After how many seconds will they meet?

63. A body tends to move with equal speeds in two directions inclined to each other at 120°. Find its path and resultant velocity.

64. One man A is standing at a point 1500 ft. east of another man B. At the same instant both men start to walk east, B at the rate of 4 mi. an hour, and A at the rate of 3 mi. an hour. How long will be required for B to overtake A?

65. A particle has an initial velocity of 20 cm. a second, and receives an acceleration of 8 cm. per second every second in the same direction. What distance will it travel in 15 sec.?

66. A wind is blowing from a point intermediate between north and east. The northerly component of its velocity is 10 mi. an hour, and the easterly component is 36 mi. an hour. Find the whole velocity.

67. Write in the margin of your book the special form which Eqs. 15, 16, and 17 assume when the particle starts from rest.

CASE II. *Uniform Motion in a Circle: Direction of Velocity Variable; Speed Constant*

38. Let us now consider an entirely different case *where the velocity of a particle varies from instant to instant only in direction.* Such a motion is almost perfectly illustrated by the motion of a heavy particle at the end of a light thread as indicated in Fig. 22 — a device known as a conical pendulum. The motion of the earth about the sun is also a close approximation to uniform circular motion.

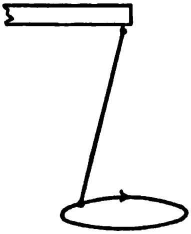

FIG. 22. — Conical pendulum.

Let the circle in Fig. 23 represent the path of the particle whose motion we are studying. The velocity of the particle at any instant evidently has a direction which is tangent to this circle.

If P_t be the position of the particle at any instant, the direction of the velocity at this instant will be represented by a line perpendicular to the radius OP_t.

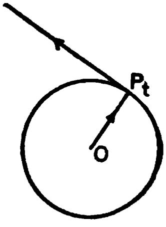

FIG. 23 — Uniform circular motion.

Passing now to Fig. 24, let P_1 and P_2 be two positions of this particle separated by a small interval of time, t. Then OV_1 and OV_2 drawn at right angles to OP_1 and OP_2 will, as before, represent the velocities of the particle in the positions P_1 and P_2 respectively. The length of the vector OV_1, as also that of OV_2, must of course be numerically the same as that of the constant speed, S, of the particle.

Denote the angular speed of the radius OP_1 by Ω, and its length by R; then the value of the linear speed of the particle, that is, of the point P_1, is ΩR; while the linear speed of the point V_1 is ΩS.

We can now determine at once the acceleration of the particle; for the total *change* in the velocity of the particle during the time, t, is completely represented by the vector $\overline{V_1V_2}$, this being the velocity which must be added to $\overline{OV_1}$ to give $\overline{OV_2}$. And hence, by definition.

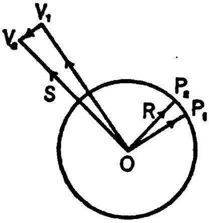

FIG. 24. — Radial acceleration.

$$\text{Acceleration of particle at } P_t = \left[\frac{\overline{V_1V_2}}{t}\right]_{t\doteq 0}$$

But $\left[\frac{\overline{V_1V_2}}{t}\right]_{t\doteq 0}$ measures the speed of the point V_1; it is therefore as we have just found equal to ΩS and has a direction at right angles to OV_1. Hence

$$\text{Acceleration of particle} = \Omega S$$

and is always directed from the particle towards the center.

Remembering that $S = \Omega R$, we may also write

$$\left.\begin{array}{l}\text{Acceleration of particle}\\ \text{moving at constant}\\ \text{speed in circle}\end{array}\right\} = \Omega S = \Omega^2 R = \frac{S^2}{R}. \qquad \text{Eq. 18}$$

Or if we denote by T the time required for one revolution of the particle in its circular path — an interval which is usually called the **period** of the motion — then $T = 2\pi/\Omega$, and Eq. 18 may be written as follows: —

$$\text{Acceleration towards center} = \frac{2\pi S}{T} = \frac{2\pi R\Omega}{T}.$$

Thus in the case of the earth which moves about the sun with a motion which is nearly circular and nearly uniform we have T = one year and $R = 15 \times 10^{12}$ cm. Hence the earth is accelerated toward the sun at the rate of 0.6 cm./sec.². In like manner the moon, which is 4×10^{10} cm. distant from the earth, is accelerated toward us at the rate of 0.27 cm./sec.^{2}

We have now a complete description of the motion of a particle which moves with uniform speed in a circle. The student should be able to put into words the meaning of each of these five different expressions. For example, the third form, $\frac{S^2}{R}$, says that "the acceleration of a particle moving with uniform speed in a circle is constant, and is numerically equal to the square of the linear speed of the particle divided by the radius of the path."

General Case of Linear Acceleration

39. We have now considered, in some detail, two special cases of acceleration, in one of which the speed alone varies, in the other of which the direction alone varies.

The importance which attaches to these two cases arises partly from the fact that the acceleration of any particle whatever, moving at any given instant in any path, however tortuous, may always be resolved into two components, one along the path, and one normal to the path; and these two components belong respectively to the two special cases which we have just studied. We have thus the general result: —

$$\left.\begin{array}{c}\text{Total acceleration of}\\ \text{any particle}\end{array}\right\} = \left\{\begin{array}{l}\text{Vector sum of the acceleration along}\\ \text{the path and the acceleration normal}\\ \text{to the path.}\end{array}\right.$$

The advanced student will discover in Vector Analysis a much more elegant method of treating this subject.

Addition and Subtraction of Acceleration

Since acceleration is a vector quantity, it follows at once that accelerations are added and subtracted in the same manner as velocities and other vectors. See § 14.

40. When one considers any spinning body, such as a grindstone or the propeller shaft in a steamer, it is at once evident that the angular velocity with which any of these rigid bodies is rotating may change in either of two ways. (i) The rate at which the body is spinning about its own axis may vary, as, for instance, the angular speed of the propeller when the engine is just starting or stopping. And (ii) the direction of the axis of rotation may change; for example, the direction of the propeller shaft when the steamer is changing her course without changing the speed of her engines.

We proceed to consider these two cases more in detail.

CASE I. *Direction of Axis of Spin Constant; Rate of Spin Variable*

41. Here we have a case in which the acceleration is constant so far as direction is concerned. The ideas are therefore precisely the same as those involved in Eqs. 15, 16, and 17, except that here we are dealing with angular speed; there, with linear speed.

Let the angular position of the rotating body be denoted by θ, its angular speed by Ω, its angular acceleration by A, and time by t, then one has

$$\Omega = \Omega_0 + At. \qquad \text{Eq. } 15'$$

$$\theta = \Omega_0 t + \tfrac{1}{2} At^2. \qquad \text{Eq. } 16'$$

$$\Omega^2 = \Omega_0^2 + 2A\theta. \qquad \text{Eq. } 17'$$

CASE II. *Rate of Spin Constant; Direction of Axis of Spin Variable*

42. We now come to the case in which the propeller shaft is changing direction; the boat in changing her course turns about a vertical axis, *i.e.* about an axis at right angles to the axis of spin. The analogue with uniform circular motion (§ 38, Eq. 18) is complete. There it was found that

$$a = \left\{\begin{array}{l}\text{Acceleration of}\\ \text{particle in trans-}\\ \text{lation}\end{array}\right\} = \left\{\begin{array}{l}\text{Linear speed of}\\ \text{particle multiplied}\\ \text{by the rate at}\\ \text{which its direc-}\\ \text{tion is changing}\end{array}\right\} = S\Omega. \qquad \text{Eq. } 18$$

Here it is found that

$$A = \left\{\begin{array}{l}\text{Acceleration of}\\ \text{rigid body in}\\ \text{rotation}\end{array}\right\} = \left\{\begin{array}{l}\text{Angular speed of}\\ \text{body multiplied by}\\ \text{rate at which its}\\ \text{axis of spin is}\\ \text{changing direction}\end{array}\right\} = \Omega\omega. \qquad \text{Eq. } 18'$$

Observe that just as in uniform circular motion the three quantities a, S, and Ω are vectors each one at right angles to the other two, so here, in Eq. 18′, A, Ω, and ω are three mutually rectangular vectors.

The motion here considered is easily illustrated by the ordinary balanced gyrostat.

The student will find it an interesting exercise to prove that angular and linear accelerations at any point on a rotating rigid body are connected by the following equation:—

$$a = rA, \qquad \text{Eq. } 8'$$

an equation which is strictly analogous to Eq. 8, p. 30.

This subject is one whose detailed discussion at this point would lead us too far afield; but it is hoped that every student will at least get a fair grasp of the exact analogy between translation and rotation in Kinematics, and thus reduce the complexity of the subject by at least one half.

ALGEBRAIC SUMMARY OF KINEMATICS

Linear	**Angular**
Fundamental Quantities	
Length and time; p and t.	Angle and time; θ and t.
Position	
Defined by a vector whose length measures a distance.	Defined by a vector whose length measures an angle.
Velocity	
$V = \left[\dfrac{\overline{P_1P_2}}{t_2 - t_1}\right]_{t_1 \doteq t_2}$	$\Omega = \left[\dfrac{\theta_2 - \theta_1}{t_2 - t_1}\right]_{t_2 \doteq t_1}$
$V = r\Omega.$	
Acceleration	
$a = \left[\dfrac{v_2 - v_1}{t_2 - t_1}\right]_{t_2 \doteq t_1}$	$A = \left[\dfrac{\Omega_2 - \Omega_1}{t_2 - t_1}\right]_{t_1 \doteq t_2}$
$a = rA.$	

Special Case I. Direction Constant: Speed Variable

$S = S_0 + at,$	$\Omega = \Omega_0 + At,$
$p = S_0t + \frac{1}{2}at^2,$	$\theta = \Omega_0t + \frac{1}{2}At^2,$
$S^2 = S_0^2 + 2ap.$	$\Omega^2 = \Omega_0^2 + 2A\theta.$

Special Case II. Direction Variable: Speed Constant

$a = S\Omega.$	$A = \Omega\omega.$

Problems

68. Twenty seconds after the current is turned on an electric motor, the armature, starting from rest, has acquired an angular speed of 600 radians per second. Find the average angular acceleration.

69. The fly wheel of an engine is subject to a negative angular acceleration of 4 radians per second per second. If the wheel is making 220 R.P.M. when steam is shut off, how long will it take the fly wheel to come to rest?

70. A particle is accelerated vertically upward with an acceleration of 3, and horizontally eastward with an acceleration of 4. Find the total acceleration of the particle.

71. A particle on the surface of a fly wheel of 120 cm. radius, rotating with an angular speed of 4 radians per second, is accelerated toward the center at what rate?

72. How fast must a bicycle wheel be made to revolve in order that a small particle of mud attached to the tire may experience an acceleration of 12 cm./sec.^{2}? It is assumed that the diameter of the wheel is 70 cm.

73. What data would you need in order to compute the acceleration of the moon toward the earth as it revolves about the earth in an orbit which is practically a circle?

74. A grindstone is set in motion with an angular speed of 3 radians per second. When left to itself, it rotates through an angle of 60 radians before stopping. Find the angular acceleration.

75. The shaft of a hoisting windlass is set into motion (starting from rest), under an angular acceleration of 200 radians per second per second. What angle will it have turned through by the end of 3 seconds?

76. Show that, if we take any equal intervals of time and count from the beginning of a motion uniformly accelerated in one direction, the distances traversed during these successive instants will bear to one another the same ratio as the series of odd numbers, 1, 3, 5, 7, etc. [Galileo.]

77. Prove that, when a particle which is uniformly accelerated in one direction traverses any two distances, measured from a common initial point, in any two time intervals whatever, these time intervals bear to one another the same ratio as one of the distances bears to the mean proportion of the distances. [Galileo.]

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> "Philosophy has since Bacon's time ceased to inquire into final causes."
> VISCOUNT BRYCE, *South America.*

CHAPTER II

SIMPLE HARMONIC MOTION

CASE I. LINEAR MOTION

43. Another motion which one meets with extraordinary frequency is that of a particle hanging at the end of a spiral spring and vibrating in a vertical line. Such a particle is represented by *P* in Fig. 25.

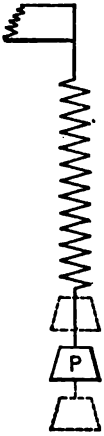


FIG. 25.

There is perhaps no one motion in nature more widespread, and no department of physics in which it does not make its appearance. The motion of any point on a guitar string which has just been plucked and let go furnishes another illustration of this motion. If a body be suspended by a rubber band and then slightly displaced up or down and suddenly released, its motion also will be that which we are about to study.

In each of these cases observe that, when the body vibrates freely: —

(*a*) The motion is one of translation; because the particles of the body describe equal and similar paths.

(*b*) The path is a limited straight line.

(*c*) The particle moves most rapidly at the middle of its path, and comes to rest at each end of its path.

(*d*) The motion gradually dies down, unless kept up by some " outside " means.

Such would be a popular and qualitative description of the motion.

CASE II. ANGULAR MOTION

44. In the case of solid bodies having a fixed axis of rotation, we frequently meet a similar motion, and one with which, again, all students are familiar in a general way.

The pendulum of a clock is approximately a rigid body whose angular position is constantly changing. It typifies the motion we are about to describe. The motion of the balance wheel in a watch movement also illustrates the motion under consideration. If a slender lath be clamped at one end, while the free end is pulled aside and suddenly let go, this free end will vibrate with practically the same motion as that of a pendulum left to itself.

Observe that, in each case, when the body vibrates freely: —

(*a*) Its motion is an angular one.

(*b*) The amount of swing (angular displacement) never exceeds, in any one case, a certain limited angle.

(*c*) The angular speed is greatest at the middle, and vanishes entirely at the end of the swing.

(*d*) The vibration gradually dies down, unless kept up by some means external to the system we are studying, as, for instance, by the weight of the clock or the coiled spring of the watch.

These two motions, the linear and the angular, are each close approximations to what is called **Simple Harmonic Motion**; and are often indicated by the letters S.H.M.

The student should note here, once for all, that a mere name of a physical quantity or phenomenon, such as the above, while exceedingly convenient, does not define it. We now proceed to the exact

DEFINITION OF LINEAR SIMPLE HARMONIC MOTION

45. Imagine a point P (Fig. 26) to move with uniform speed in the circumference of a circle. Imagine any diameter drawn in the circle. And from the point P, in each of its successive positions, imagine a perpendicular let fall upon this diameter. The motion of the foot of this perpendicular, H, to and fro along the diameter, is a **simple harmonic motion.** It is evident that the speed of the point H near either end of the diameter will be much less than at the center O. In this respect the motion of H is like that of a pendulum bob.

The same definition may be put more elegantly, as follows: **Simple harmonic motion is the projection of uniform circular motion upon a diameter of the circle.** We shall presently see that this motion can be accurately described (defined) in a manner even simpler than this.

The student frequently confounds simple harmonic with circular motion, and should, therefore, at this point, carefully distinguish between these two motions. S.H.M. is a to-and-fro motion in a straight line; circular motion is curved. S.H.M. is oscillatory; uniform circular motion never turns back but carries its particle along at uniform pace in its curved path. The *actual* motion of one of Jupiter's satellites is nearly circular while its *apparent* motion is practically simple harmonic. S.H.M. is merely uniform circular motion seen edgewise.

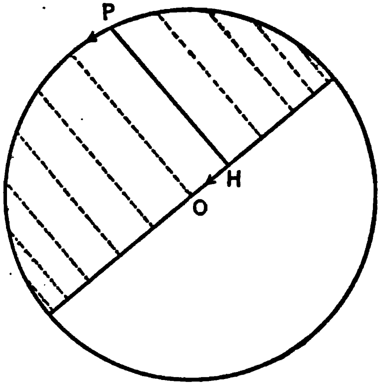


FIG. 26. — Simple harmonic motion.

DEFINITION; CIRCLE OF REFERENCE

46. The circle in which we have imagined the point P (Fig. 26) to move with uniform speed is called the circle of reference. We shall also employ the center of this circle O as our **point of reference.**

DEFINITION; AMPLITUDE OF S.H.M.

47. The radius of the circle of reference is the maximum distance to which H can recede from O in either direction. This maximum distance, which will be denoted by A, is called the **amplitude** of the S.H.M. **Accordingly the amplitude of S.H.M. is defined as the radius of the circle of reference.**

DEFINITION; PERIOD OF S.H.M.

48. Let us denote by S the uniform speed of the point P in the circle of reference. In one complete trip around the circle this point will travel a distance $2\pi A$. The time required for this round trip will, therefore, be $\frac{2\pi A}{S}$ seconds. This interval of time, generally denoted by T, is a constant, and is called the period of the S.H.M. **Accordingly the period is defined as the time occupied in one round trip of the actual moving point *H* to and fro across the diameter of the circle.** The reciprocal of the period T is called the **frequency,** and is generally denoted by n.

Definition; Phase of S.H.M.

49. Let H (Fig. 27) be the particle which is moving along the diameter BC with simple harmonic motion; let P be the corresponding point moving in the circle of reference with uniform speed. The angle, θ, which the radius vector OP makes with the line of reference OX is called the **phase** of the S.H.M. at the time t.

The phase of a S.H.M. is therefore defined as the angle between the line of reference for angles and the radius vector drawn to the corresponding point in the circle of reference. The phase angle, θ, is usually expressed in radians.

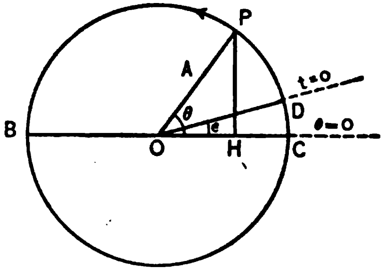


Fig. 27.

Definition; Epoch of S.H.M.

50. Let us now suppose that the moving point P occupied the position D at the instant when $t = 0$, and that it travels about the circle of reference in a counter-clockwise sense. Then the radius OD will be the line of reference for the measurement of time. It is evident that the phase at the time $t = 0$ is the angle DOC, which we shall denote by ϵ. **This angle ϵ, which is the value of the phase at the time $t = 0$, is called the epoch of the S.H.M.**

51. If we denote by ω the angular speed of the radius, OP, it follows, Eq. 6, that during any interval, t, this radius will describe an angle which is measured by ωt, and hence the phase at the end of this interval, t, will be $\omega t + \epsilon$. Or one may express this algebraically by writing

$$\theta = \omega t + \epsilon.$$

Thus when $\theta = \frac{\pi}{2}$, we understand that the particle, H, has completed one quarter of a vibration since it was in the position C at the end of its swing.

DEFINITION; DISPLACEMENT OF S.H.M.

52. *The point which is actually moving with S.H.M is represented by H in Fig.* 27. **The distance of this moving particle, at any instant, from the center of the circle of reference is called its Displacement.** We shall denote this displacement, OH, by x. Then in the right-angled triangle OPH, Fig. 27, it is evident from the definition of a cosine that

$$\cos\theta = \frac{x}{A},$$

or

$$x = A\cos\theta.$$

But by the definition of the phase angle, θ, we have

$$\theta = \omega t + \epsilon,$$

and hence

$$x = A\cos(\omega t + \epsilon). \qquad \text{Eq. 21}$$

Or if angles and times be measured each from the same line of reference, namely, OX, then $\epsilon = 0$ and we shall have

$$x = A\cos\omega t. \qquad \text{Eq. 22}$$

Equation 21 tells the whole story concerning the displacement. Every possible value which x can assume is easily computed as soon as we know the corresponding value of the variable t and of the constants A, ω, and ϵ.

To completely describe a S.H.M. along any given straight line we must, then, tell the following four things about it: —

(1) The **amplitude**, A, *i.e.* the radius of the circle of reference.

(2) The **period**, T, or what is the same thing, ω; for

$$\omega = \frac{2\pi}{T}.$$

(3) The **phase** at any time t, namely, $\omega t + \epsilon$.

(4) The **sense of motion** of the point P in the circle of reference, *i.e.* the sign of ω must be given.

These having been given, nothing can be added that will make the description more definite.

LINEAR ACCELERATION OF S.H.M.

53. In our previous study of circular motion, we found that whenever a particle moves in a circle with uniform speed, it is always accelerated toward the center. The amount of this acceleration we found (§ 38) to be $\omega^2 A$, where A is the radius of the circle and ω is its angular speed.

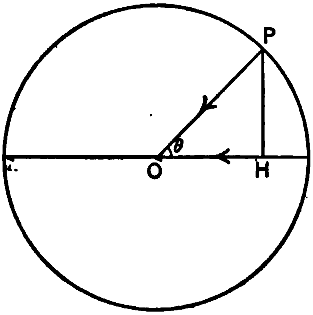


FIG. 28.—Acceleration of S.H.M.

In other words, the acceleration in uniform circular motion is represented by a vector of length $\omega^2 \cdot PO$ drawn to O in the direction PO.

Hence the acceleration in S.H.M., which is the projection of uniform circular motion, is represented by the projection of this vector, that is, by a vector of length $\omega^2 \cdot HO$ drawn to O in the direction HO. See Eq. 2, § 14.

Let us denote by a this acceleration of S.H.M., then

$$a = \omega^2 PO \cos\theta = \omega^2 \cdot HO = -\,\omega^2 x. \qquad \text{Eq. 23}$$

This equation shows us, when once we know the *displacement* of any S.H.M., how to obtain the corresponding *acceleration.* This is the fundamental equation of S.H.M., and may perhaps be written in the following form: —

$$\frac{\text{Acceleration}}{\text{Displacement}} = \text{A negative constant.}$$

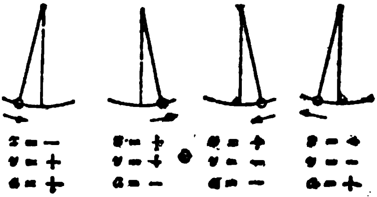
In other words, the acceleration is proportional to the displacement and opposite in sense.

The relation between displacement, velocity, and acceleration will be clear from Fig. 29.

This equation (23) also gives us an exceedingly useful expression for the period of a S.H.M. For, since

$$T = \frac{2\pi}{\omega} \text{ and } \omega^2 = -\frac{a}{x}, \text{ we have } T = 2\pi\sqrt{\frac{x}{-a}}. \qquad \text{Eq. 24}$$

On account of damping such as that due to frictional resistance, the amplitude of all *actual* vibratory motions, except those in which energy is supplied from outside the vibrating system, continually diminishes. Hence we do not meet any motions in nature which are accurately simple harmonic; but in thousands of cases the approximation is so close that in treating these cases we may, with impunity, use the laws of S.H.M.



a and *x* always have opposite signs.

FIG. 29.

ANGULAR SIMPLE HARMONIC MOTION

As we have already seen (§ 44), bodies rotating about a fixed axis may have an angular motion which is analogous to the linear motion which we have just been studying. A most excellent illustration is to be found in the motion of the balance wheel of a watch.

The angular displacement θ of such a body is described in terms of three constants and the time exactly as in the case of linear S.H.M. Thus

$$\theta = \Theta \cos(\omega t + \epsilon), \qquad \text{Eq. 21}'$$

where the constants are

Θ = maximum angular displacement, or amplitude,

$\omega = \frac{2\pi}{T}$, where T is the period of oscillation,

ϵ = phase at time $t = 0$,

while the variables are θ and t.

Just as in the case of linear S.H.M. it was shown (Eq. 24) that the period depends only upon the ratio of displacement to acceleration, so here it may be shown in the same way that

$$T = 2\pi\sqrt{\frac{\text{Angular displacement}}{-\text{(Angular acceleration)}}} = 2\pi\sqrt{\frac{\theta}{-A}}. \qquad \text{Eq. 24}'$$

In other words, the criterion of angular S.H.M. is that the angular acceleration shall be proportional to the angular displacement, but opposite in sense.

THE HARMONIC CURVE, OR THE COMPOSITION OF UNIFORM RECTILINEAR MOTION WITH A S.H.M. IN A DIRECTION AT RIGHT ANGLES

54. If one walks along the smooth sand of the lake shore or the ocean beach, dragging his walking stick behind him and at the same time vibrating it to and fro sidewise, he will trace upon the sand a wave-shaped line which is of considerable importance in Physics and which is known as the Harmonic Curve.

It is the resultant of two motions which are at right angles to each other and which are described by the following equations: —

(i) $y = A \cos \omega t,$ Simple Harmonic Motion.

and

(ii) $x = Vt.$ Uniform Rectilinear Motion.

Eliminating t between these two equations of motion, we have the equation of the path, namely: —

$$y = A \cos \left(\frac{\omega}{V}\right)x. \quad \text{Eq. of Harmonic Curve.} \qquad 25$$

Such a motion is evidently periodic, the same value of y recurring whenever $\frac{\omega x}{V}$ changes by 2π, as illustrated in Fig. 30.

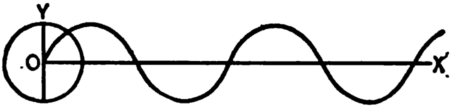


FIG. 30. — Path resulting from S.H.M. and uniform rectilinear motion.

This curve is also known in mathematics as the cosine- or sine-curve.

Problems

78. Show, either graphically or algebraically, that when the particle H (Fig. 28) is passing the center and has its maximum speed, the acceleration is zero.

79. Show that when the particle H (Fig. 28) reaches the end of its swing and for an instant stands still, the acceleration is a maximum.

80. The displacement in a certain S.H.M. at a given instant is -32. At the same instant the acceleration is $+\frac{1}{2}$. Find the period of the S.H.M. *Ans.* 16π.

81. The period of a S.H.M. is 6.28 seconds, and the acceleration at a given instant is -4. What is the displacement at the same instant?

82. In any S.H.M. when is the displacement a maximum? When is it a minimum?

83. At what part of its path does a particle in S.H.M. move most rapidly?

84. A man walks at a uniform rate in a circular track *ABCD*, Fig. 31. Another man starts from *A* at the same time and walks along the diameter *AC* so that the line joining them is perpendicular to *AC*. What kind of motion will the second man have? Where will he walk the fastest? The first goes clear around in 20 minutes. What is his angular speed? What is the periodic time of the second man?

Shearer's *Questions*, 407.

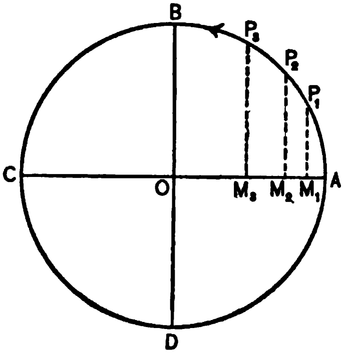


FIG. 31.

85. Imagine a particle to move in a straight line with constant acceleration. Draw a curve which shall represent such a motion, using abscissæ to represent *time* and ordinates to represent *acceleration*.

86. If a particle starts from rest and moves in a straight line under constant acceleration, what curve will represent the relation between the *speed* of the particle and the *time*? Draw such a curve on the blackboard.

87. A particle starts from rest and moves in a straight line under constant acceleration; draw a curve in which the abscissæ represent *times* and the ordinates the corresponding *distances* traveled by the particle. What name have mathematicians given to such a curve?

88. Suppose T to be the period of a S.H.M. such as that shown in Fig. 31. Take $\frac{T}{24}$ as the unit of time and imagine the particle M to start from A at the instant when $t = 0$. Plot a curve in which the abscissæ represent *times*, ranging from 0 to T and the ordinates represent the corresponding *displacements* of the particle M.

89. Plot a curve similar to the preceding except that while abscissæ represent *times*, the ordinates represent the corresponding *accelerations* of the particle M.

90. Put Fig. 29 on the blackboard and interpret the signs of the three quantities there mentioned.

> **"A theory of matter is a policy rather than a creed."**
> **SIR J. J. THOMSON,**
> ***Corpuscular Theory of Matter.***

CHAPTER III

SOME GENERAL PROPERTIES OF MATTER

55. Up to this point we have been considering some of the motions of particles or bodies, and have confined our attention entirely to the motion, not at all to the moving object. We next proceed to study some of those properties which all pieces of matter have in common; properties possessed alike by bodies at rest and by bodies in motion, and by bodies of every possible chemical composition. Properties of this kind are called **general properties.**

We shall consider four of these, viz., **Inertia, Gravitation, Capacity for Energy,** and **Elasticity.** Afterwards we shall investigate some of the **special properties** of inanimate matter, such as hardness, magnetic quality, transparency, color, and we shall then see how these special properties are employed to classify the different kinds of bodies.

Matter is something with which we are familiar *in a general way* from our earliest years; on the other hand, investigators in physical science have spent centuries in studying the various peculiarities of matter, and have not yet succeeded in defining it in terms of anything simpler. Much has recently been discovered to commend the view that matter will ultimately be found to consist of electricity in motion. This is, at present, the most helpful theory of matter ever proposed.

I. INERTIA

56. One of the commonest experiences of life is that of " sizing up " a body by pushing, pulling, or " hefting " it.

When one sees a barrel being rolled along the street, he can always tell whether it is empty or is filled with something that has considerable weight. The empty barrel rebounds to a greater extent, as it passes over a small stone, and is more easily turned aside from a straight course.

Even a wagon drawn by a team behaves so differently when empty and when loaded that an observer does not need to look into the box of the wagon to see whether it contains a load or not.

Imagine three balls, each six inches in diameter, each painted black, one a rubber football, one a wooden ball from a bowling alley, one an iron cannon ball. Imagine them all started rolling with the same speed over a moderately smooth sidewalk. Any boy or girl can tell instantly, by observing the behavior of the three balls, which one is filled with air, which one is made of wood, and which one of iron.

Everybody knows that the empty barrel is more easily set in motion than the full one, and that it is more easily stopped than the full one. Any one can tell, by kicking the barrel, whether it is full or empty. In like manner, any one can go up to the three black balls, each of the same size and color, each at rest on the ground, and can distinguish, by a push with the foot, the iron ball from the wooden ball, the wooden ball from the football. But just how is one able to arrive so quickly at the correct conclusion? The answer is that every one knows from his previous experience with matter these two facts: —

(i) That it requires an *effort* to put matter into motion when the matter is already at rest; that it requires an effort to stop matter after it has once been set in motion.

(ii) That, with any given body, the *amount* of effort is greater in proportion as the change of motion is greater; and that, for any given change of motion, the *amount* of effort is greater in proportion as the amount of matter in the body is greater.

The first of these facts is described by saying that **all matter possesses inertia**. Now inertia is simply the Latin word for laziness; but this laziness of matter differs in one essential respect from that exhibited by ourselves — on certain more or less rare occasions — in that matter hesitates quite as much to stop when once in motion as to start when once at rest. Does your own experience indicate that liquids and gases, as well as solids, possess inertia?

In all the experience of the human race no exception has been found to the statement that all matter possesses inertia. How, then, shall we explain the fact that a marble started rolling on a level blanket so soon stops? Here the blanket forms a slight hill in front of the marble, and hence exerts an effort, so to speak, against the marble. But suppose the marble put into motion on a smooth level table; here the marble rolls much farther before it stops, but the very slight roughness of the table is sufficient to bring the marble to rest. The effort which a sheet of level plate glass can oppose to the motion of the marble is slighter still, yet sufficient to stop the marble by and by. The smoother the road and the freer the bearings are from friction, the longer a bicycle will run when coasting on level ground.

Suppose now that a bicycle wheel is entirely free from friction, not only the friction of the bearings, but the friction of the air. How long will it continue to spin when the frame is held above the ground and the wheel set in motion? **All our experience goes to show that, under these ideal conditions, the wheel would never stop.** The earth in its rotation on its axis comes very nearly being such a body; the earth is, indeed, so perfectly free from any effort to stop it, that it keeps on moving, year after year, with essentially the same speed.

So extraordinarily constant is this angular speed of the earth, that Sir George Darwin says, " Attempts have been made to estimate the actual amount of retardation of the earth's rotation but without much success. *Ency. Brit.*, art. " Tides," p. 945.

In like manner, if a body has a motion of pure translation (§ 15), the natural and logical inference is that the body will keep on moving indefinitely *if no effort is exerted against it.* This inference was first made by Galileo on the ground of the following experiment.

A body is allowed to roll down the inclined plane *AF*, of fixed height *AB* (Fig. 32). The body on reaching the foot of the plane *AF* with a constant and definite velocity, is allowed to roll up a second incline of variable slope. Galileo observed that as the slope of the second plane diminished it required a longer and longer time for the ball to reach any given elevation which, of course, must be less than AB. From this he argued that if friction were eliminated and the ball were allowed to roll along a perfectly smooth and horizontal plane FG, the time required would be infinite. In other words, there would be no retardation along such a plane, and hence the motion would be uniform in direction and speed.

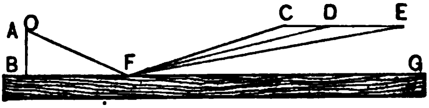


FIG. 32.

57. **But as a matter of fact in all cases of actual motion we find that there is some outside influence at work which interferes with the uniformity of the motion.** Thus, in the case of a bullet fired vertically upwards, we find that the bullet does not continue to move upwards forever, but soon returns to the earth. In the case of a kite, on the other hand, the combined effort of the string and the weight of the kite are not sufficient to bring the kite down. An equally peculiar case is that of an empty bottle corked and then immersed under water. The bottle at once rises to the surface.

In order to explain such various motions in a really simple and comprehensive way it will be necessary first to define two new terms, viz., *Mass* and *Momentum*.

COMPARISON OF MASSES

58. Since one does not know what matter is, he finds it manifestly impossible to take any one body by itself and determine just " how much matter " there is in it. But we have already seen that one can compare, in a rough way at least, the amount of matter in one body with that in another by simply shaking or pushing the two bodies in succession. **In physical science the " mass of a body " is a phrase used to denote the inertia of that body or, if you prefer, the amount of matter in that body compared with that of another body taken as standard.**

Naturally the next problem is to find some quantitative method for comparing the mass of one body with that of another taken as standard. Perhaps the first solution which occurs to the student is to put the two bodies on an ordinary beam balance and weigh them; and this is, indeed, the very best of all known methods. But weighing is, *as yet*, open to this objection, namely, one does not know *without further evidence* that the earth pulls equally hard on all substances, *i.e.* upon all kinds of matter. One does not know, *before the event*, but that copper may be more strongly attracted by the earth than is nickel; that mass which we call a "pound of feathers" may weigh more than that mass which we call a "pound of lead" simply because the earth exerts a stronger attraction for feathers than for lead; just as a piece of iron exerts a stronger pull upon a magnet than upon a piece of brass having the same size, shape, and position as the magnet.

As a matter of fact, Galileo, Newton, and Bessel proved — by methods which will be considered later — that the earth accelerates — attracts — all bodies equally. **Hence weighing is the approved method for comparing masses.**

Since however the mass of a body is something quite different from its weight, and since the idea is fundamental to every branch of physics, we must consider **a method of comparing masses which is independent of weight.** The following is such a process. Let two small metal cars be placed upon a smooth horizontal table and let the cars be attached to each other by means of a stretched rubber band *AB*, as indicated in Fig. 33.

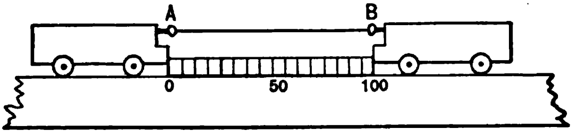


FIG. 33. — Fundamental method for measurement of mass.

The cars should be made to run with very little friction and should be held apart at a distance of, say, 100 centimeters by means of a meter stick. When this stick is deftly removed and the wagons approach, we may fairly assume *that the rubber band pulls with equal strength upon each car regardless of the substance from which the car is made and of the substance with which it is loaded.* This assumption corresponds, of course, to that which is tacitly made when one compares masses by weighing, namely, that the earth's attraction for a substance does not depend upon the chemical composition of that substance.

From the moment when the two cars are released until they collide, each will acquire an increasing — and hence accelerated — speed. Let M_A and M_B indicate the respective masses of the two wagons, including their loads, while a and b indicate the respective accelerations of the wagons at any instant. Then the *ratio* of the masses, M_A and M_B, is, *by definition*, equal to $\frac{b}{a}$, a fact more simply expressed by the following equation: —

$$\frac{M_A}{M_B} = \frac{b}{a}. \qquad \text{Eq. 26}$$

Defining equation for ratio of masses.

But since the distance which a body travels from rest is always proportional to its acceleration, one may also write

$$\frac{M_A}{M_B} = \frac{p_B}{p_A}, \qquad \text{Eq. 27}$$

where p_A and p_B are the respective distances traversed by the two wagons before collision.

To compare the masses of two bodies without weighing we have therefore only to take the ratio of their accelerations when the motion of each body is accelerated by the same external influence. This definite quantitative idea of inertia seems to have been introduced into physics by Sir Isaac Newton (*Principia,* Book I, Def. III).

It was quite a century after the publication of the *Principia* in 1687, that Lavoisier proved that the mass of a given body is an invariable quantity. No man has succeeded by any means, chemical, thermal, or other, in creating or annihilating a single particle of matter. But really all those who from the earliest times to the present have used the steelyard or other beam balances have been comparing masses while they were weighing. It was not therefore a new method of determining mass which Newton introduced, but a new idea, a new conception of inertia, a new interpretation of the result of weighing on a beam balance, according to which the mass of a body has the same numerical value whether determined on one part of the earth or another or even on another planet.

UNIT OF MASS

59. Having once learned how to compare (*i.e.* measure) masses, we have only to select a certain standard of mass, in order to be able to determine the masses of other bodies.

There is a piece of matter preserved in the International Bureau of Weights and Measures at Sèvres, near Paris, a piece of platinum-iridium, weighing about two pounds, which the scientific world has arbitrarily agreed to call the standard kilogram, *i.e.* 1000 grams. **The gram, which is therefore one thousandth part of this standard of mass, is taken as the unit of mass.**

When hereafter we shall speak of the mass of a body as "25 grams," we shall mean that the body has 25 times as much matter in it as there is in one gram; that the effort required to start it moving with a given speed is 25 times that required to start one gram moving with the same speed; or, what happens to be the same thing, that it weighs 25 times as much as one gram.

DIGRESSION ON FUNDAMENTAL AND DERIVED UNITS

60. The standard of mass, and also the standards of length and time, have been defined in a purely arbitrary way. See §§ 9 and 20. They do not depend upon any other standards for their value. **A unit which is chosen in this arbitrary manner, without reference to other units, is called a fundamental unit. In modern physics there are practically only three fundamental units, viz., the unit of mass, the unit of length, and the unit of time.**

All other units are on a different basis, as will be seen by considering one of them, say the unit of speed. The whole scientific world uses as unit speed, that speed with which a particle will traverse unit distance in unit time. In the metric system we say unit speed is one centimeter a second. The unit of speed thus depends upon the units of time and length. It is, therefore, said to be a **derived unit.**

Derived units are defined as those which depend for their value upon the fundamental units. In like manner, the unit of acceleration, the unit of area, and the unit of volume are all derived units. So are all the units employed in this textbook, except the three fundamental ones just mentioned. Units which are based upon the centimeter, gram, and second are spoken of as " C.G.S. units."

The engineer uses a different set of fundamental units, which are as follows: the foot is his fundamental unit of *length,* and the second his fundamental unit of *time;* but the inertia of a pound he takes as his unit of *mass* and the weight of a pound as his unit of *force.* This last step is sometimes convenient, though unnecessary, and the practice is not followed either in physical science or in electrical engineering; for as we shall see when we come to consider the subject of force the weight of a pound is variable while the mass of a pound is constant.

THE INTERNATIONAL METRIC SYSTEM

61. Since 1875 when the International Bureau of Weights and Measures was established at Paris and placed under the control of an international committee, the whole scientific world has been using the same standards of length and mass. This International Bureau has distributed to each of the nations represented in it copies of the original meter and the original kilogram preserved at Paris. In 1901 our government established a Bureau of Standards at Washington, and this bureau, under the directorship of Professor S. W. Stratton, is charged with the care and use of these " American prototypes," as our copies are called. This, however, is but one of the many functions of the Bureau of Standards; for this institution has prepared and holds ready for use many electrical standards, radioactive standards, standards of temperature, etc. Besides producing and duplicating these standards, the bureau also investigates many methods and questions of fact arising in chemistry, physics, and engineering. The results of these investigations are published in the *Bulletin of the Bureau of Standards.*

The British nation preserves its standard pound — the Imperial Pound Avoirdupois — in the Standards Office at Westminster, London. In Germany, France, and Italy the metric system is used everywhere. The German *pfund,* the French *livre,* and the Italian *libbra,* while retaining their original names, are each exactly 500 grams. The English pound unfortunately contains only 453.5924 grams.

Recently Professor A. A. Michelson has measured the length of the standard meter at Paris in terms of a wave length of red light, by a method so accurate that if the original meter were destroyed it could nevertheless be duplicated with an error probably not to exceed one part in two million. The standard kilogram if lost could probably be restored with even greater accuracy. Evidently modern precision is of a higher order than that with which the old English pound was defined, in the reign of Henry III (1266), as 7680 " wheatcorns in the midst of the ear." *

DEFINITION OF DENSITY

62. It is a household fact that the mass of a cubic inch of lead is greater than that of a cubic inch of pine wood; it is a matter of common knowledge that copper is denser than water, and that a copper coin will sink in water. **In physics the " density of a body " is a phrase used to denote the ratio of the mass of that body to its volume.** Accordingly if we denote the mass of a body by M, its volume by V, and its density by D, then

$$D = \frac{M}{V}. \quad \text{Defining equation for average density.} \qquad \text{Eq. 28}$$

The gram was intended to be, and is very nearly, equal to the mass of one cubic centimeter of water at a temperature of 4° C. The *mass* of any body of water is, therefore, numerically equal to its *volume,* provided we use the centimeter and the gram as units; that is, the density of water is unity in the metric system. Accordingly for this substance Eq. 28 reduces to $M = V$.

The more accurate value for the mass of one cubic centimeter of water at 4° C. is 0.999980 gram.

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* Chisholm, H. W., *Weighing and Measuring* (Macmillan, 1877), p. 55.

DIGRESSION ON MOMENT OF MASS AND CENTER OF MASS

63. Let us consider any rigid body of mass M as made up of a large number of particles. Call the masses of these particles m_1, m_2, m_3, etc.; their positions in the body may be stated by means of rectangular coördinates $(x_1y_1z_1)$, $(x_2y_2z_2)$, $(x_3y_3z_3)$, etc., respectively. Next form the sums indicated in the three following equations: —

$$\left.\begin{aligned} m_1x_1 + m_2x_2 + m_3x_3 + \text{etc.} &= \Sigma(mx), \\ m_1y_1 + m_2y_2 + m_3y_3 + \text{etc.} &= \Sigma(my), \\ m_1z_1 + m_2z_2 + m_3z_3 + \text{etc.} &= \Sigma(mz), \end{aligned}\right\} \qquad \text{Eq. 28 A}$$

Each of these products is called a "moment of mass." Those in the first equation are the moments of mass with respect to the YZ-plane; those in the second equation are the moments of mass with respect to the XZ-plane; while the moments of mass, for each particle, with respect to the XY-plane are given in the third equation.

Let us now choose a point whose distance $\bar{x}$ from the YZ-plane is such that

$$\bar{x} = \frac{\Sigma(mx)}{M}, \qquad \text{Eq. 28 B}$$

and whose distance, $\bar{y}$, from the XZ-plane is such that

$$\bar{y} = \frac{\Sigma(my)}{M}.$$

In like manner, make the distance from the XY-plane such that

$$\bar{z} = \frac{\Sigma(mz)}{M}.$$

The point thus determined by these three rectangular coördinates $(\bar{x}, \bar{y}, \bar{z})$ is called the **center of mass** of the body, a name introduced by Daniel Bernouilli (1700–1782).

If the origin of coördinates be chosen at the center of mass, it follows that $\bar{x} = \bar{y} = \bar{z} = 0$; and hence

$$\Sigma(mx) = \Sigma(my) = \Sigma(mz) = 0. \qquad \text{Eq. 28 C}$$

It follows, therefore, from Eq. 28 C that the **center of mass of any body is the point about which the moment of mass is zero.** If we pass a plane through the center of mass, it does not follow that there are as many particles of unit mass on one side the plane as the other; but it *does* follow that the moment of mass of the particles on one side is the same as the moment of those on the other. A briefer and clearer way of viewing this subject is perhaps the following. Let each particle m be completely located with reference to any fixed origin by means of a position vector $\bar{r}$. Then the position vector of the center of mass $\bar{R}$ is by definition expressed in the following equation: —

$$\bar{R} = \frac{\Sigma(m\bar{r})}{M}. \qquad \text{Eq. 28 D}$$

As will be seen later, in the discussion of gravitation, the center of mass is practically identical with the center of gravity.

DEFINITION OF LINEAR MOMENTUM

64. The product obtained by multiplying the mass of a body by its linear velocity is one so frequently employed in physics that a special name has been given to it, viz., Linear Momentum.

Since velocity is a vector quantity and mass a scalar quantity, it is evident that their product, momentum, is a vector quantity, and that its direction is the same as that of the velocity factor. The product of the mass and linear *speed* of a body gives the merely numerical part, the scalar factor, of momentum.

The momentum of a bicycle ridden north at the rate of six miles an hour is by no means the same as that of the same wheel ridden east at six miles an hour. The mere numerical values of these two momenta are the same; the difference of momenta in these two cases is keenly appreciated when we consider the difficulty involved in avoiding a collision with another wheel going south. For instance, in a crowded street, when the wheels are proceeding from north to south, one after another, the danger to another wheel moving east or west six miles an hour is enormously greater than the danger to the same wheel going north or south six miles an hour.

Let m denote the mass of a particle and $\bar{v}$ its velocity, then

Linear momentum of a particle $= m\bar{v}$. Defining eq. for linear momentum. Eq. 29.

The reasonableness of this definition will be seen when we recall that the mass of a body is one of the factors which measure the difficulty of starting or stopping the body. But, as every one knows, the difficulty of starting or stopping a body in translation depends also upon how fast you start it going, or upon how fast it is moving when you attempt to stop it. It depends, as Newton said (Definition 2, *Principia*), upon the "quantity of motion" in it; or, as we now say, upon its **linear momentum.**

Since, then, the difficulty of stopping a body in translation depends both upon its mass and upon its velocity, we say,

$$\text{Linear momentum} = \text{Mass} \times \text{Velocity}.$$

ROTATIONAL INERTIA AND ANGULAR MOMENTUM

65. Two children who are seesawing, one at each end of a board, know very well that the seesawing goes on more smoothly and with greater uniformity when they are out near the ends of the board than when they are in near the middle. But when they are near the middle, the board is much more quickly started or stopped by a third party taking hold of the end. The mass of the board and the children is the same in each case; only the distribution of the mass is different.

The children and the board constitute a typical rotating system. The axis of rotation is the line of contact of the board and the log over which the board is balanced. The point to which attention is here directed is that **the resistance which a body offers to being set in rotation, *i.e.* its rotational inertia, is not measured by its mass merely, but depends also upon the distribution of its mass,** *i.e.* upon the position of the mass.

Every one who has used a sewing machine knows that it would run in a very jerky manner were it not for the heavy fly wheel which has most of its mass in the rim and therefore steadies the motion. Imagine two fly wheels to have equal masses. In the first wheel we shall suppose the mass to be situated mostly near the axis of rotation; in the second wheel the mass is placed far out near the rim. Every one knows that the second wheel will be the more effective in producing steady motion.

But if now this rotational inertia does not depend merely upon the mass of the wheel, upon what else does it depend? The answer to this question is simple but it cannot be satisfactorily given until we have, in a later section, considered the subject of energy. For the present we can only borrow from the future and ask the student to take for granted, what we shall prove a moment later, that the inertia of any particle with respect to its axis of rotation is measured by the product of its mass, m, and the square of its distance r from that axis.

$$\text{Rotational inertia of a particle} = mr^2. \qquad \text{Eq. 30}$$

$$\text{Rotational inertia of a rigid body} = \Sigma(mr^2) \qquad \text{Eq. 31}$$

This product which measures the rotational inertia of a body is called its **moment of inertia**, an unhappy name first introduced by the Swiss mathematician Euler (1707–1783), and now generally denoted by the letter I.

Just as we defined

Linear momentum = Linear inertia × Linear velocity,

so we may now define

Angular momentum = Rotational inertia × Angular velocity,

or more briefly Angular momentum $= I\omega$. Eq. 32

66. We have now increased our vocabulary by the definition of four physical quantities of fundamental importance, namely:—

(1) Linear Inertia, commonly called Mass.
(2) Rotational Inertia, commonly called Moment of Inertia.
(3) Linear Momentum.
(4) Angular Momentum.

A mere passing acquaintance with these four ideas will not suffice. The student should know them intimately.

DIGRESSION ON THE DIMENSIONS OF UNITS

67. Before going farther with the principal discussion it is necessary to consider for a moment what is meant by the " dimensions " of a unit. As stated above, there are in mechanics, only three units which are chosen arbitrarily, namely, those of *length*, *mass*, and *time;* these units are generally indicated by their initial letters M, L, and T; and all other units are expressed in terms of these three.

Now the powers to which the fundamental units enter into any derived unit are called the " dimensions " of that unit.

As soon therefore as one knows the definition of a physical quantity he can write its dimensional equation. Thus if we denote speed by S and define it as the ratio of distance to time, we have

$$[S]=\left[\frac{L}{T}\right]=[LT\ \ ].$$

Maxwell introduced the square brackets to indicate that not the values of the quantities, but merely the dimensions of their units, are involved in these equations. The student will be interested to obtain for himself the " dimensions " listed in the following table. All the data needed are the definitions of the quantities.

PARTIAL LIST OF DIMENSIONS

QUANTITY	DIMENSIONS	QUANTITY	DIMENSIONS
Distance	L	Angular speed	T^{-1}
Area	L^2	Angular acceleration . .	T^{-2}
Volume	L^3	Mass	M
Time	T	Moment of inertia . .	ML^2
Linear speed	LT^{-1}	Density	ML^{-3}
Linear acceleration .	LT^{-2}	Linear momentum . .	MLT^{-1}
Angular displacement.	L^0	Angular momentum . .	ML^2T^{-1}

The idea of " dimensions " is useful in detecting any lack of homogeneity in the equations of physics. Thus take Eq. 16 or any other equation in this book and it will be found that in each term of the equation the fundamental units enter to the same power. Imagine yourself called upon to write from memory the expression for the distance traversed by a freely falling body. You might inadvertently write down

$$p = S_0t^2 + \tfrac{1}{2}at^2.$$

But you could at once detect the error: for each term on the right-hand side must be of the same "dimensions" as p, which is a length. The expression as it stands, however, contains a term S_0t^2 which has the dimensions $[LT]$; but this is impossible, and the equation is therefore wrong. All equations must be homogeneous; in other words only like things can be equal to each other. The above doctrine of "dimensions" was first formulated by the French physicist Fourier in 1822. The advanced student will find a still more important application in the principle of "dynamical similarity."

Problems

91. How is it that, in a fly wheel weighing one ton, the effect of the wheel in steadying the motion of the engine is very much increased by placing most of the mass out near the rim of the wheel?

92. If an extra pound is to be added to a bicycle wheel, where will it prove to be the greater hindrance to starting, when placed in the tire or in the hub? Why?

93. Why is it that in the balance wheel of a watch most of the material is placed in the rim of the wheel? Sketch such a wheel on the blackboard.

94. An iron hoop, circular in shape, and very thin, weighs 200 g. Its radius is 15 cm. Find its rotational inertia about an axis which passes through the center of the hoop and is perpendicular to the plane of the circle.

95. The angular momentum of a certain fly wheel is 25,000,000 C.G.S. units. Its angular speed is 20 radians per second. Find its moment of inertia.

96. A light pendulum rod whose mass may be neglected has clamped on it two heavy particles; one is a mass of 12 g. at a distance 4 cm. from the axis, the other a mass of 20 g. at a distance of 35 cm. from the axis. Find the moment of inertia of the system.

97. We shall presently find that the time, T, required for one vibration of a simple pendulum is given by the following equation: —

$$T = 2\pi\sqrt{\frac{\text{Length of pendulum}}{\text{Acceleration of gravity}}}.$$

Show on the blackboard that the "dimensions" of each member of the equation are the same.

THE MEANING OF FORCE

68. Up to this point we have been considering some of the simpler motions which bodies may acquire; but we have not concerned ourselves with the manner in which any body has acquired its motion.

Bodies act upon each other in a great variety of ways. An incandescent lamp may illuminate a page, it may heat one's hand, it may, through rise of temperature, set the surrounding air into motion; a walking stick coming into contact with the hand of a boy is very apt to acquire motion, and it may also undergo a change of shape; the steam in the cylinder of a locomotive sets the piston head in motion.

But of all these various effects there are two with which we are especially concerned in the study of mechanics. (i) The first is **the production of motion.** In fact no other process is known by which motion may be gained or lost except by the mutual action of one body upon another. (ii) There is a second effect of one body upon another which involves no change of motion but which is nevertheless commonly described in terms of "force"; this is **a change of shape or size.** When the butcher places a piece of meat on the pan of his spring balance, the whole system quickly comes to rest and the only effect of the meat is to stretch the spring of the balance by a certain amount. *The shape of the spring is altered.* The meat is said to exert a "force" upon the spring; while the spring holding up the meat is said also to exert a "force" upon the meat. Again consider a system made up of a beam balance and the earth. When one buys two pounds of tea from the grocer, a two-pound brass piece is first placed upon one pan of this beam balance; then the grocer places in the *other* pan just enough tea to restore the configuration which the system had before the balance was loaded and to bring the pointer back to zero. The earth is then said to exert the same "force" upon the tea as upon the two-pound mass. In a similar manner, the boy who uses a pump to inflate the tire of his bicycle compresses the air in the cylinder of the pump, that is, he changes the *size* of the inclosed air, and we therefore say that he exerts a "force" upon this air. So also the air in the tube is said to exert a "force" upon the tube, keeping it stretched to a size larger than is natural.

In general when one body acts upon another in such a way as to change either its motion or its size and shape the one body is said to exert a force upon the other. So much for a qualitative definition of "force." We now proceed to define it quantitatively.

THE MEASURE OF FORCE

69. In looking over the accomplishments of the ancients in the domain of mechanics one is struck by the fact that up to about the year 1600 A.D. — say to the time of Shakespeare, Queen Elizabeth, or the settlement at Jamestown —no one had made any successful study of *change* of motion. Thus Vitruvius, the distinguished Roman engineer and architect, in his interesting treatment of mechanics, describes various pumps, windlasses, cyclometers, etc., each involving only uniform motion; when he comes to catapults and ballistæ, he confines his attention to the structure of the machine, not at all to the variable motion of the projectile.

And so it came about most naturally that, until recent times, the only practical standard of force was the pull which the earth exerted upon a selected standard body. Among the Romans this standard body was called a *libra*. Its mass was 327.25 grams. The common instrument by which these measures were made was, of course, the steelyard. Forces of all kinds, such for instance as that required to break a rope, were measured in terms of this weight. This was a method which answered every purpose until late in the seventeenth century. Between the years 1671 and 1672 Huygens and Richer discovered that the weight of a body is a variable, instead of a constant, quantity. These men showed by experiment that the weight of a body diminishes by something like one part in three hundred when the body is transported from the latitude of Paris to that of the equator, and that this loss is regained when the body is brought back to Paris. It became at once evident that if forces were to be accurately measured they must be compared with something more constant than the weight of a standard mass. For the comparison of masses the balance remained still the same superb instrument which it had always been.

In the meantime Galileo (1564–1642) had made a highly original study of variable motion. He had found that a body which is left to itself — free from others — is never accelerated; even though the body be in motion when left to itself, its motion does not change. He experimented mainly with falling bodies, *inquired especially into the rate at which they changed momentum,* and discovered by experiment that this rate is approximately constant for any one body falling freely. Besides this, Galileo laid especial emphasis upon his definition of constant acceleration, which is essentially the one we use to-day.

The next and the final step in the modern definition of force was taken by Newton and his contemporaries still before the end of the seventeenth century. They proposed to adopt the method of Galileo and measure the action of one body upon another, at any instant, by the rate at which the momentum of this other body is changing at that instant. **Accordingly the modern measure of the force acting upon any body is the time rate at which the momentum of that body is changing.** Change of momentum is therefore the criterion of force; and its time rate, the measure of force.

To express this definition in the shorthand of algebra let us denote the momentum of a body, at the instant t_1, by M_1; and at the next instant t_2 let its momentum be M_2. Then the force F_a, acting upon the body during the interval $t_2 - t_1$, is given by the following equation: —

$$F_a = \frac{M_2 - M_1}{t_2 - t_1}. \qquad \text{Defining equation for average force.} \qquad \text{Eq. 33}$$

Or if we choose the interval $t_2 - t_1$ indefinitely small and pass to the limit: —

$$F = \left[\frac{M_2 - M_1}{t_2 - t_1}\right]_{t_2 = t_1} \qquad \text{Defining equation for force at any instant.} \qquad \text{Eq. 34}$$

And since we have already agreed (Eq. 12) to call the rate of change of velocity by the name of acceleration, we may indicate the mass of the body by m, its acceleration by a, and write:—

$$F = m\left[\frac{v_2 - v_1}{t_2 - t_1}\right]_{t_2 \doteq t_1} = ma. \qquad \text{Eq. 35}$$

Thus **force** comes to be defined **as the product of mass by acceleration**, a definition equivalent to that given above. Of the two factors in this defining equation one, m, is a scalar and the other, a, is a vector quantity. **Hence force itself is a vector quantity**, and can, therefore, be represented in direction, sense, and amount by a straight line of limited length.

UNIT OF FORCE: THE DYNE. THE POUND

70. Since the force acting upon any body is the product of the acceleration which it produces in this body multiplied by the mass of the body, we have no liberty of choice as to a unit of force. For, provided we wish to be at all consistent, we must maintain our previously employed units of acceleration and mass. But these we have already defined in §§ 33 and 59. Hence the **unit of force, which is called a "dyne," is that force which will produce unit acceleration in a mass of one gram.**

Or, what is the same thing, unit force is that force which is capable of changing the momentum of a body at the rate of one unit per second.

Or, what is the same thing still, we may define the unit of force as that force which will change the velocity of a mass of one gram at the rate of one unit (that is, one centimeter per second) per second. Note that "dyne" is simply the Anglicized form of the Greek word for force. The student will be interested in proving that the dimensions of force are MLT^{-2}.

The engineer employs as a unit of force the weight of a one-pound mass at London; in other words, his unit of force is that force which the standard pound exerts upon the bottom of the box which contains it at the Standards Office in London.

In order to distinguish between the mass of a pound and the force of a pound, it is well to write the former "*lbs.*" and the latter "*pounds.*"

In practice one meets a great variety of units of force. It is all important, therefore, for the student to remember that the simple form of Eq. 35 holds true only when F, m, and a are all measured in the same system of units. In general,

$$F = kma,$$

where k is a constant to be definitely determined. If mass is expressed in grams and acceleration in cm./sec.^{2}, then

$$F = ma \textit{ dynes},$$

Or if mass is expressed in pounds and acceleration in ft./sec.^{2}, then

$$F = ma \textit{ poundals},$$

where evidently the "poundal" is defined as the force which will produce an acceleration of one foot per second per second in a mass of one pound.

If, however, force and mass are each measured in pounds, then

$$F = \frac{1}{32{\cdot}2} \cdot ma \textit{ pounds}, \text{ approximately.}$$

Here the proportionality constant k has the value of 1/32.2.

Again if F and a are expressed in the C.G.S. system and m in pounds mass, instead of grams, then

$$F = 453.592\, ma \textit{ dynes},$$

from which it follows that k has the value 453.592.

Definition of Torque

71. In a precisely analogous manner, **torque is the name given to the time rate at which the angular momentum changes.** Let L denote the torque acting upon a rigid body whose rotational inertia (moment of inertia) is I and which is capable of rotation about some given axis; let ω be the amount by which its angular velocity is changed during the time t; then the

Average Torque

acting upon the body during the interval t is given by the following equation: —

$$L = \frac{I\omega}{t} = I \times \text{rate of change of angular velocity,}$$

or, Eq. 13, $$L = IA,$$ Eq. 36

where A denotes the angular acceleration of the rigid body about the given axis.

RELATION BETWEEN FORCE AND TORQUE

72. As a matter of fact many bodies are set into rotation by means of "a single force" applied to the body at what is practically a single point. Thus, one starts a grindstone or a coffee mill by means of a push of the handle; the beam of a balance is set into rotation by means of a force applied at the knifeedge.

The effect of such a force acting on any such body is in general to change not only its motion of translation, but also its motion of rotation. The amount of this change in rotational speed will obviously depend upon two things: (1) the magnitude of the force applied, and (2) the distance of the line in which it acts from the axis of rotation.

Thus, if one wishes to shut a door and can exert only a limited force, he knows that this force will be much more effective when applied near the edge of the door than when applied near the hinge line.

In like manner, when one wishes to turn a heavy wheel, he always takes hold near the rim and not near the hub. He does this in order to make the perpendicular distance between the force and the axis as great as possible.

Hence, the effect of a force in producing rotation about any axis can be measured by the following product, viz., Fr, where F is the force acting, and r is the perpendicular distance from the axis to the direction of the force. See Fig. 34.

Let us denote the moment of force by L, then

$$L = rF. \qquad \text{Eq. 37}$$

This simple relation between force and moment of force, $L = rF$, should be thoroughly mastered by every student; for it is the connecting link between force and torque, and is generally considered as the defining equation for torque. Outside of engineering circles torque is generally called "moment of force," because "moment" is a Latin word which means importance; and since the product Fr measures the importance of the force F in producing rotation, it is appropriately called moment of force. The word is used here then in the same sense in which Shakespeare employs it when he speaks of "enterprises of great pith and moment."

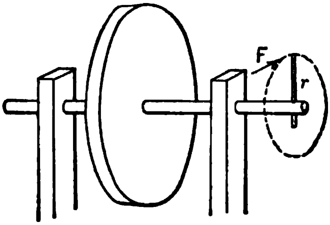

Fig. 34.—Moment of a force acting on a particle, at the end of an arm r.

The idea is one which appears first to have been introduced into physics by the Italian mathematician Benedetti (1530–1590).

NEWTON'S LAWS OF MOTION

73. Some years before Newton was born, Galileo had shown that he understood how bodies behave when they are not acted upon by any forces. In computing the path of a projectile fired from a gun, he showed himself equally familiar with the behavior of a body when acted upon by several forces at once. But it was not at all clear to him as to what happens when two bodies collide, such for instance as two billiard balls. This last problem seems to have been solved largely by Wallis, Wren, and Huygens, contemporaries of Newton.

The first man, however, who succeeded in formulating the few fundamental principles which govern all the cases mentioned above, and indeed all the cases occurring in mechanics, was Newton himself. These simple descriptions of how bodies behave under the action of forces are contained in the introductory chapter of the *Principia*, and are known as Newton's *Laws of Motion.*

They are, in modern terms, as follows: —

74. LAW I. **If a body in translation is not acted upon by any outside force, its linear momentum remains constant.**

This fact is described algebraically by the following equation: —

$$mv = \text{constant}, \qquad \text{Eq. 38}$$

where m is the *mass* and v, the *linear velocity* of the body.

In like manner *if a body in rotation is not acted upon by any external torque, its angular momentum remains constant.* Or, more briefly,

$$I\omega = \text{constant}, \qquad \text{Eq. 39}$$

where I is the *moment of inertia* and ω the *angular velocity* of the body.

Experiment shows that these two statements are true for bodies at rest as well as for bodies in motion, and that they are true for a body of any size, shape, or composition. But they hold only under the conditions stated; namely, that the body or the system of bodies is affected by no outside action. Before one can say whether or not a body is acted upon by outside influences, he must be very careful to define just what the body is which he is studying. This done, the distinction between "internal" and "external" forces is easy.

If, therefore, a heavy fly wheel, set in rotation and "left to itself," gradually diminishes in angular speed, we may fairly expect to find the wheel is *not* really left to itself and is *not* entirely free from external influences. In such an expectation no one has yet been disappointed.

If a skater coasting on level ice finds himself moving more and more slowly, he suspects resistance to his motion in the air, and in the frictional resistance between the ice and the skates.

If a baseball, thrown vertically into the air, changes its velocity (as it does, first diminishing, then stopping, and then increasing in speed), we may rest satisfied that an outside influence is at work. In this case it is popularly called "gravity."

Since, then, bodies left to themselves move with uniform velocity, we may say that motion is a state of matter quite as natural as rest. For bodies in motion and left alone keep in motion; while bodies at rest and left alone remain at rest.

75. Law II. The change in the linear momentum of a body is proportional to the force acting upon the body; and the direction of the change is the same as the direction of the force.

The algebraic description of this fact is the same as the defining equation (p. 75) for force, namely: —

$$\text{Force} = m\left(\frac{v_2 - v_1}{t_2 - t_1}\right) = ma, \qquad \text{Eq. 35}$$

where a is the *acceleration* of a body whose *mass* is m.

A similar law describes the behavior of a body in which there occurs a change of *angular* momentum.

The change in the angular momentum of a body is proportional to the torque (or moment of force) acting upon the body; and the axis and direction of the change are the same as the axis and direction of the moment of force.

The mathematical form of this law is given by Eq. 36, namely,

$$\text{Torque} = \frac{I\omega}{t} = IA,$$

where A is the *angular acceleration* produced in a body whose *moment of inertia* is I.

Nothing connected with Newton's second law is more striking than its generality. It covers the behavior of all bodies except possibly electrons moving at very high speeds. Ordinary bodies may be at rest or in motion; may be hot or cold; may be liquid, solid, or gaseous; may be acted upon by other forces at the same time or not; in every case the law has been found to hold. The fact that any given force always produces the same effect whether other forces be acting or not gives us immediately the rule for the addition of forces which will be considered in § 78.

76. Law III. To every force there always exists a corresponding force which is equal and oppositely directed.

So also to every torque there always exists an equal corresponding torque directed about the same axis, but in an opposite sense.

The meaning of the third law can hardly fail to become clear to one who carefully studies the following problem: A railroad train is running on a level track at the uniform rate of 24 miles an hour, there is no change in the speed or in the direction of the velocity; accordingly the acceleration of the train is zero. This conclusion is plain, simple, unavoidable. But how can this be, when the engine is exerting a force of millions of dynes on the train?

The answer is simply this: The pull of the engine is only one among many forces acting upon the train. In these forces we must include the friction of the rails, the friction of the bearings, and the friction of the wind. These forces are all opposing the pull of the engine. If now the train be running *uniformly* at the rate of 24 miles an hour, we conclude that these opposing forces are exactly equal and opposite to the pull of the engine; and hence that the total force acting upon the train is zero. If steam be cut off the engine, the opposing forces retard the motion of the train in such a way as to diminish its speed. When the train comes to rest, the opposing forces become zero; and hence the total forces are again zero and the acceleration is again zero. When the engineer lets on steam, the forward pull of the engine is greater than the backward pull of the opposing frictional forces. Hence the motion of the train is accelerated, this time in such a way that its speed is increased. This acceleration will continue until the frictional forces are exactly equal and opposite to the pull of the engine. At this point the acceleration is again zero.

From this illustration the student will observe that great care is necessary to include in any system under discussion all the various parts of that system. The track and the air retarding the train are essentially parts of the system.

When the forces which aid any uniform motion are added to those which oppose the motion, the sum is always zero. But even when the motion is *not* uniform, it is found that the mass which is being accelerated exhibits a force of reaction just opposite in sense and equal in amount to the resultant of all the other forces acting upon the body. This is an observed fact of nature which Newton was the first to accurately describe. **This he did by saying that action is always equal and opposite to reaction;**

or the mutual actions of any two bodies are always equal and oppositely directed.

This statement is known as Newton's Third Law of Motion. This force of reaction in accelerated bodies is beautifully illustrated by the following experiment: —

With a piece of light string support an iron ball, from two to four inches in diameter, as indicated in Fig. 35. To the lower side of the ball attach a piece of the same kind of thread.

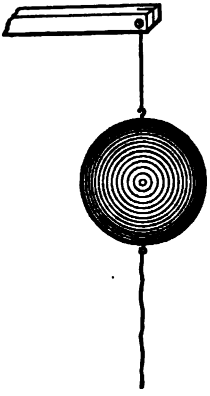

FIG. 35. — Illustrating reaction which is measured by mass × acceleration.

Taking hold of this lower thread, one may by pulling break either thread he likes. If he pulls slowly, the lower thread will be affected only by the pull of the hand, while the upper thread will be stretched both by the pull of the hand *and* by the weight of the iron ball. In this case, of course, the upper string will break first.

If, on the other hand, one pulls the lower string quickly, it will as before feel the pull of the hand alone. But the upper string will be stretched practically by the weight of the ball only; for in this case the ball will be so rapidly accelerated that the reaction of the ball (its mass × its acceleration) will be sufficient to break the lower string before the ball has moved through a distance sufficient to appreciably stretch the upper string.

The student will find it an excellent habit to look at every force from these two points of view.

When a body, or system of bodies, is not accelerated, we may consider it is acted upon by two or more forces whose sum is always zero, or by two or more moments of force whose sum is always zero.

When a body *is* accelerated, we may consider the force of reaction as one of the forces acting upon the body; and *from this point of view,* which is due to D'Alembert, all forces are balanced forces.

RELATION OF NEWTON'S LAWS TO EACH OTHER

77. The first law describes the behavior of a body when there is no external force acting upon it; or if you prefer, it states the conditions under which there are no external forces. The second law describes the behavior of bodies when external forces are acting. It is clear, therefore, that the first law is merely a special case of the second law, telling us what happens when the external forces have a particular value, namely, zero.

To clearly grasp the relation of the third law to the other two it is helpful to recur to the idea of Maxwell that when two bodies interact there is a "dynamical transaction" taking place between them. "Just as in commercial affairs the same transaction between two parties is called Buying when we consider one party, Selling when we consider the other, and Trade when we take both parties into consideration" (*Matter and Motion*, Art. 39).

The third law tells us then what happens when we include in our system all the bodies whose action we are considering. Under these conditions, that is, under no external forces, the third law says that the forces remaining are equal. The "forces remaining" may be called *internal* forces.

Put in other words, the first law furnishes one, but not the only, criterion for the presence (or absence, if you like) of force. The second law tells us how to measure force, an idea due to Galileo. The third law expresses the fact that forces always occur in pairs which are equal and opposite.

ADDITION OF FORCES, AND TORQUES

Case of a Particle

78. Since each force produces its own effect quite independently of any other (Law II) and since forces are directed quantities, they are added according to the ordinary rule for vector addition; that is, in the same manner as displacements, velocities, momenta, and accelerations, viz., by placing end to end the lines which represent the single forces. **The line joining the extremities of the broken line so formed represents the resultant force, or simply the resultant.**

Imagine the three vectors F_1, F_2, F_3, indicated in Fig. 36, to represent in direction and in amount three different forces acting upon any particle O. The line $\overline{OA}$ in Fig. 37 will then represent a force which alone will change the momentum of a body at the same rate as will the three forces F_1, F_2, F_3 combined. This is all that is meant by a resultant.

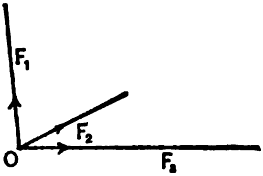

FIG. 36.

Imagine the sense of the force F_3 to be reversed; the line $\overline{OA}$, in Fig. 38, will then be the resultant. The numerical values of the forces are the same in Fig. 38 as in Fig. 37; so are their directions; but the resultants in these two cases are very different. Accordingly, the student should bear in mind that the sense of a force is, in general, quite as important a factor as its amount, or its direction. When only two forces act upon a particle, it is evident that these two forces together with their resultant will form a plane triangle.

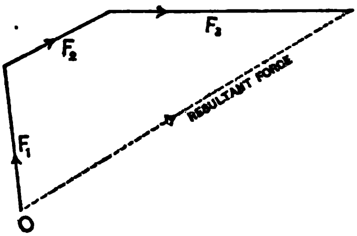

FIG. 37.

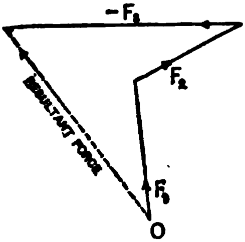

FIG. 38.

Computation of Resultant

Imagine a particle acted upon by a number of forces, R_1, R_2, R_3, etc., all lying in one plane, and let θ_1, θ_2, θ_3, etc., represent the angles which these respective forces make with the axis of X.

Then, if X is the sum of all the components of force along the direction of the X-axis, Y the sum of the components along the Y-axis, we shall have for the components of the resultant: —

$$\left.\begin{aligned} X &= R_1 \cos \theta_1 + R_2 \cos \theta_2 + R_3 \cos \theta_3 + \text{etc.} \\ Y &= R_1 \sin \theta_1 + R_2 \sin \theta_2 + R_3 \sin \theta_3 + \text{etc.} \end{aligned}\right\} \qquad \text{Eq. 40}$$

The beginner will need to use care in giving to each term in the equation its correct sign. Knowing the components, X and Y, one can easily compute the magnitude of the resultant $\sqrt{X^2 + Y^2}$, and also its slope, Y/X. It is important to understand that the algebraic method represented in Eq. 40 is strictly equivalent to the graphical method described above.

Case of a Body

It is a very curious and remarkable fact that when a given force acts upon any body for a given length of time, that force always produces the same change of *linear* momentum in the body, regardless of the point at which the force is applied. Consequently the resultant of the forces acting upon a body is found in the same way as for a particle.

But when the rotational effect of a force is concerned, the matter is very different. Here as we have seen, § 72, the value of the torque is the product of the force by the perpendicular distance from its line of action to the axis of rotation.

Imagine the drum of a hoisting engine at the mouth of a mine; let F_1, F_2, F_3, etc., be the various forces acting upon it; let p_1, p_2, p_3, etc., be the respective perpendiculars let fall from the directions of the forces to the axis of the drum. Then the resultant torque on the drum, L, will be given by the following expression: —

$$L = F_1p_1 + F_2p_2 + F_3p_3 + \text{etc.} \qquad \text{Eq. 41}$$

In a case of this kind, where the various torques all act about one axis, it is seen that they are added according to the rules of ordinary arithmetic; but when they act about different axes, one must, since a torque is a directed quantity, add them according to the rule for vector addition, or if you please, according to Newton's Second Law of Motion.

Definition of Equilibrium

79. Very often the resultant of a given system of forces is zero; in this case the body acted upon is said to be in equilibrium, as regards translation.

Thus, when a metal ball rests suspended by a spiral spring, as shown in Fig. 39, the weight of the ball, acting downwards, is exactly balanced by the pull of the spring upwards; and either of these forces is to be measured by the acceleration it would produce in the ball if the other force were absent.

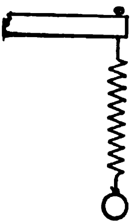

Fig. 39.

One might think at first glance that the criterion of equilibrium is rest; but this is not true, as one may easily see by setting the metal ball of Fig. 39 into vibration up and down. Just at the instant when the ball is at the upper end of its excursion it is at rest; but this is the very position in which the weight of the ball most exceeds the upward pull of the spring. Again, at the middle of its path, the ball is moving most rapidly; but this middle position is precisely the one in which the pull of the spring just balances the pull of gravity. The criterion of equilibrium is therefore the absence of acceleration, not the absence of motion.

In like manner, when the resultant of all the torques acting upon a body is zero, that body is said to be in equilibrium as regards rotation. A ship floating at rest, on even keel, is acted upon by many torques, some of which tend to make her list to one side, others to the other side; but since these torques add up to zero, the ship remains upright.

THE COUPLE

80. An interesting case of equilibrium occurs when all the translational forces acting on a body add up to zero, while the torques do not. Such a case occurs when one takes a lead pencil and rolls it between the palms of his hands in such a way that the axis of the pencil remains fixed in space while the pencil simply rotates about this axis. Indeed, one of the most important combinations of forces occurring in nature is that in which a body is acted upon by two forces, F_1 and F_2 such that $F_1 + F_2 = 0$, while the torques F_1p_1, and F_2p_2 are such that $F_1p_1 + F_2p_2 \neq 0$. These two forces can produce no change in the linear velocity of the body, but do exert a torque whose value is $F_1(p_1 - p_2)$, say, F_1r, where r is the least distance between the direction of the two forces.

To this combination the name **couple** was given in 1803 by the French mathematician Poinsot. The effect of a couple is then to produce angular acceleration without affecting the translation of the body upon which it acts. Evidently then two equal forces cannot be said to be in complete equilibrium unless they act along the same straight line as well as in opposite senses.

STATICS

81. That large branch of mechanics and engineering which deals with forces that are in equilibrium is called **Statics**.

The statics of a rigid body rests on the following assumptions: —

(i) *The sum of all the forces acting upon a body in equilibrium is zero.*

(ii) *The sum of all the torques acting upon a body in equilibrium is zero.*

(iii) *Any two forces acting along lines which intersect may be replaced by their resultant acting through this point of intersection.*

(iv) *A force may be supposed to be applied indifferently at any point in its line of action.*

The last of these assumptions is the only one which here calls for comment. The following simple but highly instructive experiment will illustrate the evidence which justifies this assumption.

Take a pair of circular wooden disks and drive them on a brass rod, just as car wheels are driven on their axle. Then wrap a string about the axle in the manner shown in Fig. 40. When this system is placed upon a smooth level table top, we may consider it as a single rigid body acted upon by a single force — the pull of the string — which shall be denoted by *F*.

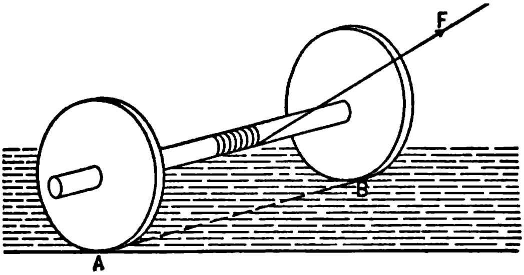

FIG. 40.

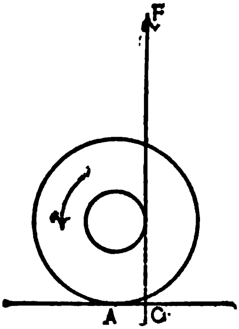

FIG. 41.

As the system rolls along the table, the axis of rotation at each instant is, not the central line of the brass axle, but the dotted line which joins the point of contact A to the point of contact B.

Let us suppose first that the pull on the string is vertical, as shown in Fig. 41. The perpendicular distance from the axis to the direction of the force is AC. The value of the torque will therefore be $F \cdot \overline{AC}$. The effect is that the wheel rotates in a counter-clockwise sense and the string unwinds.

Next pull the string in a horizontal direction, as shown in Fig. 42. Now the string winds itself up and the rotation of the wheel is clockwise. The sense of the torque is different from that of the preceding case, because the line of action of the force passes on the other side of the axis through A. The value of the torque is $F \cdot \overline{DA}$. Lastly suppose the pull of the string to be in such a direction as to pass through the axis A, that is, through the line of contact between the wheel and the table. In this case the string neither winds up nor unwinds. The system *slides* along the table instead of rolling. The arm through which the force F acts is zero; hence the torque is zero.

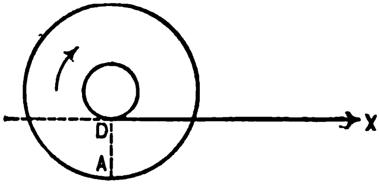

FIG. 42.

In each of these three cases we may clearly consider the force F to act *at any point in the wheel through which the direction of the stretched string passes.* This is our fourth assumption mentioned above. Every student should try this experiment for himself, even if he has to take the knobs off his door, temporarily, to make up a pair of wheels and an axle.

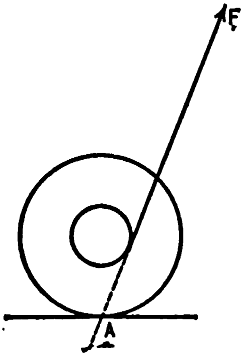

FIG. 43.

The Solution of Problems in Statics

82. A few illustrations will show how the preceding principles are used in practice.

(i) *The problem of parallel forces. Given a rod of length l, and of negligible weight, carrying a load of 3 pounds at one end and 5 pounds at the other. Find the value of the equilibrating force, R, and the position, x, at which it must be applied in order that the system may be in equilibrium.* Here the body under consideration is the rod AB of Fig. 44. For the origin of coördinates we shall choose the point A. Forces acting vertically upwards will be taken as positive; in like manner torques which act in a clockwise sense will be considered positive; counter-clockwise negative.

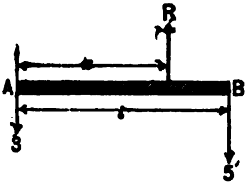

FIG. 44.

Equating now the sum of all the forces to zero we have: —

$$\Sigma(F) = -3 - 5 + R = 0.$$

Placing the sum of the torques equal to zero, one has: —

$$\Sigma(L) = -Rx + 5\,l = 0.$$

Eliminating R between these two equations and solving for x, one finds $x = \frac{5}{8}l$, or $\frac{x}{l-x} = \frac{5}{3}$. In words, the equilibrating force, R, is to be applied at a distance from A which is $\frac{5}{8}$ of the length of the rod. The student may for himself easily prove the general theorem that the equilibrant of any two parallel forces is to be applied at a point such that its distances from the lines of these forces are in the inverse ratio of the forces themselves.

To find the value of this balancing force one has only to solve the first equation for R.

In case the weight of the rod were not negligible, one would merely have to introduce another force — a vertical force — equal to the weight of the rod and acting through its center of gravity.

(ii) *Particle acted upon by three forces. Imagine a clothesline stretched between two posts A and B, and a weight W attached to* *the line at the point P. Find the tensions T_1 and T_2 in the two respective parts of the line.*

Here the body under consideration is the particle of the clothesline, at P, Fig. 45, to which the weight W is attached. Let P be the origin of coördinates and let angles be measured from the horizontal (dotted) line through P. Let θ_1 and θ_2 denote the directions of the two parts of the clothesline respectively; then since the particle P is not accelerated either to the right or the left, we may equate to zero the sum of the horizontal components of the forces, thus:—

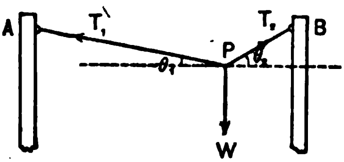

FIG. 45.

$$\Sigma(X) = -\ T_1 \cos\theta_1 + T_2 \cos\theta_2 = 0.$$

For the same reason the vertical components add up to zero.

$$\Sigma(Y) = T_1 \sin\theta_1 + T_2 \sin\theta_2 - W = 0.$$

To obtain T_1, eliminate T_2 between these two equations and solve for T_1, which gives

$$T_1 = W\left(\frac{\cos\theta_2}{\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2}\right) = W\left(\frac{\cos\theta_2}{\sin(\theta_1+\theta_2)}\right).$$

Similarly, $$T_2 = W\left(\frac{\cos\theta_1}{\sin(\theta_1+\theta_2)}\right).$$

This problem might also be solved graphically by drawing the force diagram shown in Fig. 46. The problem being one of equilibrium, the triangle of forces is closed; the side W and the angles are given to find the length of the sides T_1 and T_2.

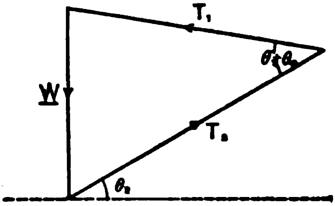

FIG. 46.

(iii) *The problem of a bridge truss. A bridge and its load, weighing* 4000 *lb. are symmetrically placed between two piers, so that each pier carries* 2000 *lb. The member T which is under tension is horizontal; the slanting member P which is under compression makes an angle of* 60° *with the horizontal. Find the forces acting along P and T.*

Here the body under consideration is the iron pin *B*, at, say, the right-hand end of the bridge shown in Fig. 47. This pin passes through an eye in each of the members *P* and *T* and bolts them together. Take *B* as the origin of coördinates. Then equate to zero first the horizontal and then the vertical components of the forces, using the small diagram to the right.

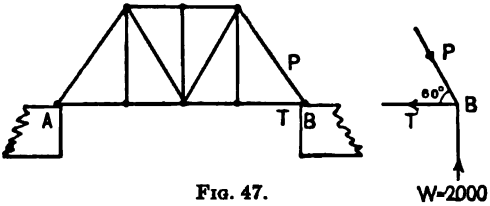

FIG. 47.

$$\Sigma(X) = -T + P\cos 60^\circ = 0.$$
$$\Sigma(Y) = W - P\sin 60^\circ = 0.$$

To obtain *T* eliminate *P* between these two equations, and obtain

$$T = W\left(\frac{\cos 60^\circ}{\sin 60^\circ}\right) = \frac{W}{\sqrt{3}} = \frac{2000}{\sqrt{3}} \text{ pounds.}$$

From the first equation one may now obtain: —

$$P = 2\,T = \frac{2\,W}{\sqrt{3}} = \frac{4000}{\sqrt{3}} \text{ pounds.}$$

The student should now solve this same problem by the graphical or geometrical method.

83. Much labor will be saved, in the solution of statical problems, by observing the one method which has been employed in each of the three preceding examples. The steps of that method are as follows: —

First clearly delimit the one body or particle which you are studying. Next decide upon the origin and axes of coördinates from which you propose to measure distances and directions. Thirdly, write the equations which express the fact that the sum of the forces and the sum of the torques acting upon this body are zero. The last step is to eliminate between these equations all the unknown quantities except the one whose value you desire.

Problems

98. An iron rod 4 ft. long weighs 10 lb., and has suspended from one end a weight of 4 lb., from the other end a weight of 8 lb. At what point must the rod be supported in order to be in equilibrium?

99. A pendulum bob weighing 2 lb. is pushed aside by the hand until the suspension string makes an angle of 30° with the vertical. What horizontal force must the hand exert in order to do this?

100. An iron bar carrying a weight of 3 lb. at one end balances when supported at a point 4 ft. from that end. If the bar weighs 2 lb. per foot length, what is the total length of the bar?

101. A man can just close a door against a strong wind by using a horizontal force of 50 lb. at a distance of 2 ft. from the hinge line. What force will be required at a distance of $2\frac{1}{2}$ ft.

102. Find the center of mass of a uniform right-angled triangle.

103. Prove that when a particle is in equilibrium under the action of three forces, the resultant moment of these three forces about any line is zero.

104. Two parallel forces, of which one is 5 pounds, have a resultant of 6 pounds, acting at a distance of 2 feet, from the larger force. Find the distance between the parallel forces. *Ans.* 12 feet.

105. A square sheet of metal 8 cm. on the side has a square of 2 cm. on the side cut out of one of its corners. Find the center of mass of the remaining figure.

106. Find the center of mass of four masses of 3, 6, 8, 4 lb., respectively, which are placed in a straight line with the following distances separating them, 6 in., 2 ft., and 3 ft., respectively.

107. Three men are carrying a log. The rear end is carried by one man; the forward end is supported over a stick, each end of which is carried by one of the other two men. Where must this stick be placed under the log to divide the load equally between the three men?

108. What couple will two equal parallel and opposite forces of 3 dynes produce when acting at opposite ends of an arm 0.1 millimeter long?

109. A string 80 inches long is stretched by a force of 5 pounds. What force must be applied at the middle of the string to depress it 1 inch? Neglect the weight of the string.

110. An I-beam is supported by two pillars, one at each end, the load on the respective pillars being 100 and 200 lb. When the pillars are shifted so that each stands 1 ft. from the end of the beam, the loads are 90 and 210 lb. respectively. Find the length of the beam.

111. Prove that when the arms of a balance are unequal, the true weight of a body may be found by observing its apparent weight, first in one scale pan, then in the other, and afterwards taking the square root of the product of their apparent weights.

112. A crane having the dimensions shown in Fig. 48 carries a load of 800 lb. Find the stretching force on the tie rod *BC* and the force of compression along the strut *BA*, neglecting the weight of the crane itself.

113. Solve the preceding problem on the assumption that the strut *BA* (Fig. 48) weighs 200 lb.

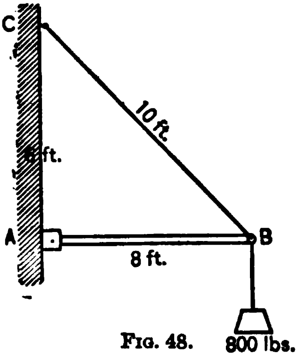

Fig. 48.

114. Explain by means of a diagram the action of the toggle-joint employed in the air brake of a locomotive.

115. A particle in equilibrium is acted upon by three forces, 4, 5, and 6. Draw the force diagram and measure with a protractor the angles between the directions of the forces.

Two Interesting Special Cases

I. *Centripetal and Centrifugal Force*

84. Every one has from his early boyhood a lively sense of the pull exerted by a body which he swings around his head at the end of a string. But curiously enough it was more than sixteen centuries after the beginning of the Christian era before any one succeeded in explaining just how the amount of this pull depends upon the length of the string and the rate of rotation. The explanation, first given by Huygens, the great Dutch physicist, in 1669, is as follows.

We have found that, in general, the force acting upon any body is to be measured by the product of the mass of the body and its acceleration; and when a particle moves in a circle with constant speed, we have found (§ 38) that it is always accelerated toward the center with an acceleration which is measured by the square of its linear speed divided by the radius of the circle.

$$a = \frac{v^2}{r} = \omega^2 r. \qquad \text{Eq. 18}$$

If now we wish to know what force F is required to keep this particle moving in a circular path, we have only to multiply the mass of the particle m by its acceleration. Thus: —

$$F = m\frac{v^2}{r} = m\omega^2 r = mv\omega. \qquad \text{Eq. 42}$$

This expression gives the force which one must exert upon a string in order to change the direction of the linear momentum mv at the rate ω.

This force is sometimes called **centripetal force** because it measures the pull of the string towards the center; it is sometimes called **centrifugal force** because it measures the pull of the stone outward. Both names are perhaps permissible; since according to Newton's third law there is always a reaction corresponding to any action. On the other hand, since the acceleration is *always toward the center*, the name "centripetal" is distinctly preferable. Note that this centripetal force is not limited to rigid bodies, but is equally true of liquids and gases and all other bodies when in rotation. Mr. John Aitken has shown (1878) by an exquisite experiment that even a limp chain when set into rapid rotation will roll along the floor as if it were a solid ring of one piece. And thus the clown at the circus gives his soft-felt hat a quasi-rigidity by whipping it into rapid rotation.

II. *The Gyroscope; Precession*

85. Is there any lad or lassie whose curiosity has not been fired by watching the axis of a spinning top swing round and round on its pivot instead of falling over in response to gravity? Is there any bicycle rider who has not experienced that curious sensation which comes from holding one of the wheels in his hand and attempting to change the direction of the axis while the wheel is spinning? The explanation of this phenomenon — for the two just mentioned are precisely the same — is strictly analogous to that of centrifugal force.

Corresponding to the linear momentum of a rigid body is the angular momentum of a rigid body, such, for instance, as that of the detached bicycle wheel, shown in Fig. 49. Such a bicycle wheel held in one hand can be set spinning rapidly with an angular velocity which may be denoted by Ω. The angular momentum of the wheel will, by analogy, then be $I\Omega$. And the torque L required to change the direction of this angular momentum at the constant rate ω, will, by the same analogy be $I\Omega\omega$; or in terms of algebra: —

$$L = I\Omega \cdot \omega. \qquad \text{Eq. 43}$$

If the spinning wheel is supported by both handles with its axle horizontal, there is no torque acting upon it, and hence no tendency for the axle to change direction; but if the wheel be set spinning about a horizontal axis and held lightly by one handle, then the weight of the wheel exerts a torque about another horizontal axis perpendicular to the axis of spin, and the torque L makes the axis of spin, therefore, rotate about a vertical axis. This rotation of the axis of spin is called **precession**. The advanced student will find this phenomenon beautifully exhibited by the earth, upon the equatorial belt of which the moon exerts a torque.

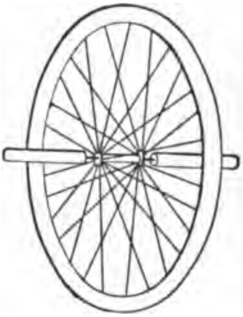

FIG. 49. — Bicycle wheel detached so as to easily show precession.

The simplest illustration is the ordinary peg top; but some of the various forms of gyroscope, usually found on sale at the expositions and toy stores, are more convenient.

Problems

116. A bicycle rider exerts a force of 10 million dynes on a pedal. Suppose the force to be at right angles to the pedal crank, and imagine the crank to be 18 centimeters long, what moment of force does the rider exert? *Ans.* $L = 180 \times 10^6$.

117. At one end of a board which is 20 ft. long is placed a stone weighing 40 lb. At the other end of the board is placed a stone weighing 90 lb. Neglecting the weight of the board, find the point at which it must be supported in order that these two stones may just balance each other. *Ans.* $6\frac{2}{13}$ ft. from end.

118. When a bicycle is being ridden through a wet or slushy street, how is it that the water collects in a little ridge at the middle of the tire?

119. In opening my pocket-knife I catch the blade at a point 8 cm. from the pin about which the blade turns; and, at this point, I exert a force of 500,000 dynes. What moment of force is required to open the knife?

120. A mass of 20 g. is revolving uniformly once in 5 sec. in a circle of 30 cm. radius. Find the centrifugal force.

121. A skater weighing 70 kg. describes a circle of 30 m. radius with a speed of 6 m. per second. Find the horizontal force tending to throw him over. Why does he incline himself toward the center of the circle?

122. How many times a minute must a mass of 3 grams revolve in a horizontal plane at the end of a string 50 centimeters long in order that the force on the string may equal 1000 dynes?

123. An angler, fishing with rod and line, hooks a fish. Will the fish appear to pull harder if the rod is a long one or a short one? Why?

124. Taking the period of the moon's revolution around the earth as 28 days and its distance from the center of the earth as 240,000 mi., calculate its acceleration.

125. A mass of 500 grams is acted upon by a force of 3500 dynes. Find the acceleration produced in the mass. Starting from rest, under this acceleration, what speed will the mass acquire in 8 seconds?

Ans. $a = 7$, $v = 56$.

126. A force of 20 dynes is able to produce an acceleration of 2 in a certain body. What is the mass of the body? *Ans.* $m = 10$ g.

127. What force would be required to change the momentum of a body from 25 to 150 in 12.5 sec.? *Ans.* $F = 10$ dynes.

128. A boy swings about his head, in a circle of 1 m. radius, a bullet whose mass is 20 g. What is the pull on the string when the bullet is making 1 revolution per second? *Ans.* $8000\pi^2$ dynes.

129. A locomotive whose mass is 100,000 kg. is rounding a curve of 400 m. radius at a speed of 10 m. per second. What is the force which the locomotive exerts upon the outer rail? This problem is similar to the preceding: the only difference is that here the flange of the wheel, or the outer rail, holds the locomotive in its circular path, while in the preceding the string holds the bullet in its circular path.

Ans. 2500 million dynes.

130. Why does a bicycle rider "lean in" as he rounds a corner?

131. How does a cyclist "right" himself by turning his front wheel toward the side to which he is falling? The centrifugal force, *i.e.* the righting force, acting upon the wheel and rider is measured by $\frac{mv^2}{r}$. This may be increased by increasing v, or by diminishing r.

132. An engine fly wheel is spinning about a horizontal axis with an angular momentum of 50,000,000 C.G.S. units. How great a torque will be required to rotate the axle of the fly wheel (still spinning) about a vertical axis at the rate of ½ radian per second? About what axis must this torque be exerted?

133. A wagon is longer than the platform of a set of hay scales. Show whether the correct weight can be obtained by weighing one end at a time and adding the two weights. Ferry's *Dynamics*, p. 181.

134. On account of the spheroidal shape of the earth the mouth of the Mississippi River is farther from the center of the earth than is the source. That is, the river flows up hill. How is this possible? Ferry's *Dynamics*.

135. If a body traverse a circumference with the same speed which it would gain in falling from a height equal to one quarter the diameter, the centrifugal force acting on the body will be equal to the weight of the body. Huygens, 1669.

II. SECOND GENERAL PROPERTY OF MATTER: GRAVITATION

86. Children at a very early age learn to look toward the floor for things that leave their hands. This indicates what is probably their first acquaintance with the great fact that the earth attracts all bodies to itself. This single fact was practically all that men knew of gravitation until the sixteenth century, when Galileo, by his researches along this line, instituted the era of modern physics.

Before the time of Galileo (1564–1642), the ideas of men concerning gravitation were simply chaotic. They had observed that some bodies, such as stones, fall very rapidly; that others, such as feathers and thin paper, fall very slowly; that heavy bodies fall with a constantly increasing speed; that lighter bodies, such as small raindrops, or feathers, fall with a practically uniform speed; that in some media certain bodies do not fall at all, as, for instance, wood in water, or iron in mercury. Galileo was the man who first harmonized these apparently contradictory facts, and described them all in a very simple way.

Up to this period nothing was known as to how far a body would fall in a given time, or as to what speed it would acquire in a given time, or as to what speed it would attain in falling a given distance.

The difficulty in getting at these facts, *i.e.* the difficulty in discovering *how* a body falls, lay partly in the circumstance that the ancients had no accurate method for measuring small intervals of time, partly in the circumstance that the speed of falling bodies, after a minute interval of time, is liable to be very rapid, and partly in the fact that the predecessors of Galileo had little or no idea of obtaining knowledge by experiment.

For measuring small intervals of time, Galileo employed an ingenious water clock, consisting of a large vessel of water with a small opening in the bottom. Just before the beginning of the observation, this opening was closed by the finger. At the beginning of the observation the finger was released and the water allowed to flow until the end of the observation, when the opening was again closed. The water flowing out was collected in a cup and weighed, the weight of the water being proportional to the duration of the observation. To prevent the attraction of the earth from giving the falling body too high a speed, he allowed the body to roll down a smooth inclined plane of small slope.

But first he satisfied himself that all kinds of matter would fall the same distance in the same time. One ball might be made of copper, another of iron; the time of descent was the same in each case. The resistance of the air was practically eliminated when bodies of the same size and nearly the same density were let fall. But, in many cases, large bodies and small bodies occupy *practically* the same time in falling from the same height. On the other hand, as every one knows, a bunch of cotton does *not* fall as rapidly, in air, as does a bullet of lead.

But Galileo's conclusions were that, in all such cases, the difference in speed was due to the resistance of the air. He regretted that he had no means of producing a vacuum, so that he might appeal to a direct experiment; accordingly he employed a series of fluids of different densities, such as mercury, water, air, and found that as the density of the fluid diminished the bodies descending in these fluids fell more nearly at the same rate, *i.e.* the same distance in the same time. **He argued, therefore, that, in the limit (*i.e.* in a vacuum), where the density is**

zero, all bodies, when starting from rest, whatever their size, shape, or composition, fall through equal distances in equal times.

How thoroughly Galileo was justified in this conclusion was proved later, about the year 1650, when the air pump was invented by Otto von Guericke. One of the first experiments to be tried in the newly discovered vacuum was to place a feather and a coin in the same tube, and see whether they would fall at the same rate in a region devoid of air. The result was such as to show that Galileo was perfectly correct in thinking that the earth impresses the same velocity upon all falling bodies, if allowance is made for the resistance of the air.

87. Galileo was now ready to begin his experiment on the inclined plane, to which we have referred above. This is very conveniently repeated in the following form (see Fig. 50): From one side of a room to another stretch a brass or steel wire of one or two millimeters diameter. The wire should have a slope of about one foot in eight or nine, and should be stretched very taut.

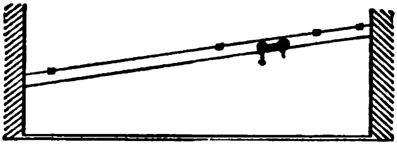

FIG. 50. — Galileo's experiment.

A small carriage to run on this wire track can be easily made by soldering to a small brass bar the straps of two small and very light wooden pulleys (see Fig. 51). The straps of the pulleys must be sawed through on the other side, so that they may slip over the wire. A small piece of metal, say a brass sphere, hooked on the lower part of the carriage, will keep the whole upright on the wire track.

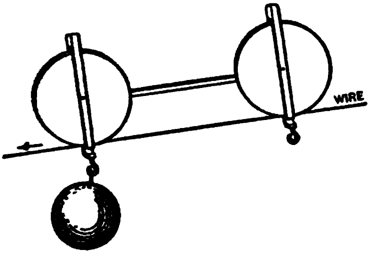

FIG. 51. — Carriage for Galileo's experiment.

A metronome, a clock, or a telegraph sounder should beat seconds in the room. Parallel to the stretched wire, and about six inches above it, should be stretched a string, on which some small indices of cardboard are strung, as shown in Fig. 50.

On the beat of the seconds pendulum the carriage is released at the upper end of the inclined track. The position of the carriage at the end of one second is marked by sliding a cardboard index along the string to the proper place. A second observer, or a very soft buffer, to catch the carriage at the lower end of the track, is convenient. Another trial enables one to mark the position of the carriage at the end of the second second, a third observation gives the position of the carriage at the end of the third second, and so on. Having thus marked off, on the inclined plane, the positions of the falling body at the ends of successive seconds, one may arrange his results as they are given in the first two columns of the following table. From these the acceleration along the plane is easily computed by means of our general equation for acceleration in terms of distance and time, Eq. 16. These results are given in column 3 and are, it will be noticed, nearly constant, thus indicating that *the acceleration along the plane does not vary, although the velocity increases rapidly.*

Time since Carriage was Released t	Total Distance passed over by Carriage in Time t	Acceleration a along Inclined Plane: computed from Formula $x = \frac{1}{2}at^2$	Acceleration in a Vertical Direction: computed from Formula $g = 12.7\,a$
1 sec.	30 cm.	60	762
2 sec.	118 cm.	59	749
3 sec.	273 cm.	61	775
4 sec.	478 cm.	60	762
5 sec.	726 cm.	58	737

Observer — E. W. Gray.

Another way of stating the result which Galileo obtained by making a large number of observations of this kind, is to say that the constancy (approximate, of course) of the figures in the third column means that the distance traversed by a body sliding freely down an inclined plane varies directly as the square of the time, a result which is graphically expressed in Fig. 52.

We must next recall that the acceleration a which we have been measuring is only one component of the total acceleration, namely, that component which acts along the inclined plane. Knowing this component, BD in Fig. 53, and knowing that the total acceleration of gravity, which we shall hereafter denote by g, is vertical, we can easily find the value of g. For the triangles BCD and AOX are similar; and hence

$$\frac{g}{a} = \frac{AX}{AO} = 12.7 \left\{ \begin{array}{l} \text{in the experiment} \\ \text{recorded above.} \end{array} \right.$$

The values computed from a in this manner are given in the fourth column of the table above.

In this manner Galileo inferred that this particular acceleration, namely, the acceleration produced, upon a freely falling body, by the earth, is also a constant. This is a capital discovery; a fact with whose consequences we are so familiar that we are perhaps a trifle blind as to its importance.

As every one knows, gravity accelerates a body most rapidly along the vertical direction, and does not accelerate a body at all in a horizontal direction. Indeed, all that we mean by a "horizontal" direction is one in which the acceleration of gravity has no component.

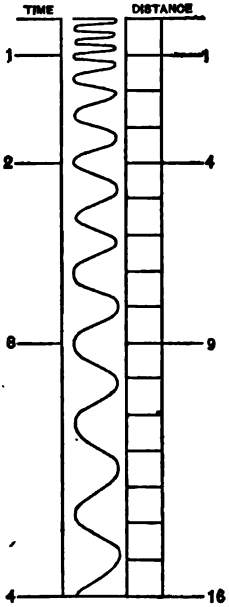

FIG. 52.

This device of Galileo's is, therefore, competent not only to show that the acceleration of gravity is a constant, but also to give us the numerical value of this constant. Since, however, no allowance has here been made for friction, or for the sag of the wire, or for the fact that a part of the body is rolling instead of sliding, the results are about 25 per cent too small.

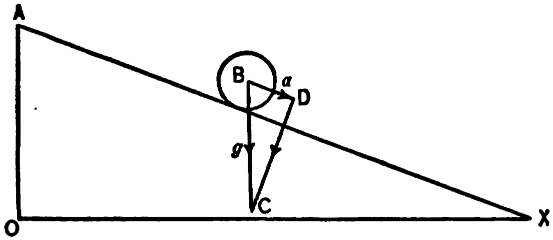

FIG. 53.

How nearly constant the acceleration of gravity is at various parts of the earth's surface will be shown by the following table of values derived from pendulum experiments.

PLACE	ALTITUDE IN METERS	OBSERVED VALUE OF GRAVITY IN CM./SEC.^{2}
Key West, Fla.	1	978.954
Charlottesville, Va.	166	979.938
Princeton, N.J.	64	980.178
New York, N.Y.	38	980.267
Cambridge, Mass.	14	980.398
Chicago, Ill.	182	980.278
Madison, Wis.	270	980.365
Colorado Springs, Colo. . . .	1841	979.490
Pikes Peak, Colo.	4293	978.954
San Francisco, Cal.	114	979.965
Seattle, Wash.	74	980.725
Minneapolis, Minn.	256	980.597
Washington, D.C. (Bureau of Standards)	103	980.095

U. S. Coast Survey. Special Publication No. 12, p. 10. (Wash. 1912.)

88. Having found that the acceleration of gravity is constant, Galileo at once made the following inference; namely, that the speed at the end of any interval of time is proportional to the first power of the time. This consequence, we have already seen (Eq. 15, p. 37), follows in *all* cases of constant acceleration. The inference may be tabulated as follows: —

Relation between Time of Fall and Speed Acquired

TIME	SPEED
1	g
2	$2g$
3	$3g$
4	$4g$
5	$5g$
6	$6g$
etc.	

By definition, g is the number by which the speed of a freely falling body is changed in each second. Consequently, in t seconds, the total *change* in speed will be gt; or, if the body starts from rest, the results in the preceding table are expressed algebraically as follows: —

$$S = gt.$$

If the falling body has, at time $t = 0$, a speed of S_0, our expression for the *actual speed* at any instant t becomes

$$S = S_0 + gt.$$

Summary of Laws of Freely Falling Bodies

89. These are exactly the laws of a uniformly accelerated particle which we have studied in Chapter I, only here the numerical value of the acceleration g is practically always 981 centimeter/second^2 units.

Relation between Speed and Time: —

$$S = S_0 + gt. \qquad \text{Eq. 15}$$

Relation between Distance and Time: —

$$x = S_0t + \tfrac{1}{2}\,gt^2. \qquad \text{Eq. 16}$$

Relation between Distance and Speed: —

$$S^2 = S_0^2 + 2\,gx. \qquad \text{Eq. 17}$$

90. But when a body is *not* falling freely, one obtains its acceleration by the general method of first finding the resultant of all the forces acting upon it and then dividing this resultant by the mass of the body.

Thus if two masses m_1 and m_2 be strung over a light frictionless pulley by means of a thin string, as shown in Fig. 54, the mass which falls is not a "freely falling body." The same may be said of a body which rolls or slides down an inclined plane. Referring to Fig. 54 which represents the essential feature of Atwood's machine (a classical instrument devised for the purpose of diluting gravity, so to speak), we proceed to write the equations of motion for the two bodies m_1 and m_2 there represented.

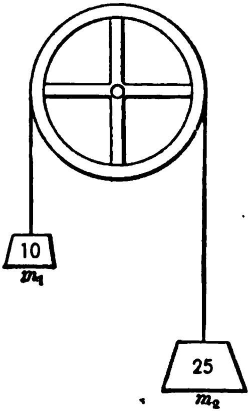

FIG. 54.

The data of the problem are, say, the masses of the bodies and the acceleration of gravity g. It is required to find the acceleration a of the system and the force T with which the string is stretched.

Consider first the body m_1; all the forces acting upon it are: —

(i) the tension T of the string acting upward,
(ii) its weight m_1g acting downward; and
(iii) the reaction of the mass m_1 against its upward acceleration, which amounts to m_1a, acting downward.

These three are the only forces acting upon m_1, and they all act along the same straight line; hence its equation of motion is: —

$$T - m_1g - m_1a = 0.$$

Considering now the mass m_2, the forces acting upon it are: —

(i) the tension T, acting upward — the tension being the same in every part of the string,
(ii) the weight m_2g acting downward, and
(iii) the reaction of the mass m_2 acting upward against its downward acceleration a; the acceleration for each of the two bodies being the same in amount but opposite in sense.

Since these are the only forces acting upon m_2, and since they all act in the same straight line, the equation of motion for m_2 is

$$T - m_2g + m_2a = 0.$$

These two equations, each involving the two unknown quantities, are competent to give us the value of each of them. Eliminating between them first T and then a, we obtain: —

$$a = \frac{m_2 - m_1}{m_2 + m_1} \cdot g, \qquad \text{Eq. 44}$$

and

$$T = \frac{2\,m_1m_2}{m_1 + m_2} \cdot g. \qquad \text{Eq. 45}$$

A method identical with this gives one the solution of practically any problem of bodies falling over frictionless inclined planes or pulleys, or combinations of these two. A little later we shall learn how to take friction into account.

91. On the other hand a body may be falling freely and also have an initial velocity in a horizontal direction, as, for instance, a bullet fired from a gun which is held horizontal. If we take the muzzle of the gun as the origin of coördinates, the equations of motion for the bullet are

$$x = Vt, \qquad \text{Eq. 5}$$

and

$$y = -\tfrac{1}{2}gt^2. \qquad \text{Eq. 16}$$

Between these two equations one may eliminate t and thus obtain the path of the bullet, namely: —

$$y = -\frac{g}{2V^2}x^2, \qquad \text{Eq. 46}$$

an equation which the advanced student will recognize as that of a parabola.

If the initial velocity is not horizontal, then one must substitute for V the horizontal component of the initial velocity; likewise the vertical component of the initial velocity will enter into the equation for the vertical motion. But these are matters which, like the resistance of the air, must be left for later study.

Problems on Gravitation

In considering the following questions, the resistance of the air is to be neglected and the acceleration of gravity is to be taken as 981, using the centimeter and second as units.

136. A stone let fall from a window 20 m. high will strike the ground with what speed? *Ans.* 1980 cm. per second.

137. A boy throws a baseball to a height of 39 m. With what speed does the ball leave his hands? With what speed will it return to his hands? *Ans.* 27.66 m. per second.

138. A tuning fork makes 250 vibrations a second. How many vibrations will it make while it falls from rest a distance of 10 cm.? How many while falling 20 cm.? In the latter case, compare the number of vibrations performed in the first and the second decimeters of the fall. *Ans.* 35.7 vibrations; 50.4 vibrations.

139. A gun is held horizontal at a distance of 300 cm. above the ground. How long after the gun is fired until the bullet reaches the ground? During this interval, how far will it travel in a horizontal direction if it leaves the muzzle of the gun with a speed of 500 m. a second? *Ans.* 0.78 sec.; 390 m.

140. A block is allowed to slide down a smooth inclined plane of 30° slope. What speed would it acquire at the end of 4 sec. if its motion were not opposed by friction? *Ans.* 1962 cm. per second.

141. Determine the acceleration and the tension in each of the following diagrams: — *Shearer*, 157.

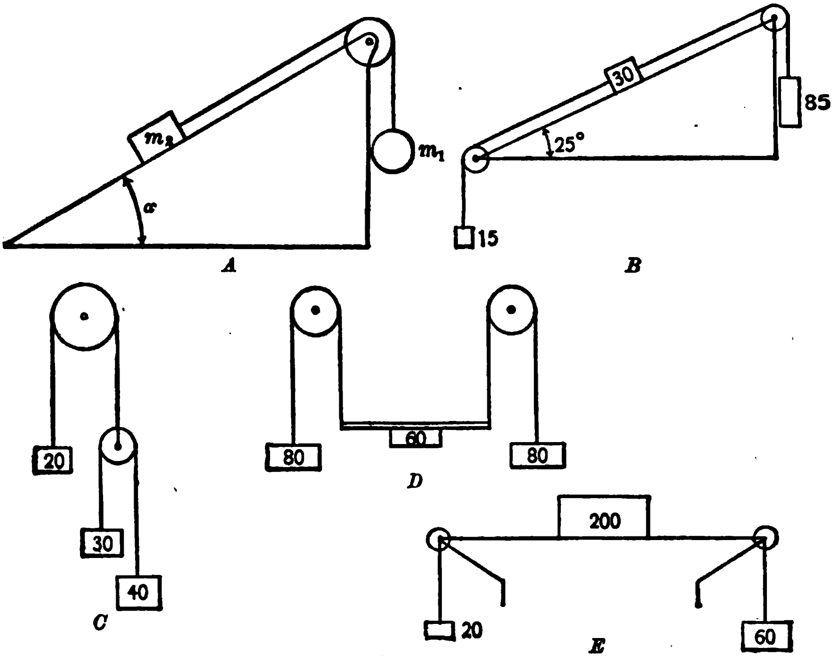

Fig. 55.

142. A man who weighs 150 lb. steps into an elevator just as it is starting up with an acceleration of 200 C.G.S. units. By how much will his weight appear to increase?

143. Starting from rest, an automobile travels 400 ft. during the first ten seconds. Suppose the engine to exert a constant pull, what must be the inclination of a hill in order that the machine will ascend it at constant speed?

144. Two weights whose sum is 4 lb. are strung over a massless and frictionless pulley; the difference between the two weights is 2 oz. Find the acceleration which gravity will produce in the system.

145. Find the time required for a body to slide from rest down an inclined plane whose length is 32 ft. and which makes an angle of 30° with the horizontal.

146. An ore bucket descending a mine shaft breaks the cable at an instant when the bucket is moving with a speed 20 ft. per second and is at a height of 240 ft. from the bottom of the mine. How long after the break will the bucket strike the bottom?

THE PENDULUM

92. The first *accurate* information concerning the *amount of the earth's acceleration* was obtained by Huygens (1629–1695), the inventor of the pendulum clock.

Galileo had already rediscovered what had previously been known and what is now familiar to most boys, that a pendulum takes practically the same time to vibrate, whether the angle through which it swings be large or small. **In other words, he showed that the period of a pendulum does not depend upon its amplitude.** Mersenne, in 1644, had determined the length of a pendulum which would beat seconds. But Huygens went a step farther, and showed how this period *does* depend upon the acceleration of gravity, and how a pendulum may be used to measure this acceleration with accuracy. His method is as follows.

DETERMINATION OF THE PERIOD OF THE SIMPLE PENDULUM

93. DEFINITION. **A heavy particle suspended from a fixed point by means of a thread whose mass is negligible is called a simple pendulum.**

When such a pendulum is pulled aside through an angle θ (Fig. 56), the particle is urged back toward its position of equilibrium with an acceleration which may be easily computed as follows: —

The simple pendulum is represented by the line OB, the heavy particle by B, its position of equilibrium by D.

Let the vertical line $\overline{BC}$ represent the acceleration of gravity. This may be replaced by two components: one, $\overline{AC}$ acting along the direction of the string, balanced by the pull of the string toward the support O, and hence not affecting the particle at B, the other component $\overline{BA}$ acting practically at right angles to the string and urging the particle back into its position of equilibrium. And since the triangles ABC and OBD are similar, this latter acceleration a bears to g the same ratio that

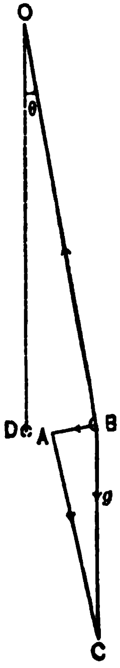

FIG. 56.

$\overline{BA}$ bears to $\overline{BC}$, and hence

$$\frac{a}{g} = \frac{\overline{BA}}{\overline{BC}} = -\sin\theta.$$

The acceleration represented by $\overline{BA}$ we call negative because it is toward the left.

But when the angle θ is small, we may replace $\sin\theta$ by θ without appreciable error. Hence $a = -g\theta$, where θ is, as usual in physics, measured in radians.

Let l denote the constant length of the pendulum, $l\theta$ will then be the linear displacement of the particle from its position of rest. This displacement we call positive because it is toward the right. Accordingly, we have

$$a = -g\theta = \text{acceleration},$$

and $$x = l\theta = \text{displacement}.$$

Eliminating θ, we have $a = -\left(\frac{g}{l}\right)x$,

where $a \propto x$; for g and l are constants.

That is, the acceleration varies as the displacement, and is opposite in sense; since when the particle is displaced to the right the acceleration is to the left, and *vice versa*.

This motion therefore satisfies the criterion for simple harmonic motion. And hence we know that the particle B moves to and fro with a S.H.M. in a path which, for small amplitudes, is practically straight.

Since the motion of the pendulum is simple harmonic, we may apply to it at once the principles of S.H.M. and write for the period T

$$T = 2\pi\sqrt{\frac{\text{Linear displacement}}{-\text{Linear acceleration}}} \qquad \text{Eq. 23}$$

$$= 2\pi\sqrt{\frac{l\theta}{g\sin\theta}}$$

$$= 2\pi\sqrt{\frac{l}{g}}, \text{ approximately.} \qquad \text{Eq. 24}$$

This simple expression enables one to compute the value of g. For T and l are easily measured; and then we have for the computation of g the following laboratory equation: —

$$g = 4\pi^2\frac{l}{T^2},$$

where it is to be noted that T is the time of one "complete vibration," *i.e.* of a swing "to *and* fro," not "to *or* fro."

DIGRESSION ON CENTER OF GRAVITY

94. One would naturally next proceed to a discussion of the actual physical pendulum, such as is employed in clocks and in the laboratory. But before this can be intelligently taken up, we must add to our vocabulary another term, namely, **center of gravity.** Nothing is easier than to balance a table spoon, a hammer, or walking stick on the tip of one's finger and thus discover experimentally the position of **the point through which the total weight of the body — considered as a single vertical force — acts.** And this is all that is meant by the center of gravity.

For any body of moderate size, the direction of gravity is the same for every part of the body. The problem of finding the center of gravity is therefore the problem of locating the resultant of a set of parallel forces; for the weight of a body is the sum of the weights of the various parts of the body. Let us for instance string two heavy masses M_1 and M_2 (Fig. 57), upon a light rod of negligible weight. We then know, from the rule for the addition of forces, that the weight of the system, R, is the sum of the weights, W_1 and W_2, of the two masses.

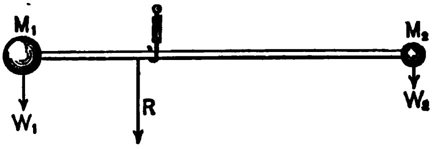

FIG. 57.

A spring balance attached to any part of the system whatsoever will indicate the same weight; but there is only one point on the bar at which we can place the balance and have the bar remain horizontal. When once we have placed the balance at this point, we have discovered the point through which the resultant R acts.

For bodies such as one ordinarily meets in the factory or the laboratory, the problem of finding the center of gravity is the same as that of finding the center of inertia, which has already, § 63, been solved. One has merely to multiply each member of Eq. 28 B by g, which gives

$$Mg\,\bar{x} = \Sigma(mgx),$$

and two similar equations for $\bar{y}$ and $\bar{z}$. Now Mg is the total weight of the body; and mgx is the moment of force of the particle m. These equations, therefore, mean that the sum of all the moments of force tending to rotate the body are equivalent to the torque of the total weight of the body acting at the center of mass $(\bar{x}, \bar{y}, \bar{z})$. But such a point is, by definition, also the center of gravity.

CENTER OF GRAVITY BY EXPERIMENT

95. If a body is straight and slender, as in the case of a wire or a walking stick, we may locate the center of mass at once by balancing the body over a knife-edge, thus discovering the point about which the moment of force is zero.

But if the body be a lamina, *i.e.* a thin flat plate, we may infer from symmetry that the center of mass lies somewhere in the plane of the body. And by balancing the body about a knife-edge we may determine in this plane a straight line $\overline{AB}$ (Fig. 58), which contains the center of mass. If, now, we choose another position for the knife-edge, say $\overline{A'B'}$, and balance the body on it again, we shall locate the center of mass on another straight line.

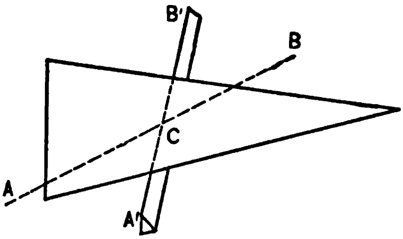

FIG. 58. — Determination of center of mass by experiment.

Since now we know that the center of mass lies on each of these two straight lines, we know that it must lie at their intersection,

C. We shall hereafter frequently denote center of mass by the letters *c.m.*

In the case of an extended body, *i.e.* one for which each of the three dimensions is considerable, we should need to balance the body about three different lines; for in this case, each balance would determine one vertical *plane* in which the *c.m.* lies; and three such planes would be required to locate, by their intersection, the *c.m.* of the extended body.

In many cases of this kind the *c.m.* can be located by symmetry, as in the case of a sphere or a parallelopipedon or a cylinder; but only where the density of the body is uniform or symmetrical. In the case of "loaded" dice, for instance, the center of mass would *not* in general coincide with the center of figure.

It is a general principle of dynamics, first formulated by Galileo, that the center of mass always seeks the lowest possible position in space. This we now recognize as a special case of a still more general principle, — that the potential energy of a body tends to become a minimum.

But however this principle is stated, it suggests still another slightly different method for getting the *c.m.* of a body extended in all three dimensions; namely, instead of balancing on a knife-edge, suspend the body from some one *point* by a thread; the *c.m.* will seek its lowest position, and will, therefore, lie in the straight line determined by the thread.

If now the body be suspended from another point, a second line will be determined in which also the *c.m.* lies. The intersection of these two lines must occur, else the body would have *two* centers of mass. This intersection must therefore be the *c.m.* In this way two observations, instead of three, suffice to locate the *c.m.* of a three-dimensional body such as a chair.

From this digression, we return now to the

DETERMINATION OF THE PERIOD OF THE PHYSICAL PENDULUM

96. Imagine any rigid body which may be perfectly irregular in shape, or more or less regular, as shown in Fig. 59. Let it be suspended at the point S so that it swings freely in a vertical plane. Such a body is called a physical or compound pendulum.

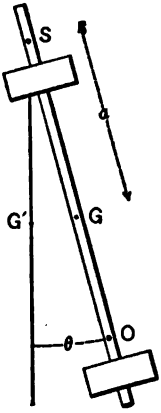

FIG. 59.

Denote the *c.m.* by G and let SG' indicate the position in which the line SG comes to rest. Denote the distance of G from S by a and the deflection of the pendulum at any instant by θ. If the mass of this rotating body is M, the only force acting upon it is its weight Mg acting through the *c.m.*, G. This force exerts a torque $Mg \cdot GG'$, equal to $Mg \cdot a\theta$, tending to restore the pendulum to its position of rest. And since this torque is proportional to the displacement and opposite in sense, the motion will be simple harmonic. We may, therefore, at once write the period, by analogy with Eq. 24:—

$$T = 2\pi\sqrt{\frac{\text{Angular displacement}}{-\text{ Angular acceleration}}} = 2\pi\sqrt{\frac{\theta}{-A}}.$$

But, Eq. 36, § 71:— $A = \dfrac{\text{Moment of force}}{\text{Moment of inertia}} = \dfrac{Mga\theta}{I}.$

Hence $$T = 2\pi\sqrt{\frac{I}{Mg \cdot a}}.$$ Period of physical pendulum. Eq. 47

If a simple pendulum of length l is to have the same period, it will be necessary that

$$l = \frac{I}{Ma}.$$ Eq. 48

This value of l is known as the equivalent length of the physical pendulum.

REVERSIBLE PENDULUM

97. The advanced student will find in the reversible pendulum a beautiful device by which the length of the equivalent simple pendulum can be directly measured in the laboratory. Here one can only say that if the pendulum vibrates with equal periods about two points S and O (Fig. 59), which include the center of gravity G in the straight line joining them, then the distance SO is the length of the equivalent simple pendulum, a fact discovered by Huygens in 1673.

Problems

147. Shortly after the invention of the pendulum clock, one was sent from Paris, where it kept correct time, to Cayenne, in South America. In the latter place the clock lost $2\frac{1}{2}$ minutes daily. The length of the pendulum was adjusted so that the clock kept correct time in Cayenne; and in this condition the instrument was sent back to Paris, where it now *gained* $2\frac{1}{2}$ minutes daily. Explain this phenomenon.

148. It is well known that metals expand on heating. What will be the effects of the cold weather in winter and the hot weather of summer upon the rate of a clock having a brass pendulum?

149. A clock having a pendulum 60 cm. long keeps correct time. The pendulum is lengthened to 60.5 cm. How many seconds will the clock lose each day? *Ans.* 362 sec.

150. The period of a given pendulum at Denver is exactly 1 sec. The period of the same pendulum at Chicago is 0.9995 sec. The acceleration of gravity at Denver is 979 C.G.S. units. What is its value at Chicago? *Ans.* 979.97 C.G.S. units.

151. There are 454 g. in 1 lb. What is the force, in dynes, with which the earth attracts a mass of 1 lb., at a place where $g = 980$ C.G.S. units? *Ans.* 444,920 dynes.

152. Find the weight of 7 kg. in dynes; $g = 960$.
Ans. 6,720,000 dynes.

153. A spring balance is made and graduated at a town where $g = 983$. At another town, where $g = 979$, the spring balance is used in a meat market. Who gains by this error, the man who buys the meat or the man who sells? *Ans.* The buyer.

154. From the basket of a balloon hangs a spring balance. This balance carries a mass of 100 g. The balloon ascends with an acceleration of 220 C.G.S. units. What will be the apparent weight of the 100 g. during the ascent? *Ans.* 122.4 g.

155. A ball thrown vertically upward is caught by the thrower 5 sec. later. Neglect the resistance of the air, and compute the height to which the ball was thrown. *Ans.* 30.66 m.

156. The radius of the earth is 636,000,000 cm. At what linear speed must a body move along the surface of the earth to lose all apparent weight?

Ans. Eq. 18, $\frac{v^2}{636 \times 10^6} = 981$, $v = 7899$ m. per second.

157. In which of the following cases does the *c.m.* lie inside, and which outside the body: a teacup, a straight copper wire, the same wire bent into the arc of a circle, a hoop, a silver half-dollar, a baseball, a football?

158. Locate, by geometry, the *c.m.* of a triangular sheet of brass. If a corner of this triangle be clipped off along a line parallel to the opposite side, the *c.m.* will be displaced along what line?

159. A meter scale (the mass of which is negligible) is divided into decimeters, and at the end of the first and eighth decimeters are attached masses of 1 and 9 k. respectively. Where is the *c.m.* of the system? *Ans.* At the end of 73d cm. on the scale.

UNIVERSAL GRAVITATION

98. Hitherto we have been considering a special case of gravitation, namely, the acceleration which the earth produces upon small bodies near its surface.

Newton's discovery of the law which governs the general case of gravitation must always be reckoned one of the most brilliant achievements of the human mind. The circumstances which led up to this discovery are something like the following: —

Copernicus (1473–1543) had suggested, on grounds of simplicity and symmetry, that the sun and not the earth was the center of what we now call the solar system; but neither he nor any of his successors up to the time of Newton had shown what it was that kept the planets in motion about the sun.

Kepler (1571–1620) in the meanwhile had succeeded in describing, with a high degree of accuracy, the motion of the planets about the sun. His description is as follows: —

(i) Each planet moves in a nearly circular ellipse, with the sun in one focus,

(ii) at such a speed that the line from the center of the sun to the center of the planet sweeps out equal areas in equal times,

(iii) while the square of the period, measured in any unit of time, bears to the cube of the planet's distance from the sun a ratio which is constant for each member of the solar system.

Next came Newton (1642–1727), who, building upon the dynamics of Galileo, argued from Kepler's second statement that *no* force from behind is necessary to drive the planet along its orbit, but only a deflecting force acting toward the sun, a force which is continually balanced by the centrifugal reaction of the planet. His next step was to infer from Kepler's first statement that the value of this deflecting force varies inversely as the square of the distance which separates the planet from the sun. While from Kepler's third statement he inferred that the force of attraction between the planet and the sun is proportional to the mass of the planet, the constant of proportionality being the same for each of the planets. Following this, he concluded that, unless there be something more peculiar than mere size in the mass of the sun, this attraction must be proportional also to the mass of the sun.

Before summarizing these results of Newton, let us recall the fact that the dimensions of each of these planets are so small compared with their distances from the sun that they may be treated as particles. Newton showed by mathematical computation that if we suppose the sun to attract each of the planets with a force which varies directly as the mass of the planet, and inversely as the square of the distance between the planet and the sun, this supposition will explain, in the most satisfactory manner, all the motions of the planets. Going one step farther, he generalizes and supposes that *any* two particles, or masses which may be considered as particles, m and m', attract one another with a force depending only on the product of the masses mm', and the square of their distance r. Expressing this in the language of algebra: —

$$F = G\frac{mm'}{r^2}\text{ dynes,} \qquad \text{Eq. 49}$$

where G is a constant whose numerical value is very nearly 6.66×10^{-8}. The physical meaning of this constant, G, is easily seen by imagining a small lead shot of one gram mass to attract an equal mass at a distance of one centimeter. Then $m = m' = r = 1$; and $F = G$. Or, G is the force of attraction which one gram exerts upon another at a distance of one centimeter. Remembering that this force is only $\frac{1}{15000000}$ of a dyne, one realizes what a feeble force gravity is in comparison with most others.

The clever methods by which Cavendish, Boys, Poynting, and others have determined the value of this "constant of gravitation" G is a matter which must be postponed for advanced study.

Newton also proved that this supposition would explain the circular path of the moon. And by an elegant demonstration, he showed that ordinary bodies at the surface of the earth are heavy because the earth attracts them just as it does the moon. The whole earth acts as if its mass were concentrated at its center, so that bodies at the surface of the earth act as if attracted by a particle whose mass is that of the earth, and whose distance is 4000 miles. What we call the weight of a body thus becomes a special case of the forces described in Eq. 49, which is known as Newton's Law of Universal Gravitation. This description of gravitation is absolutely unique among the laws of nature; for it includes apparently the whole universe and is unlimited by any condition of any kind. As to the accuracy with which the attraction varies according to the inverse square, Professor E. W. Brown of Yale has recently shown, from researches on the motion of the moon, that the exponent of r in Eq. 49 does not differ from 2 by a fraction greater than $\frac{1}{400000000}$. **Let the student bear in mind, however, that this law simply describes the behavior of a mass *m* in the presence of another mass *m'*, and, as Newton himself clearly pointed out, does not explain gravitation even in the slightest degree.** *Why* bodies attract one another is as much a mystery as ever. *How* bodies attract one another Newton discovered and described in the most perfect manner.

THE BEAM BALANCE

99. In this instrument the gravitational attraction of the earth is employed for the purpose of comparing masses.

The essential features of the ordinary balance are: —

1. A rigid beam carrying three knife-edges, one near each end, and a third, as nearly as possible, midway between these two.
2. Two scale pans, supported, one on each knife-edge, at the end.
3. The beam when carrying the knife-edges and the pans must be so adjusted as to come to rest in a definite position, usually horizontal. This is accomplished by so placing the knife-edges and so distributing the mass in the beam that

the *c.m.* of the whole system lies just *below* the central knife-edge. Why would not neutral equilibrium answer quite as well as stable?

4. The whole system must be sensitive to a small moment of force. This is accomplished by making the distance from the central knife-edge to the *c.m.* very small. How does this secure the desired result?

5. A pointer, or index of some kind, generally fastened to the beam, for the purpose of indicating the position of equilibrium.

PRINCIPLE OF THE BALANCE

100. In physics the balance is employed to compare two masses by comparing their weights. This is done as follows: The position of equilibrium is observed before any mass has been added to either pan, that is, when the pans are empty. The body whose mass is required, having been placed in one pan, standard masses are then placed in the other pan until the moments of force which their weights exert on the beam are just equal. Since these two masses tend to rotate the beam in opposite senses with equal moments, the total moment will be zero. This equality and opposition of moments is indicated by the pointer coming back to the same position of equilibrium which it had when the pans were empty. **Thus the immediate purpose of the balance is simply to tell us when we have equal moments of force.**

Now if, in addition, the arms of the balance (that is, the respective distances between central and end knife-edges) are equal, we know that the *weights* of the two bodies in their respective pans are equal.

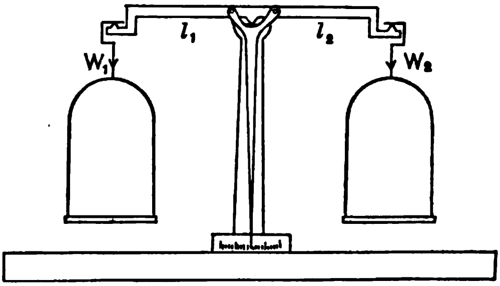

FIG. 60.

Let l_1 and l_2 (Fig. 60) indicate the length of the arms, W_1 and W_2 the weights of the two masses M_1 and M_2. Then, by proper adjustment of the masses, we make

$$W_1l_1 = -W_2l_2,$$

so that the total moment L is zero.

$$L = W_1l_1 + W_2l_2 = 0.$$

If, now, $l_1 = -l_2,$

then $W_1 = W_2,$

or $M_1g_1 = M_2g_2.$

But Newton and Galileo have shown that, **at any one point on the earth's surface, the acceleration of gravity is the same for all bodies.** Expressed in a more useful form, *i.e.* in an algebraic form, this would read: —

$$g_1 = g_2,$$

and hence $M_1 = M_2.$

So that really the equality of masses is an inference from the equality of moments. **And this inference would not be allowable if the earth acted as a magnet, attracting different bodies with a force depending upon the kind of material as well as the mass of material in them.**

The earth, indeed, is a great magnet, directing the compass needle, as we all know; but, as we shall see later, it does not attract the needle as a whole. It simply *rotates* the needle into one position, but does not *translate* it; since the lines of magnetic force at any one point on the earth's surface are practically parallel.

Aside from the Tower of Pisa experiments by Galileo, and from the consistency of the results obtained by the balance, Newton furnished conclusive evidence by the use of a hollow pendulum that gravity acts equally upon all substances without regard to their chemical composition. For a description of this experiment see the *Principia*, Book III, Prop. VI.

Problems

160. At the surface of the sun the acceleration of a falling body is 26 times as great as at the surface of the earth. A man who weighs 150 lb. at the earth would weigh how much at the sun?

161. The rotation period of Jupiter is 10 hr.; its radius is 11 times that of the earth, and its gravitational acceleration 2.6 times that of the earth. Compute what fraction of its weight a body would lose in being carried from the north pole to the equator.

162. The earth is nearly 3 times as far away from the sun as the planet Venus. If the earth were transferred to the orbit of Venus and made to move in it, what length of year would be necessary in order that the centrifugal force should just balance the sun's attraction? Assume the orbit circular. See Eq. 42, § 84.

163. Prove that the "dimensions" of the "gravitational constant" are $M^{-1}L^3T^{-2}$.

164. If a hole were drilled through the earth along any diameter, the attraction of the earth on any body placed in this tube would vary directly as the distance of the body from the center of the earth. Neglecting friction, what kind of motion would be executed by a body dropped into this tube at the surface of the earth?

165. In the preceding problem prove that a body would fall through the earth, so to speak, in approximately 42½ minutes.

166. A fly is inclosed in a corked bottle. The bottle and fly are weighed with great accuracy (i) when the fly is standing still on the bottom of the bottle and (ii) while the fly is flying about inside the bottle but without touching the glass. How will these two weights compare?

167. Sound travels at the constant rate of 1100 ft. a second. At what height above the surface of the earth must a body be let fall so as to reach the earth at the same instant as the sound of a pistol shot fired at the same place and time from which the falling body starts?

III. Matter as a Vehicle of Energy

101. Work is something of which every one has, or ought to have, a fairly definite idea. The farmer knows that it requires twice as much work to plow a two-acre field as to plow a one-acre field. For while the horses exert no more force on the plow in the first case than in the second, the total length of furrow is twice as great in the first case as in the second. In like manner, we all know that it requires twice as much work to carry 2000 bricks to the top of a house as to carry 1000 bricks to the same height. For while the distance is the same in each case, the weight of 2000 bricks is twice that of 1000 bricks.

In general, the quantity of work done depends upon two factors, and upon two factors only: (1) the force exerted, and (2) the distance through which this force is exerted.

DEFINITION OF WORK

102. Consequently, the term **work** is employed in physics to denote one definite quantity, viz., **the product of a force multiplied by the distance through which it is exerted, both distance and force being measured in the same direction.**

A mass *B* (Fig. 61) is attracted by the earth with a force of 1000 dynes. What work will be required to roll this mass up an inclined plane whose length $\overline{OA}$ is 24 centimeters? If work were defined simply as "the product of force by distance," the answer to the problem would be $1000 \times 24 = 24{,}000$ units of work. But using the proper definition, we observe that since here the force, *i.e.* the weight of the body *B*, is vertical, we must measure the distance also in a vertical direction.

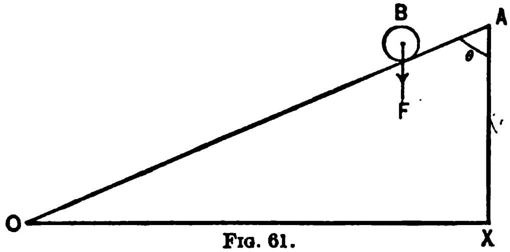

FIG. 61.

Evidently, therefore, we must know the height $\overline{AX}$ before we can solve the problem at all. In other words, we must know how steep the inclined plane is before we can estimate the work required to carry the body *B* through the distance $\overline{OA}$.

Suppose the height $\overline{AX}$ to be 12 centimeters. The work in this particular case will then amount to 1000×12 units of work.

So, in general, to obtain the work done we have merely to multiply the force exerted by the distance resolved along the direction of the force; or, what amounts to the same thing, multiply the distance by the force resolved along the direction of the displacement.

Let us denote the work by W, the force by F, and the displacement by l. Then the defining equation for work may be written as follows: —

$$W = Fl \cos \theta = Fx, \qquad \text{Eq. 50}$$

where θ is the angle between the directions of the two vectors, F and l; and x is the component of l in the direction of F. This algebraic definition is of course identical with the one given above in words.

In what precedes it has been assumed that the force F has the same value at every point of the distance through which it is exerted. In many cases, however, — as, for example, in stretching the spring of a butcher's balance — the force varies with the distance. In this particular case, the force varies directly as the first power of the elongation; or $F = kx$, where k is a constant of proportionality, whose value is that of the force required to produce unit elongation. It is the average value of F which one must employ in Eq. 50. Here F varies uniformly from 0 to kx; its average value is therefore $\frac{kx}{2}$. Accordingly the work done in elongating such a spring by an amount x is $\frac{kx^2}{2}$.

103. Rotational Analogue. The student will find it an interesting exercise to prove that, in like manner, the work done by a torque L acting about any given axis is measured by the product of the torque, and the angular displacement θ about this same axis. Thus

$$W = L\theta, \qquad \text{Eq. 51}$$

an expression which is convenient in computing the amount of work done in winding a watch, turning a dynamo, etc.

UNITS OF WORK: THE ERG AND JOULE

104. Since now the unit of force is the dyne, and the unit of distance the centimeter, **the only consistent unit of work is that amount of work which is done when a force of one dyne is exerted through a distance of one centimeter.**

This unit is of such frequent occurrence in physics that it has been given a special name. It is called an **Erg**, which is simply the Greek form of our English word "work."

A larger, and for many purposes, therefore, more convenient, unit is the **joule**, which is defined as 10^7 ergs. This unit will be found especially convenient when we come to consider the work done by an electric current.

The engineer's unit of work is, of course, the work done in exerting a force of 1 pound through a distance of 1 foot. It is called the **foot-pound**, and is equal to 1.356 joules.

Problems

168. How many ergs of work will be done in exerting a force of 6 dynes through a distance of 18 centimeters? *Ans.* 108 ergs.

169. What work will be required to hoist 10 kg. of water against an acceleration of 981 units from a well whose depth is 12 m.? *Ans.* 1.1772 joules.

170. Suppose forces to be measured in "pounds weight" and distances in feet; how many foot-pounds of work are required to carry 100 lb. of brick to the top of a building 42 ft. high? *Ans.* 4200 foot-pounds.

171. If the weight of one kilogram be taken as the unit of force and the meter as the unit of length, what will be the unit of work? *Ans.* One kilogrammeter.

172. If 2640 foot-pounds of work are required to propel a bicycle for a distance of one mile, what is the average force required to propel the wheel?

173. A magnet attracts a piece of iron with a force of 8000 dynes. How much work will be performed by the magnet in moving the iron through a distance of 0.15 millimeter?

174. A force of one million dynes is required to operate a "double-action" bicycle pump. If the length of stroke is 12 cm., how much work will be required to make 50 complete strokes of the pump?

ENERGY

105. We shall see later that a body may be able to do work in consequence of

1. Its position.
2. Its speed (angular or linear).
3. Its temperature.
4. Its electrical condition.
5. Its chemical composition.

Etc.

Energy may be defined therefore as **the capacity of doing work,** and a system is said to possess energy when it can do work.

In view of the fact that a body or system may possess a large store of energy which is not available for work, the following definition is perhaps better.

Energy is that property of the system which diminishes, when the system does work, by an amount equal to the work so done.

Since the work which a system can do is the measure of its energy, **the same unit is employed for energy as for work, viz., the erg.**

If we were to attempt to study all the forms of energy which a system may possess, we should find ourselves at once in the midst of the entire subject of physics. For the idea of energy is the only one except those of space and time which is common to all the different branches of physics. Accordingly, we shall here consider only the energy which a mechanical system may have by virtue of

1. The relative position of its parts; and
2. The relative speed of its parts.

Every one is familiar with many instances of these two forms of energy.

The boy who starts at the top of a hill on a bicycle knows that his energy of position is such that, without pedaling, it will carry him to the bottom of the hill and some distance beyond. If, coming in the opposite direction, he approaches the foot of the hill with sufficient speed, the energy due to his motion may be ample to carry him to the top without any pedaling. In this case the system consists of the bicyclist and the earth.

An uncoiled spring will not run a watch; but if, using a key, we change the relative position of its parts, by coiling it up, it will run the watch for the next twenty-four hours. When one drives a nail, he gives the hammer a high speed. The energy due to this speed does the work of forcing the nail into the wood.

In the language of physics, the energy which a body possesses in virtue of its position is called Potential Energy; while the energy which a body possesses in virtue of its speed is called Kinetic Energy.

The energy of the Niagara River just at the top of the falls is nearly all potential, but just before reaching the bottom of the falls it is nearly all kinetic. The energy of the iron block — the ram — used in a pile driver is all potential at the top of the stroke, but all kinetic just before striking the pile at the bottom of the stroke.

The question now arises as to how much work a body can do in virtue of its position, and how much work a body can do in virtue of its speed. In other words, what are the proper measures for potential and kinetic energy respectively? Before answering these two questions we must digress for a moment to consider the

DISSIPATION OF ENERGY

106. It is well known to every one that a ball, or a bicycle, which is allowed to run down one hill from a certain height h (Fig. 62) will not, with the speed thus acquired, ascend an adjoining hill to the same height. The ball, or the wheel, always stops short of its original elevation. The explanation of this is something which does not now concern us. The important fact is that the kinetic energy of the wheel at the bottom of the hill is not quite sufficient to do the work of carrying the wheel to its original height.

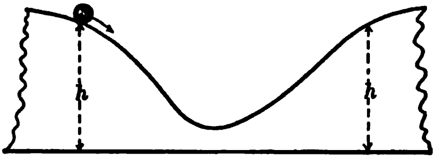

FIG. 62.

The same is observed to be true of a pendulum as it swings from its position of rest at A to its position of rest at C (Fig. 63).

The point C is always a little lower than the point A, *i.e.* the kinetic energy of the pendulum at B is not quite sufficient to do the work of carrying the bob to its original height.

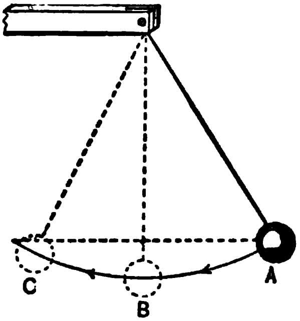

FIG. 63.

In every case of this kind it has been observed that we never get out of a machine all the work which we put into it. Some of the mechanical energy of a system is always wasted when we attempt to use that energy by transforming it, or transferring it.

As we shall learn later, this unavoidable waste of energy is brought about by friction; a portion of the mechanical energy is transformed into heat energy, where it is of less use to us than before. This tendency of energy to assume a more and more useless form is known as the principle of the Dissipation of Energy.

CONSERVATION OF ENERGY

107. But the most important discovery along this line, perhaps the most important discovery of the nineteenth century, is the following: that *when allowance has been made for unavoidable wastes,* the sum of the kinetic and potential energies of a body or system of bodies never changes, unless through some external influence.

"Waste of energy" is a very common expression; but, as we shall learn by and by, this never means destruction of energy. There is no evidence that the slightest bit of energy has ever been annihilated. Energy is, in this respect, like matter. No human being has ever succeeded in creating or destroying any amount of either one. When, therefore, we speak of allowing for "unavoidable wastes," we refer to that energy which has been unavoidably changed into heat or some other form of energy which, for our purpose, is useless.

The law of the Conservation of Energy expresses the fact that the sum total of the energy in any isolated system remains the

same. This energy can change from one form to another, it tends constantly to become less and less available; but, so far as is known, it never changes in quantity.

That this grand principle was more or less clearly suspected and grasped by Galileo, Stevinus, Leibnitz, Huygens, and Newton there can be little doubt; but it is also very certain that the clear and definite establishment of the principle is due to Helmholtz, Joule, and Kelvin, about the middle of the nineteenth century. If a single date be asked, perhaps the year 1847, in which Helmholtz's *Memoir on the Conservation of Energy* appeared, will mark more nearly than any other the rise of the doctrine of the conservation of energy. And yet the French Academy of Sciences many years before (1775) had announced that it would not receive any further communications upon the subject of perpetual motion; which practically affirms their belief that energy cannot, by any human power, be created.

Let it be clearly understood, however, that there is no inherent contradiction between the principle of the Conservation of Energy and the existence of perpetual motion, except in so far as any ordinary body in motion is always losing energy by friction. Robert Brown, the British botanist, discovered in 1827 that the molecules of a liquid are, indeed, in perpetual motion. The motions which these molecules impart to small solid particles, suspended in the liquid, are easily observed through a microscope. These so-called Brownian movements show that the negation of perpetual motion applies to masses, not to molecules.

The evidence for and the full import of this principle of the Conservation of Energy is something the student can hardly expect to comprehend fully until he is more or less familiar with the subjects of heat, electricity, and chemistry.

MEASURE OF KINETIC ENERGY OF TRANSLATION

108. By the application of this principle of Conservation to many special cases, the proper measure of the kinetic energy of a body in translation has been found to be as follows: —

If a force F acts upon a mass m initially at rest, for a time t, it will produce in it a speed of translation s, such that (§ 69)

$$F = ma = m \cdot s/t.$$

If the force F remains constant, then the acceleration will remain constant, and the mass m will travel, during this time, a distance x, such that (§ 37) $x = \frac{1}{2} at^2 = \frac{1}{2} (s/t) t^2 = \frac{1}{2} st.$

But the work done by a constant force F exerted through a distance x is Fx, and therefore the value of this work is

$$Fx = \tfrac{1}{2}\, ms^2. \qquad \text{Eq. 52}$$

Now this work has all gone toward putting the mass m in motion, since, by supposition, there has been no waste from friction or other causes. We see, therefore, that the right-hand member of this equation, half the product of the mass by the square of the speed, is the proper measure of the work put into the body. Relying now upon the conservation of energy, it is seen that this product also measures the amount of work which we shall be able to get out of the body, *i.e.* measures its kinetic energy of translation. Let us denote this energy by K.E., then

$$\text{K.E.} = \tfrac{1}{2}\, ms^2. \qquad \text{Eq. 53}$$

ANALOGUE. KINETIC ENERGY OF ROTATION

109. Consider any particle m of a rigid body which is rotating about an axis O (Fig. 64) with a constant angular speed ω. The linear speed of the particle m will be $r\omega$, where r is the distance of the particle from the axis of rotation, Eq. 8. Hence according to Eq. 53 the kinetic energy of the particle m will be $\frac{1}{2}\, mr^2\omega^2$. Obviously we may consider the entire rigid body as made up of such particles as m; let us call them m_1, m_2, m_3, etc., and suppose them situated at distances r_1, r_2, r_3, etc. Then the total kinetic energy of the body will be the sum

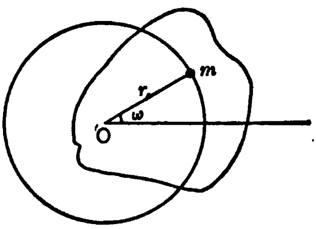

FIG. 64.

$$\text{K.E. of body} = \tfrac{1}{2}\, m_1 r_1^2 \omega_1^2 + \tfrac{1}{2}\, m_2 r_2^2 \omega_2^2 + \tfrac{1}{2}\, m_3 r_3^2 \omega_3^2 + \text{etc.}$$
$$= \tfrac{1}{2}\, \Sigma(mr^2\omega^2).$$

But since, by hypothesis, the body is rigid, every part of it is rotating with the same angular velocity, a fact which may be expressed by writing

$$\omega_1 = \omega_2 = \omega_3 = \text{etc.} = \omega.$$

Hence $\text{K.E. of body} = \frac{1}{2}\,(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2)\omega^2 = \frac{1}{2}\,\omega^2 \Sigma(mr^2)$.

Let us now denote the quantity in the parenthesis, which for any given body is a constant, by I. We may then write

$$\text{K.E. of body} = \tfrac{1}{2} I\omega^2, \qquad \text{Eq. 54}$$

which is strictly analogous to Eq. 53.

The quantity I, whose defining equation becomes

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \text{etc.} = \Sigma(mr^2), \qquad \text{Eq. 55}$$

is, of course, the rotational inertia already described in § 65, which the student should now reread. The principal object of a heavy fly wheel on an engine is to supply energy of this type, which shall keep the machinery going while the piston rod is in the neighborhood of the "dead point," where it exerts a relatively small torque.

MEASURE OF POTENTIAL ENERGY

110. We turn next to a system whose parts have a definite relative position, and ask how much work can be gotten out of it. For instance a coiled watch spring, a stretched rubber band, a pond of water at a definite height above a waterwheel, a stone raised to a certain height above the ground. How shall we measure the potential energy of each of these systems?

Experiment has answered this question in a clear and simple manner. **The amount of work done in bringing the system into its present position is, barring frictional losses, exactly equal to the potential energy now in the system.**

The weight of the water in the pond F multiplied by the average height x through which it was pumped, is at once the work required, Fx, to give the water its present potential energy and is also the measure of its present potential energy; or, denoting potential energy by P.E., we have

$$\text{P.E.} = Fx.$$

It must not be forgotten that here allowance is to be made for the energy wasted in friction in the pump.

The student will observe that in like manner the energy of the coiled watch spring is simply the average moment of force which one exerts upon the key L multiplied by the angle through which he turns the key, say θ. Or if the torque L be spent entirely in producing angular velocity, and if the body, say a fly wheel, start from rest,

$$L\theta = \tfrac{1}{2} I\omega^2,$$

the analogue of Eq. 52.

As a single, simple, concrete illustration of the manner in which the law of the conservation of energy is used quantitatively, imagine a ball let fall from a second-story window whose height above the ground is h. The total energy of the ball just before it is allowed to drop, taken with reference to the ground, is mgh, where m is the mass of the ball and g the acceleration of gravity. Therefore, according to the law, mgx is the energy at any intermediate height, say x, where the velocity of the falling ball is v. Hence

$$\underline{\tfrac{1}{2} mv^2} + \underline{mgx} = mgh = \text{constant},$$

an equation which may at once be used for computing the velocity of the ball at any given height x, the value of h being given as one of the data of the problem.

EQUILIBRIUM IN TERMS OF ENERGY

111. We have already, § 79, learned that a body is in equilibrium when its acceleration is zero, and we know that this condition is satisfied when the sum of all the forces and the sum of all the torques acting on the body are each equal to zero, that is, when

$$\Sigma(F) = \Sigma(L) = 0.$$

If we confine our attention to systems in which friction is not the determining factor, then it is a matter of daily experience that bodies may be in equilibrium in three ways. A lead pencil may, with a little care, be made to stand upon an end which has not been sharpened, and it is said to be in equilibrium; but a breath of wind or the slightest blow is sufficient to "start" it, and when once started, it falls over, "of itself," we say. If the pencil be a round one, it will rest on its side in *any* position we may give it, so long as it lies on a horizontal plane. Here, again, it is said to be in equilibrium; but if it be displaced slightly, it will still be in equilibrium in its new position; for there is no force or moment of force acting upon it in any position so long as it lies on its side.

Let us again consider such a body as a brick lying on any one of its sides. We all know that if the brick be slightly lifted at one end, *i.e.* if the brick be slightly rotated about one edge, it will not go on and " of itself " rotate still farther; but, if released, will return to its original position of equilibrium.

These three cases typify *all* the cases of equilibrium met with in nature. Their essential features are the following: —

1. *Unstable Equilibrium.* When a body is supported in such a way that its center of mass is lowered by a slight displacement, its equilibrium is said to be **unstable**; for the slightest displacement will introduce a moment of force tending to rotate the body still farther. For illustration, imagine a cone inverted and balanced on its apex. The *c.m.* when the cone is ever so little inclined moves *down* a spherical surface, and hence tends to move farther, of itself.

2. *Neutral Equilibrium.* If a body is supported in such a way that its *c.m.* remains at the same height above the point or line of support, its stability is said to be **neutral**; for when disturbed no moment of force will be introduced to displace it farther or to restore it to its original position. Such a case is represented by a right cone lying on its side in a level plane. As the cone is rolled, the *c.m.* moves on a level plane, and has, therefore, no tendency of its own to move in any direction.

3. *Stable Equilibrium.* When a body is supported in such a way that the slightest displacement will raise its *c.m.*, the body is said to be in **stable** equilibrium; for the slightest displacement will introduce a moment of force tending to restore the body to its original position. Such a case is found in a right cone standing on its base. Any small motion of such a cone starts the *c.m.* uphill, so to speak; and when left to itself, the *c.m.* tends to roll back, downhill.

It should be carefully observed that in each case where a rigid body starts to move from rest the *c.m.* of the body moves in such a way that the potential energy becomes less. If, there-

fore, in the given position the potential energy of the body is a maximum, the equilibrium is unstable; if the potential energy is a minimum, the equilibrium is stable: but, if the potential energy does not vary when the body is displaced, the equilibrium is neutral.

SUMMARY OF THE PRINCIPAL DISCOVERIES CONCERNING ENERGY

112. 1. Energy is known only in connection with matter. We know of no matter devoid of energy, and know of no form of energy which is not associated with matter, or with electric charges which exhibit inertia.

2. If no energy be given to or taken away from a system, the sum of its kinetic and potential energies remains constant.

3. When a system is left at rest, its potential energy tends to assume the form of kinetic energy. This great law of nature may be described in another way by saying that **there is a universal tendency for the potential energy of a system at rest to decrease.**

4. During any transformation of energy from one form to another, a portion of it is always degraded into a less useful form; so that the availability of the constant amount of energy in any system is always diminishing.

POWER

113. One important feature of any mechanical system is the rate at which it can do work. Two steam engines may be similar in design and yet so different in dimensions that one can do twice as much work as the other in any given time, both using steam from the same boiler.

In everyday life, **power** is a word which is employed with considerable variety of meaning; but in physics it is always used to denote **the time rate of doing work.** Accordingly the defining equation becomes

$$\text{Power} = P = \left[\frac{W_2 - W_1}{t_2 - t_1}\right], \qquad \text{Eq. 56}$$

where, of course, W denotes work and t, time.

The fundamental idea involved in power is rapidity of working. The reason it is more fatiguing to ride a wheel uphill than to ride it along a level stretch at the same rate is that, in the former case, the rate of working is higher. The main difference between a 10 horse power motor and a 5 horse power one is not that the one has done or is doing twice as much work as the other, but that the one *can* do work twice as rapidly as the other.

UNITS OF POWER

114. Three units of power are in common use: —

(i) The **erg per second**, which has no special name, and is inconveniently small,

(ii) The **joule per second**, which, in honor of James Watt, the inventor of the steam engine, is called a **watt**, and

(iii) The **horse power**, introduced by James Watt; defined as **33,000 foot-pounds per minute**, and generally indicated by the symbol HP. 1 HP = 746 watts = 0.746 kilowatt.

115. Having now completed a brief survey of the five fundamental conceptions of mechanics; namely, *inertia, momentum, force, energy,* and *power,* we pass to some simple concrete problems in energy which, it is hoped, will make the foregoing principles the intellectual property of each student.

APPLICATIONS OF THE PRINCIPLES OF ENERGY TO MECHANICS

I. *The Lever*

116. The lever is a machine which, in various forms, is familiar to every lad. It has a variety of objects. Sometimes it is employed to increase the force which one is able to apply at some particular point, as in raising a heavy stone or cracking a nut. Sometimes it is employed to multiply the motion of a point, as in the index on the dial of a steam gauge; sometimes, as in sugar tongs, pliers, and tweezers, to reach otherwise inaccessible places.

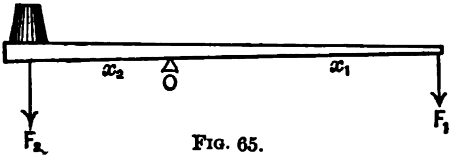

FIG. 65.

The principle of the lever was first correctly enunciated by the Greek scholar Archimedes (287–212 B.C.). The essential feature of all levers is a more or less rigid bar rotating about the edge of some body as an axis. This edge is called the **fulcrum** of the lever.

Since hardly any energy is here wasted in friction, one obtains by use of a lever practically as much work as he puts into it.

Let F_1 (in either Fig. 65 or Fig. 66) be the force exerted on the end of the lever whose length is x_1, and F_2 the force exerted upon the arm of length x_2. Since these forces are each supposed to be at right angles to the arm, the distance through which the force F_1 acts is $x_1\theta$, and the distance through which F_2 acts is $x_2\theta$, where θ is the angle through which the lever is rotated. The work put into the lever is therefore $F_1x_1\theta$. And of the work gotten out of the lever $F_2x_2\theta$ must, on the principle of conservation of energy, be the same, or

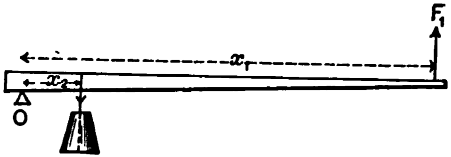

Fig. 66.

$$F_1x_1\theta = F_2x_2\theta,$$

and hence

$$F_2/F_1 = x_1/x_2.$$

The physical meaning of this equation is that the force which one can obtain by use of a lever is to the force which he applies to the lever inversely as the lengths of the arms.

The ratio F_2/F_1 is sometimes called the **mechanical advantage** of the lever.

Great care must always be observed in estimating the length of the arms of a lever. The fulcrum at O is the axis of rotation. The distance from the axis to the point where the force is applied is the one arm x_1; the distance from the axis to the point where the opposing force is applied is the other arm x_2. These definitions are illustrated in each of Figs. 65 and 66.

II. *The Pulley*

117. The pulley is simply a lever used for a particular purpose, viz., to change the direction in which a force is applied and to gain mechanical advantage.

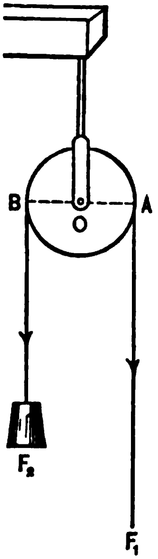

FIG. 67. — The pulley a special case of the lever.

Fixed Pulley

118. The bar constituting the lever is the diameter of the pulley $\overline{AB}$ (see Fig. 67). O is the axle (axis) about which the bar is continually rotating. And hence O is the fulcrum.

Since here the arms $\overline{AO}$ and $\overline{BO}$ of the lever are equal, it is evident that $F_1 = F_2$. **So that the only advantage offered by the fixed pulley is a change of direction**; a pull downward can be transformed into a pull upward.

Movable Pulley

119. When a pulley is arranged as in Fig. 68, it is evident that *at any particular instant* the point O may be considered as fixed, and as the fulcrum about which the lever rotates.* The force at A acts, therefore, through an arm twice as long as that of the force at B.

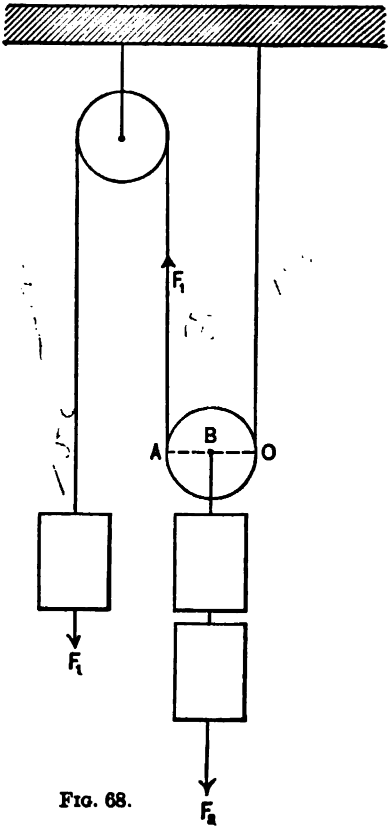

FIG. 68.

Now it is found by experiment that when any point P on the rope is lifted through a distance x_1, the point B on the pulley is lifted through a distance x_2, which is only half as great; in symbols,

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* A line drawn through the point O, perpendicular to the plane of the pulley block, is called the **instantaneous axis** of rotation. Where is the instantaneous axis of rotation of a carriage wheel rolling on the ground?

$$x_2 = x_1/2.$$

Since the work done by the force F_1 is equal to the energy stored up, we have

$$F_1x_1 = F_2x_2 = F_2x_1/2.$$

$$F_2/F_1 = 2,$$

or the mechanical advantage of a movable pulley is 2.

Block and Tackle

120. In the case of the block and tackle (Fig. 69) we secure a mechanical advantage of 2 for each movable pulley added. So that with a machine such as that figured one is able to exert a force six times as great as without its aid. For the force F_1 is exerted downward through a distance six times as great as that through which the force F_2 is exerted upward.

And, in general, to obtain the mechanical advantage of such a machine, one has only to observe the distance through which the force F_1 is exerted in order to exert the force F_2 through the distance x_2. Call this observed distance x_1.

Then it matters not what the mechanism of the block and tackle is, *provided only that no energy is lost inside*, the mechanical advantage may always be computed as follows: —

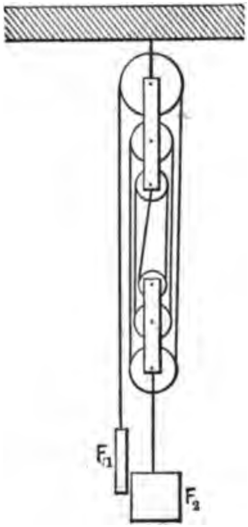

FIG. 69.

The work put into the machine is F_1x_1, the work obtained is F_2x_2, and

$$F_2/F_1 = x_1/x_2.$$

Therefore x_1/x_2, an easily observed ratio, is the required mechanical advantage. In the case represented in Fig. 69, evidently

$$x_1 = 6\,x_2.$$

The Differential Pulley

121. An interesting form of pulley and one very widely used in large machine shops is that known as the **differential pulley** and shown in Fig. 70. It consists of two parallel fixed pulleys, cast in a single piece A, and one movable pulley B, to which the weight W is attached.

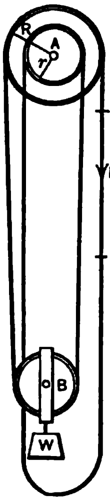

FIG. 70.

Of the two fixed pulleys one has a radius a little larger than that of the other. Let us denote their radii by R and r. Then if the force F be applied to the free side of the chain coming from the large pulley, and if any link, say P, of this chain be displaced through one circumference of the larger fixed pulley, the work done by the force F will be $F \cdot 2\pi R$.

During this same time the work done on the weight W (which is lifted by one chain and lowered by the other) is

$$\tfrac{1}{2} W(2\pi R - 2\pi r).$$

Hence, neglecting friction and employing the law of the conservation of energy, we have

$$F \cdot 2\pi R = W\pi(R - r),$$

or

$$\frac{W}{F} = \frac{2R}{R-r} = \left\{\begin{array}{l}\text{mechanical advantage}\\ \text{of differential pulley.}\end{array}\right.$$

III. *The Inclined Plane*

122. The inclined plane is a device frequently employed to avoid the direct lifting of heavy weights. Barrels are frequently loaded into wagons by use of *skids*, which are simply inclined planes. Goods are often taken aboard steamers by inclined gangways. What advantage does such a device offer? The energy required to raise a mass to a certain height above the ground is quite independent of the path by which it has been raised, *provided friction be negligible.*

Let F_1 be the force required to push the body along the incline $\overline{OA}$ (Fig. 71); and F_2 the weight of the body. Let the length of the slope $\overline{OA}$ be x_1 and the vertical height $\overline{XA}$ be x_2.

Then the work put into the mass by the force F_1 is F_1x_1, and the potential energy of the mass at the top of the plane is F_2x_2. But by the law of the conservation of energy these two are equal.

$$\therefore F_2x_2 = F_1x_1,$$

or,

$$F_2/F_1 = x_1/x_2 = 1/\sin\theta,$$

where θ is the angle AOX which measures the steepness of the inclined plane. **The mechanical advantage of the inclined plane is, therefore, $1/\sin\,\theta$; that is, the mechanical advantage varies inversely as the sine of the angle of slope.** This statement is sometimes called the law of the inclined plane.

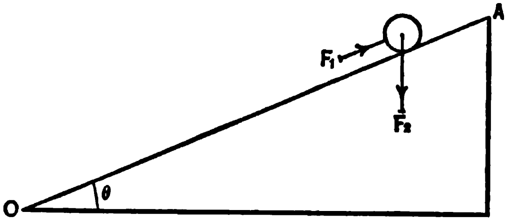

Fig. 71. — The inclined plane.

In road building the ratio of the rise to the length of the road is called its "grade"; but the ratio of the rise to the length of base is called its "pitch." The numerical value of the grade is evidently the sine of the angle of slope.

Problems

175. Show that, in the case of a windlass, such as is used for hoisting stone to the top of a building, the mechanical advantage of the windlass is R/r, where R is the radius of the crank operated by the man, and r is the radius of the axle on which the rope is wound. The windlass should be considered as a special case of the lever.

176. A lever is 14 cm. long. Where must the fulcrum be placed in order that a weight of 8 g. at one end may just balance a weight of 16 g. at the other end? *Ans.* $4\frac{2}{3}$ cm. from end.

177. A meter stick is suspended at a point 40 cm. from one end. A kilogram placed at the end of the long arm will be balanced by what weight placed at the end of the short arm? *Ans.* 1500 g.

178. When a weight is prevented from sliding down an inclined plane by a force acting along the plane, show that the ratio of this force to the normal force on the plane is the same as the ratio of the height to the base of the plane.

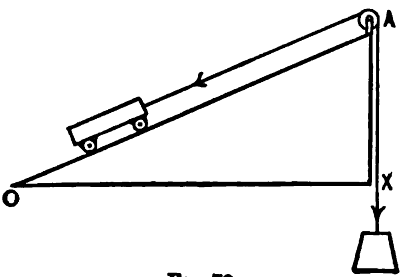

Fig. 72.

179. What is the greatest weight which a man weighing 150 lb. can lift by means of one movable and one fixed pulley? The man is not supposed to be anchored to the ground in any way.

Ans. 300 pounds.

180. A man weighing 150 lb. is lowered into a well by means of a windlass, the arm of which is 30 in. long and the axle of which is 8 in. in diameter. Find the force required to let him down with uniform velocity.

181. A screw, the pitch of which is 5 mm., is turned by means of a lever which is 120 cm. long. What force will be required to raise 2000 kg.?

182. A weight of 300 lb. is raised 3 ft. by means of a block and tackle, the block of which has three sheaves. Determine the force required and the space through which it has acted.

183. The pedal crank of a bicycle is 6½ in. long. The rider exerts a force of 100 lb. on the pedal. Find the force exerted on the chain when a 6-inch sprocket is used.

184. A body is displaced against a force which varies directly as the amount of displacement. The initial value of the force is 2 lb., the final value 10 lb., and the displacement 6 in. Find the work done.

185. A belt which drives a pulley travels at the rate of 12 ft. per second. The pull of the belt is 50 lb., the diameter of the pulley is 3 ft. Find the torque exerted upon the pulley and the power which the belt is transmitting.

Work of Friction

123. When two surfaces are brought into contact, a number of interesting phenomena are observed. There is, for instance, a certain amount of adhesion, which becomes very great in the case of two highly polished surfaces, such, for instance, as two pieces of optically plane plate glass. This adhesion is very marked in the case of objects covered with gold leaf, in the case of graphite and paper when an ordinary pencil is used, in the case of silver deposited on the back of a mirror, etc.

But there is another very different, and, so far as everyday life is concerned, more important, phenomenon exhibited when two solid surfaces are brought together, namely, one of these surfaces cannot be moved over the other, even at uniform speed, without the exertion of force. **The resistance which either one of the**

bodies offers to this motion is called the force of friction and is said to be due to friction.

Frictional forces, like all other forces, are vector quantities, and are to be compounded and resolved as are other forces.

In machinery, friction is at times a most useful feature, at other times a most wasteful one. Thus, for instance, it is by friction that belts are able to drive pulleys; but on the other hand, it is through friction that a very large portion of the energy given to an ordinary machine is wasted. And it is on this ground that a discussion of friction finds place under the head of energy.

The origin of friction doubtless lies in the interlocking of the small hills and valleys, slight rugosities, that remain on even the most highly polished surfaces. As might be expected, therefore, friction diminishes as polish increases. Note that just the opposite is true of adhesion.

Morin's Laws of Sliding Friction

124. The principal experimental facts regarding the behavior of one solid body sliding over another may be approximately summarized as follows: —

The frictional resistance between solid bodies is (i) *proportional to the force with which the two bodies are pressed together;* (ii) *is independent of the area of the surfaces in contact; and* (iii) *is, within wide limits, independent of the relative speed of the bodies.* The first of these facts may be put into useful quantitative form as follows: let N be the normal force pressing the two bodies together, and F the force of friction; then for any two given materials

$$F = fN, \qquad \text{Eq. 57}$$

where f is a proportionality constant which is known as the **coefficient of friction** for the two materials in question. Experience shows that after sliding has begun the value of f is slightly less than just before motion, a phenomenon which Sir George Greenhill describes by saying that "*friction* is slightly less than *stick-tion.*"

The following table will give some idea of the values which f assumes in practice: —

MATERIALS	COEFFICIENT OF FRICTION
Brass on cast iron	0.19
Wrought iron on cast iron	0.20
Wrought iron on wrought iron	0.14
Iron on ice (skates)	0.16 to 0.032
Oak on oak, fibers parallel	0.48
Oak on oak, fibers crossed	0.32
Leather on oak	0.27 to 0.38

A helpful point of view from which to regard the coefficient of friction is the following: If a body B be placed on an inclined plane as indicated in Fig. 73, its weight may be resolved into two rectangular components, one parallel to the plane, the other perpendicular to the plane. Let the slope of the plane be varied; and let ϕ be the angle which it makes with the horizontal at any instant: $mg \sin \phi$ will then be the component urging the body down the plane; while $mg \cos \phi$ will be the normal component. Let us now suppose the angle ϕ to increase until the body B is just on the verge of slipping, or has just begun to slip, down the plane. At this point, the friction, which acts up the plane, reaches its maximum value, and Eq. 57 becomes

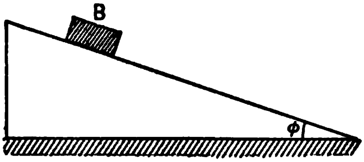

FIG. 73.

$$F = mg \cdot \sin\phi = f \cdot mg \cos\phi = fN,$$

and hence

$$f = \tan\phi. \qquad \text{Eq. 58}$$

The angle ϕ thus defined is known as the **angle of repose**.

The rate at which energy is absorbed by friction when one body is drawn a distance s over another during an interval t is then

$$\text{Power} = P = \frac{\text{Work}}{\text{Time}} = \frac{Fs}{t} = \frac{fNs}{t} = fNv, \qquad \text{Eq. 59}$$

where N is the load and v the speed.

Rolling Friction

125. Why are ball bearings, instead of ordinary bearings, used in a bicycle? The reply is, in order to substitute rolling friction for sliding friction, since rolling friction is very much less than sliding friction. But the question now arises, why should there be *any* friction in, say, a smooth iron car wheel rolling over a level smooth iron track? The reply is that the iron track is slightly and temporarily indented so that the wheel has really to run uphill all the time. Every one who rides a bicycle for any distance over a level sandy road soon becomes familiar with this phenomenon in an exaggerated form and keenly realizes the uphill features of the problem.

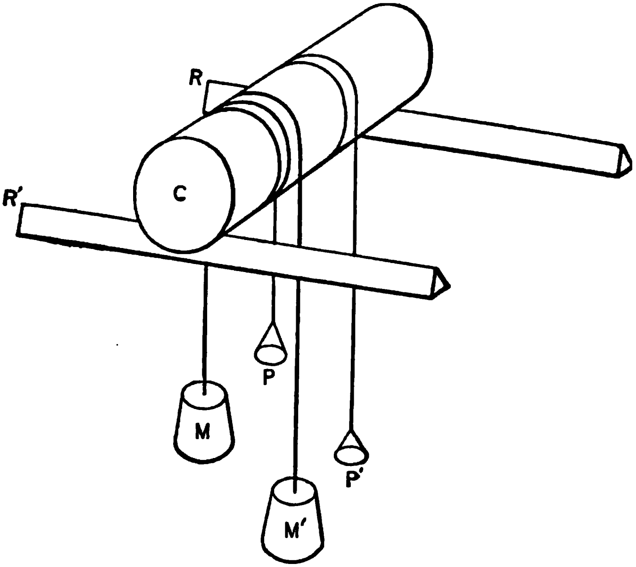

FIG. 74.

Regarding methods for observing this rolling friction quantitatively, perhaps the following is the simplest. See Fig. 74. Let a metal cylinder C roll upon a pair of rails R and R'. A pair of equal masses M and M' may be strung over the cylinder by means of string to furnish the load. A pair of equal scale pans P and P' supported by a thread, which is wrapped about the cylinder several times in the manner indicated, can receive small weights just sufficient to roll the cylinder in one direction or the other. Two forces now act upon the cylinder: —

(i) the load made up of the weight of the cylinder and the suspended masses M and M'; call this force N: and

(ii) the weight W placed in the one pan or the other.

If the cylinder is to remain in equilibrium as regards translation, the reaction of the rails Y must just balance these two forces,

or $$N + W - Y = 0.$$

In like manner, if the cylinder is to remain in equilibrium as regards rotation, the moments of all the forces acting upon it must add up to zero. We know the points through which N and W act; but as yet we do not know the point through which the reaction Y acts. Let O, Fig. 75, denote the lowest point of the cylinder, and x the distance between the lines of action of N and Y, respectively. Then taking moments of force about the point O as an axis of rotation, one has

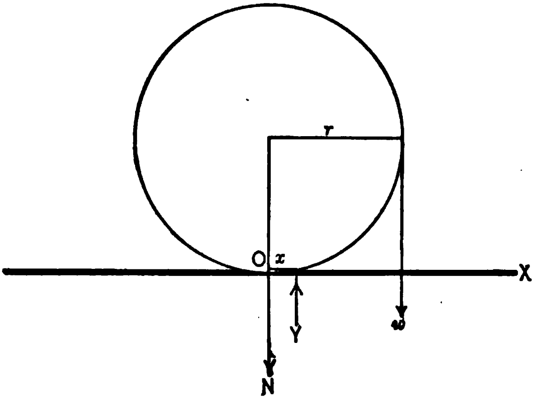

Fig. 75.

$$N \cdot O - Y \cdot x + W \cdot r = 0,$$

or, solving for x, $$x = \frac{Wr}{Y} = \frac{Wr}{N + W}, \qquad \text{Eq. 60}$$

an equation which could not be true unless the cylinder were in contact with the rail at some finite distance x from the lowest point on the cylinder, which in turn shows that the cylinder is constantly rolling uphill. The energy used up in rolling friction is therefore spent in distorting the material of the rail — a fact which will be clearer after we have considered the subject of elasticity, to which we now proceed. The entire question of fluid friction and the resistance which a fluid offers to a solid passing through it are here left untouched. Thus in the case of a falling raindrop the friction of the air soon gives rise to a retarding force equal to the weight of the drop. When the radius of the drop is one millimeter, this limiting speed is about 6 meters per second.

A much more extended study of friction is essential for all students looking forward to engineering.

Problems

186. A fly wheel whose moment of inertia is 20 million C.G.S. units is rotating with an angular speed of 240 R.P.M. A friction brake brings this wheel to rest in 20 sec. Find the torque exerted by the brake.

187. A brick lying on an oak plank begins to slide down when one end of the plank is raised so that it is 4 ft. higher than the other. If the coefficient of friction of oak and brick is 0.64, what is the length of the plank?

188. What is the total force of friction on a train which is pulled at a constant speed of 20 mi. an hour by an engine exerting 50 horse power?

189. What force of friction will be required to bring a mass of 4 gr. to rest in a space of 500 cm., the initial horizontal velocity of the body being 100 cm. per second?

190. The coefficient of friction of iron on ice is, say, 0.16. A boy is able to pull 100 pounds in a horizontal direction. What is the greatest load he can move by the use of a sled shod with iron runners?

191. It requires a force of 10 tons to move a given freight train. If the coefficient of friction between the driving wheels and the rails is 0.14, what is the least weight which the locomotive may have in order to start the train?

192. What is the angle of repose for one oak plank lying on another, the fibers being parallel? See Table, § 124.

193. What power will be absorbed in pulling a 400-lb. cake of ice over a level wooden platform at the rate of 5 ft. per second? Take coefficient of friction as 0.20.

194. A horse pulls a sleigh at the rate of 8 mi. per hour. The sleigh with its occupants weighs 500 lb. If the coefficient of friction is 0.16, what power must the horse exert?

195. Referring to Problem 141, solve the case represented in Fig. *B*, when the coefficient of friction between the mass " 30 " and the inclined plane is 0.20.

196. A locomotive can pull a train of 500 tons weight along a level piece of track. What weight can it pull up a grade of 1 in 60, the coefficient of friction being 0.08 in each case?

IV. *Elasticity*

126. In the preceding pages we have had frequent occasion to deal with a " rigid " body, by which we have meant a **perfectly rigid body**; *i.e.* one the relative position of whose parts admits of no change whatever. Experience tells us that no such bodies are met with in nature; although we have many close approximations. A strip of hard and brittle window glass may be bent with ease. Cold steel can be rolled or drawn into various shapes. While *for all ordinary operations* the metal parts of a pair of shears, the frame of a bicycle, the beam of a balance, etc., are rigid bodies, there are many cases in which we are forced to recognize the fact that each of these bodies changes form or volume. **Bodies that can be changed in either size or shape only by the application of force are said to be elastic.**

In the case of a collision, it may become painfully evident that one's bicycle frame is not a rigid body. In case one is weighing a large load, the balance beam is liable to bend slightly and thus lower the *c.m.* of the balance, and hence diminish the sensibility of the balance.

A large variety of the phenomena of elasticity is familiar to us all; one need only mention the pneumatic tire, the rubber band, the bending of an oar, the " straining " of a ship, the vibrations of a steel bridge.

127. To bring some degree of order out of this chaos of experience is the object of this chapter. And this is not very difficult. For, indeed, careful examination shows that, after all, there are only two kinds of changes which may occur in the configuration of elastic bodies; viz.: —

$$\left.\begin{array}{l}\text{1. Change of size, } \textit{i.e.} \text{ Compression or Dilatation}\\ \text{2. Change of shape, } \textit{i.e.} \text{ Distortion}\end{array}\right\}\text{Strains.}$$

When a body is changed in either size or shape, it is said to be strained. The size of a body may be strained, and the shape of a body may be strained; and these are the only strains that are known. Since all known bodies are subject to one or both of these changes, elasticity is said to be a general property of matter.

Distinction between Solids and Fluids

128. But while all bodies may be changed as indicated, they behave very differently *during* the change.

If we attempt to alter the *shape* of a piece of glass tubing by slightly bending it, we shall find that this will require a moment of force; that, if we remove the force, the rod regains its original shape. In like manner, we should find that force would be required to change the *size* of the glass rod, and that this force would need to be maintained as long as the changed size is maintained.

Now imagine a second case in which the glass tube is full of mercury or coal gas while it is being bent: the mercury will offer no permanent resistance whatever to a change of shape; it conforms perfectly, without requiring any moment of force, to the shape of the containing vessel. The mercury has, in fact, no elasticity of *shape*. The same is true of the coal gas. But if one attempts to *compress* the mercury or the coal gas into a smaller volume, he finds this operation requires force; and that this force must be maintained so long as the diminished volume is maintained. Mercury and coal gas have, therefore, elasticity of *size;* but they have no *shape* of their own.

All bodies in nature fall into one or other of the two cases we have just considered; and this fact leads to the following definitions: —

1. Bodies which exhibit elasticity both of shape and of size are called **solids.**

2. Bodies which possess elasticity of size, but not of shape, are called **fluids.**

As is well known to every one, this distinction between fluids and solids is not a distinction between different substances, but between different bodies. For the same substance may, under different circumstances, assume each of these two states — one needs only to recall the case of ice and water.

Distinction between Gases and Liquids

129. While mercury and air are alike in possessing only elasticity of size, **they differ very much in the manner in which they exhibit this elasticity of size.**

No one ever heard of a closed space partly full of air; a closed vessel of *any* size containing *any* amount of air is always full of it; and, at ordinary temperatures, the air is distributed uniformly throughout the entire vessel. Contrast this with mercury or water, which may or may not completely fill the vessel.

Definitions: —

1. "There are certain fluids any portion of which, however small, is capable of expanding indefinitely so as to fill any vessel, however large. These are called **gases**.

2. "There are certain other fluids a small portion of which, when placed in a large vessel, does not expand so as to fill the vessel uniformly, but remains in a collected mass at the bottom, even when the pressure is removed. These fluids are called **liquids**." — MAXWELL, art. "Constitution of Bodies," *Ency. Brit. Ninth Edition.*

130. We might summarize the preceding classification of bodies as follows: —

$$\text{Bodies}\begin{cases}\text{Rigid — not known in nature.}\\ \text{Elastic}\begin{cases}\text{Solids.}\\ \text{Fluids}\begin{cases}\text{Gases.}\\ \text{Liquids.}\end{cases}\end{cases}\end{cases}$$

QUANTITATIVE CONSIDERATION OF ELASTICITY

Coefficients of Elasticity

131. A builder who is putting up a roof or a bridge always finds himself between Scylla and Charybdis. The bridge must be strong enough to sustain the heaviest load easily; and, at the same time, it must contain as little useless material as possible.

In order to secure this latter result, the engineer must have definite and quantitative information concerning the elastic properties of the steel to be used in the bridge; in particular, he must know what elongation (stretch) will occur in a specimen of this steel having a definite length and a definite area of cross section, when it is given a definite load to carry.

In like manner, the engineer who designs a cotton mill must know what moment of force a steel shaft will safely transmit before he can determine the proper size of shafting to be used in the mill. The designer of a steam engine must likewise be definitely informed concerning the elastic properties of steam. A certain amount of steam has been admitted to the cylinder of an engine and the port closed; what pressure will it exert when the piston has reached the end of the stroke? Answers to such questions as these have been obtained by the following method: —

General Definition of Stress and Strain

132. When two or more forces are applied to a body in such a way that their resultant is zero, there is no acceleration and no evidence, therefore, *from the motion of the body,* that such individual forces are acting. Such forces are said to be in equilibrium. Their existence is shown by the strains which they produce in a body.

"A **stress** is an equilibrating application of force to a body.

"A **strain** is any definite alteration of form or dimension of a solid." — KELVIN.

Elasticity is measured by the resistance which a body offers to change either of size or of shape, ***i.e.***

$$\textbf{Elasticity} = \frac{\textbf{Stress}}{\textbf{Strain}}, \qquad \text{Eq. 61}$$

Elasticity of Size. I. Length

133. When a force is applied to a body in such a way as to alter its length, either by compressing it or by stretching it, the force is necessarily distributed over the cross section of the body.

The **longitudinal stress** at any point in a body is defined as **the ratio of the force to the area at that point.** Longitudinal stress, then, is not a force, and is not measured in dynes; but it is measured in dynes per square centimeter.

A longitudinal stress always changes both the length and the cross section of the body to which it is applied. But in most practical cases the change in cross section is small; and we consider only the alteration in length. The total change in length depends not only upon the stress, but also upon the length of the body. Hang a kilogram on the end of a wire one meter long: the elongation will be a certain amount. If, now, the same kilogram is hung from the end of a wire of the same kind two meters long, the elongation will be twice as great. But the wire is strained at each point in its length by the same amount in each case. Consequently **longitudinal strain** is defined as **the ratio of the total elongation to the length.**

134. In the case of a wire, let

L = length of wire;
a = area of cross section of wire;
F = stretching force applied to wire;
e = total elongation produced by F.

The longitudinal stress = F/a dynes per square centimeter and the longitudinal strain = e/L centimeters per centimeter.

Now the longitudinal elasticity of any material is measured by the ratio of the longitudinal stress to the longitudinal strain. And hence for this ratio, which is generally known as **Young's** * **modulus,** we have in the case of a wire

$$\textbf{Young's modulus} = M = \frac{\textbf{Longitudinal stress}}{\textbf{Longitudinal strain}},$$

or

$$M = \frac{\left(\frac{F}{a}\right)}{\left(\frac{e}{L}\right)} = \frac{FL}{ea}. \qquad \text{Eq. 62}$$

---

* The life of Thomas Young, a London physician, forms an interesting chapter in the history of physical science. See his biography in *Ency. Brit.*

From this equation it is evident that, in order to determine M by experiment, we must measure the four quantities in the right-hand member of the equation. As a stretching force it is often most convenient to use the weight of a mass m; viz., mg. If we employ a round wire of diameter d, its area will be $\pi d^2/4$. Accordingly Eq. 62 may be written

$$M = \frac{4\,mgL}{\pi e d^2},$$

in which all the quantities on the right are directly measurable. Such an equation we have called a **laboratory equation.**

135. The value of Young's modulus, here denoted by M, is found to be very nearly constant for any one substance, at least within certain limits; but the value of M varies largely from one kind of material to another and also varies rapidly with the temperature.

The limit within which a body can be stretched or bent and still recover its original size or shape also varies greatly from one substance to another, and likewise with the temperature. A piece of pure gum tubing becomes brittle at the temperature of liquid air, − 185° C. In the case of glass the limits within which perfect recovery of shape and size occur are quite narrow; while in the case of rubber these limits are quite wide. Hence the popular impression that rubber is highly elastic, though in fact Young's modulus is much less for rubber than for glass.

136. In such an equation as that which defines Young's modulus many important physical facts have been summarized. The student should early acquire the habit of interpreting the physical meaning of each equation which he employs.

Solving Eq. 62 for e, we have

$$e = FL/aM,$$

from which we may write at once: —

The elongation

$\propto$ directly as the stretching force (Hooke's Law).

$\propto$ directly as the length.

$\propto$ inversely as the area of cross section of the wire.

$\propto$ inversely as the longitudinal elasticity; *i.e.* the elongation depends also upon the material of which the wire is made.

Each of these predictions is easily tested in the laboratory.

Hooke's Law

The fact that, for a rod of given length, cross section, and material, the elongation varies as the stretching force is one of the earliest important discoveries in the subject of elasticity.

It was made by Robert Hooke in 1660 and published by him in 1676 under the form of the following anagram: —

ceiiinosssttuv

In 1678 he rearranged these letters and translated them as follows: "*Ut tensio sic vis;* that is, the Power of any spring is in the same proportion with the Tension thereof." Observe that at this early date he uses *power* in the sense in which we now employ the word force; while his *tension* is equivalent to our *extension.*

The best evidence for the truth of Hooke's Law is the fact first pointed out by Stokes, that all elastic solids can be made to vibrate isochronously; for equal periods of vibration, at all amplitudes, means that the force acting upon the body is proportional to the displacement.

The extreme convenience of this modulus can here be merely indicated by saying that, if an engineer wishes an iron rod of length L to carry a certain load, which we may call F, without producing elongation greater than a certain amount e, our equation tells him at once just how large a cross section, a, the rod must have; viz.: —

$$a = FL/eM.$$

As indicating the approximate numerical values of Young's modulus, at the temperature of 18° C., the following will serve: —

For Bessemer steel, M	$= 22 \times 10^{11}\ \frac{\text{dynes}}{\text{cm}^2.}$	$= 31 \times 10^{6}\ \frac{\text{pounds}}{(\text{inch})^2}$
Cast iron	$= 12 \times 10^{11}$ "	$= 17 \times 10^{6}$ "
Brass wire	$= 10 \times 10^{11}$ "	$= 14 \times 10^{6}$ "
Copper wire	$= 12 \times 10^{11}$ "	$= 17 \times 10^{6}$ "
Aluminium	$= 7 \times 10^{11}$ "	$= 9 \times 10^{6}$ "
Glass	$= 6 \times 10^{11}$ "	$= 8 \times 10^{6}$ "

Elasticity of Size. II. Volume

137. Let us denote the volume of a body by V. Imagine the volume to be subjected to pressure so that it is diminished by an amount v.

The ratio of the diminution in volume to the total volume is called the voluminal strain. Suppose further that this change of volume has been produced by applying a force F to a surface of area a; as, for instance, when air is compressed in an ordinary bicycle pump. **The ratio of the total force to the area over which it is applied is the voluminal stress; and is what we call pressure.**

In accordance with our general equation (§ 132), the **elasticity of volume** for any material is measured by the ratio of the voluminal stress to the voluminal strain. This ratio is generally known as the **bulk modulus**, and is commonly denoted by k.

$$\textbf{Bulk modulus} = k = \frac{\textbf{Voluminal stress}}{\textbf{Voluminal strain}},$$

or

$$k = \frac{\left(\frac{F}{a}\right)}{\left(\frac{v}{V}\right)} = \frac{FV}{av}. \qquad \text{Eq. 63}$$

The reciprocal of the bulk modulus is what is known as the **compressibility** of a substance.

Approximate values of certain bulk moduli are given in the following table: —

Substance	Bulk Modulus	
	In Dynes per Sq. Cm.	In Pounds per Sq. In.
Water at 20° C.	0.023×10^{12}	0.33×10^6
Brass	1.05×10^{12}	15.48×10^6
Copper	1.68×10^{12}	17.10×10^6
Steel	1.6×10^{12}	21.61×10^6
Glass	0.43×10^{12}	5.76×10^6

Elasticity of Shape

138. In a strictly analogous manner is defined the elasticity of shape, more frequently called the rigidity modulus.

$$\textbf{Rigidity modulus} = n = \frac{\textbf{Shearing stress}}{\textbf{Shearing strain}}.$$

But this subject is somewhat intricate, and must be left for later study. Before leaving it, however, we merely remark that the rigidity modulus of any material measures its strength in resisting a shear, and hence its fitness for mill shafting, for propeller shafts of steamships, etc. Without attempting to define shearing stress or shearing strain, we can only say that when a rod or shaft is twisted, a shearing stress is exerted upon it; and this produces a shearing strain, *i.e.* one surface is displaced over another, just as would happen if the rod were cut in two by a pair of shears.

For a shaft, or wire, of circular cross section, the experimental facts are summarized in the following equation, where, if the shaft be clamped at one end, θ is the amount of twist, in radians, which will be produced by a torque L, applied at the free end: —

$$\theta = \frac{2\,l}{\pi n r^4} L. \qquad \text{Eq. 64}$$

In other words, the amount of twist (angular displacement) varies

(i) directly as the torque L which produces the twist (Hooke's Law).
(ii) directly as the length of shaft l.
(iii) inversely as the rigidity modulus n.
(iv) inversely as the fourth power of the radius of the shaft r.

Eq. 64, together with the following table, will enable the reader to solve many practical problems concerning mill shafting, etc.: —

Substance	Rigidity Modulus	
	In Dynes per Sq. Cm.	In Pounds per Sq. In.
Brass	3.5×10^{11}	$4.6 - 5.8 \times 10^{6}$
Copper	$3.9 - 4. \times 10^{11}$	$5.6 - 6.7 \times 10^{6}$
Steel	$7.9 - 8.9 \times 10^{11}$	$10.6 - 11.8 \times 10^{6}$
Glass	$2.0 - 3.2 \times 10^{11}$	$3.3 - 3.9 \times 10^{6}$
Wood	$0.07 - 0.12 \times 10^{11}$	$0.1 - 0.17 \times 10^{6}$

Torsion Pendulum

139. If one suspends a metal cylinder or bar, by means of a wire as shown in Fig. 76, it will oscillate in a horizontal plane with a period T. If I denote the moment of inertia of the disk, L the torque at any instant, as defined above, and A the angular acceleration at this same instant, then since the acceleration is always in a sense opposite to that of the torque which produces the twist,

$$-L = IA. \qquad \text{Eq. 37}$$

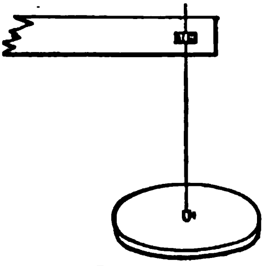

FIG. 76.

Eliminating L between this and Eq. 64, one has

$$A = -\frac{\pi n r^4}{2lI}\theta = -\text{ constant} \times \theta.$$

Hence the motion is simple harmonic and has the following period: —

$$T = 2\pi\sqrt{\frac{2lI}{\pi n r^4}}, \qquad \text{Eq. 65}$$

which is a convenient laboratory equation for measuring n, the modulus of rigidity for the material from which the wire is manufactured.

Problems on Elasticity

197. A wire whose original length was 64.4 cm. has been stretched so that it is now 70.0 cm. long. Find the longitudinal strain of the wire. *Ans.* .0869.

198. A wire has a radius of 2 mm. and carries a load of 40 kg. Find the longitudinal stress on every cross section of the wire. Express your result in C.G.S. units. *Ans.* $\frac{981000000}{\pi}$.

199. A certain metal rod has an area of cross section equal to 3 mm^2. The length of the rod is 150 cm. Young's modulus for this rod is 12×10^{11} C.G.S. How much will the rod be elongated by stretching it with a force of 8,000,000 dynes? *Ans.* $\frac{1}{30}$ cm.

200. I wish to support a weight of 10 kg. by use of a wire 2 m. long. Young's modulus for the wire is 8×10^{11} C.G.S. What cross section must the wire have in order that it may not stretch more than 1 mm. when the load is attached? *Ans.* 0.0245 cm.^{2}.

201. A very slight stress applied to a rubber band will produce a very great strain. Would you, therefore, say that rubber is a highly elastic or highly inelastic substance? What gives the popular impression that rubber is very elastic?

202. A steel rod requires a large stretching force to produce even a small elongation. Is it, therefore, very elastic or very inelastic?

203. Under ordinary conditions it requires a much larger pressure to compress a gallon of water by a certain amount than it does to compress a gallon of air by the same amount. Which, would you say, has the greater elasticity of volume?

204. Two wires suspended from a beam have each the same length and carry the same load. One wire is of brass, the other of Bessemer steel; the former has a diameter of 3 mm.; find what diameter the latter should have in order that the load will produce the same elongation in each wire.

205. What are the "dimensions" of longitudinal stress? Of longitudinal strain? Of the bulk modulus?

206. A piece of mill shafting 1 in. in diameter can just withstand a certain torque L without injury. How much stronger will be a piece of 1½-inch shafting from the same material?

207. Two torsion pendulums are alike in all respects except that the first is made of wire which is $\frac{1}{64}$ in. in diameter and 6 in. in length, while the second is made of wire $\frac{1}{32}$ in. in diameter and is 8 ft. long. Compare their periods of oscillation.

ALGEBRAIC SUMMARY OF DYNAMICS

Linear	**Angular**
Fundamental Quantities	
Length, inertia, and time.	Angle, rotational inertia, and time.
x, m, t	θ, I, t

$$I = r^2m.$$

Velocity

$v = \frac{x}{t}$	$\Omega = \frac{\theta}{t}$

$$v = r\Omega.$$

Acceleration

$a = \frac{v}{t}$	$A = \frac{\Omega}{t}$

$$a = rA.$$

Simple Harmonic Motion

$T = 2\pi\sqrt{\frac{x}{a}}$	$T = 2\pi\sqrt{\frac{\theta}{A}}$

Momentum

mv	$I\Omega$

$$I\Omega = r \cdot mv.$$

Force and Torque

$F = ma$	$L = IA$

$$L = rF.$$

Centrifugal Force and Precessional Couple

$mv\omega$	$I\Omega\omega$

Energy

K.E. $= \frac{1}{2} mv^2$ P.E. $= Fx$	K.E. $= \frac{1}{2} I\Omega^2$ P.E. $= L\theta$

Power

Fv	$L\omega$

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"Truth shall be policy enough for him."
EMERSON,
Dartmouth College Address, 1838.

CHAPTER IV

SOME SPECIAL PROPERTIES OF MATTER

140. In the preceding pages we have considered four properties of matter which are perfectly general. It is an experimental fact that matter possessing any one of these possesses all the others. This by no means exhausts the list of general properties, but it probably includes those which are most important as parts of the intellectual equipment of a liberally educated man.

PROPERTIES OF LIQUIDS

141. We shall now consider some of the special properties which are exhibited by liquids, and this from the three following points of view: —

I. Behavior of Liquids at Rest and under Gravity; Hydrostatics.

II. Behavior of Liquids in Motion under Gravity; Hydraulics. Wave motion in liquids will, however, be treated in the following chapter, along with waves in strings.

III. Behavior of Liquids at Rest and freed from Gravity; Surface Tension.

I. HYDROSTATICS

142. There are few people who are not familiar with the fact that the water in a teapot stands at the same height in the spout as in the pot. The danger of a leak in the hull of a boat arises from the fact that the boat will continue to "fill" until the water stands at the same level inside and out. Mercury in a U-tube stands at the same level in each arm, provided the diameter of the tube is uniform or large. If, in any of these cases, the level is disturbed for an instant, it is quickly restored when the force producing the disturbance is removed.

This behavior of liquids has been summarized and described in a very brief and simple manner. But we must first digress to add another word to our vocabulary; namely, *pressure.*

DEFINITION OF PRESSURE

143. In physics the term "pressure" is employed always to denote the ratio between the force applied to any surface and the area of that surface. Let us denote pressures by P, forces by F, areas by A, then

$$P = \frac{F}{A}.$$

Defining equation for *average* pressure over area *A*.

$$P = \left[\frac{F}{A}\right]_{A \doteq 0}$$

Defining equation for pressure *at any point*. Eq. 66

THE UNIT OF PRESSURE

144. (i) From the above definition it follows naturally that the **dyne per square centimeter is the C.G.S. unit of pressure**; and this is widely used in physics, under the name "*barye,*" adopted by the International Congress on Physics at Paris in 1900 (Vol. 4, p. 62). In English *barye* is generally shortened to *bar.*

(ii) But no unit can be considered thoroughly practical until it has been expressed in some more concrete form. Hence has come into use the *megabar,* which is *one million dynes per square centimeter* and is almost exactly equal to the pressure produced by 75 cm. of mercury, in any locality where the acceleration of gravity is 980.692.

(iii) The logical engineer's unit of pressure is *one pound per square foot;* but in general he uses *one pound per square inch.* Thus steam gauges are practically always graduated in pounds per square inch. In dealing with very high pressures, the engineer also uses the "atmosphere" = 76 centimeters of mercury = 14.7 pounds per square inch = 0.94 ton per square foot = 1.0132 megabars.

145. The definition of a fluid as a body which is devoid of elasticity of shape (rigidity) is equivalent to saying that a fluid cannot sustain any tangential stress. See § 128. From this definition one may infer the facts described in the first two of the following propositions — facts which are thoroughly established by experiment.

Proposition I. **The stress (pressure) across any surface drawn in a liquid at rest is normal to that surface.**

Proposition II. **At any point in a liquid at rest the stress (pressure) is the same in all directions.** Pascal's Theorem.

The design of the hydrostatic press rests upon this principle. See Fig. 77.

146. The two preceding propositions would be equally true for liquids which were not acted upon by gravity. In the case of all actual liquids, we have to consider among the forces which act upon them, not only the pressure due to the containing vessel, as indicated in Fig. 77, but also the pressure due to the weight of the liquid itself.

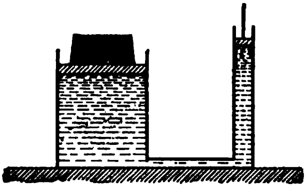

FIG. 77.

This fact is familiar to all boys who have observed when in swimming that they can dive to very moderate depths without being troubled by water getting into their ears; but that, if they exceed a depth of eight or ten feet, water almost invariably gets into their ears. Builders know that when a tank is put into the attic of a house, the walls of the tank must be made strong, *not* in proportion to the *volume* of water which it is to hold, but in proportion to the *depth* of water in the tank. These and all similar phenomena are described as follows: —

Proposition III. **The pressure at any point in a liquid at rest under gravity is proportional to the depth.**

The facts are summarized more compactly in the symbols of algebra as follows: The pressure at any point in the liquid will be the total weight of the liquid supported by any small area about that point, divided by the area. To this must be added the pressure of the air upon the free surface of the liquid. Let

P = pressure at any point in liquid (see Fig. 78);

B = pressure of earth's atmosphere on free surface of liquid;

W = weight of liquid supported by area a at the point under consideration.

Then
$$P = W/a + B.$$

But from the definition of density (Eq. 28)

$$W = ahD \cdot g,$$

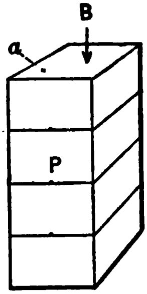

FIG. 78.

where

h = vertical distance of area a from free surface;
D = density of fluid, here supposed to be uniform;
g = acceleration of gravity.

Hence
$$P = hD \cdot g + B. \qquad \text{Eq. 67}$$

If we now neglect all pressure of the atmosphere, and consider that due to the liquid alone, we may write

$$P \propto h,$$

which is *Proposition III* stated above.

The important idea here is that in any given liquid the difference of pressure, $P_2 - P_1$, between any two points depends only upon their difference of level, $h_2 - h_1$. Thus

$$P_2 - P_1 = Dg(h_2 - h_1).$$

The fact that the pressure is independent of the form of the containing vessel is generally known as the "Hydrostatic Paradox."

147. In *Proposition II* only the variation of pressure with direction is considered. But consider a vessel filled with water in one case and mercury in another. At any given point in the vessel how will the pressure change when the liquids are changed?

This question also is definitely answered by Eq. 67, since it shows

$$P \propto D.$$

We have then as a corollary to *Proposition III* the following theorem easily verified by experiments on liquids in a U-tube.

Proposition IV. **Other things remaining the same, the pressure at any given point in a liquid varies directly as the density of the liquid.**

148. Another important experimental fact, also an inference from Eq. 67, is the following: —

Proposition V. **The pressure is the same at all points in a horizontal plane of a liquid at rest under gravity.**

For if the pressure were not the same at all points, we should have present a force tending to drive a particle from one point to another on a horizontal surface. And this we do not find in nature. When a spherical pebble sinks in quiet water, there is no lateral pressure thrusting it either to one side or the other.

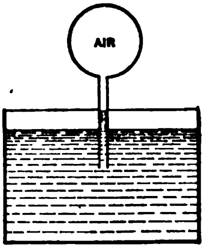

FIG. 79.

A simple experiment illustrating this fact is that sketched in Fig. 79, where the height (above the outside level) of the liquid in the inverted glass bulb indicates a variation of pressure as the lower end of the tube is raised or lowered, but shows no variation when the lower end of the tube is moved about in any horizontal plane.

Among those surfaces over which the pressure is uniform may be reckoned the top of the liquid, the bounding surface, more generally called the **free surface.**

149. If we assume the converse of *Proposition V*, which is also amply verified by experiment, then it follows as a corollary that **the free surface of a liquid is horizontal.** But all that is meant by a horizontal surface is one which stands perpendicular to the direction of the acceleration of gravity. By experiments such as that outlined below, this particular case has been generalized into the following theorem: —

Proposition VI. **Any free liquid surface is, at each point, perpendicular to the resultant of all the forces acting upon the liquid particle at that point.**

To illustrate, attach a vessel of colored liquid to a whirling table, as shown in Fig. 80. The vertical line in Fig. 81 is the axis of rotation of the whirling table; the curve is a cross section of the free surface. Owing to the fact that the inside of the glass vessel is *rough* and the liquid *viscous*, the whole mass of fluid soon comes to rotate *as if* it were a solid. The curved form which the free surface of the liquid assumes is, at every point, normal to the total force, F, acting upon it. This resultant force, F, is made up of two components; one of these is the weight of the particle, mg, the other is the reaction which the particle offers against acceleration toward the center by the centripetal force, $mr\omega^2$; the former component is vertical, the latter is horizontal. The *weight* of the liquid particle is counterbalanced by the *vertical* component of the liquid pressure, while the *centripetal force* is furnished by the *horizontal* component of the liquid pressure. The resultant force F, acting along the normal to the liquid surface, therefore makes an angle θ with the horizontal whose tangent is

$$\tan\theta = \frac{g}{r\omega^2}.$$

Free surfaces are, therefore, not always horizontal.

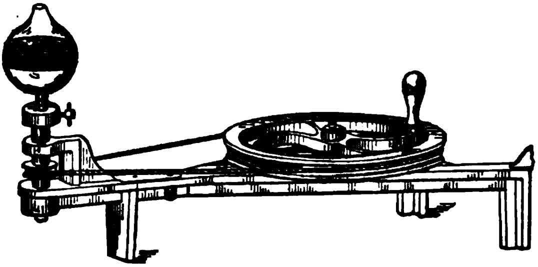

Fig. 80.

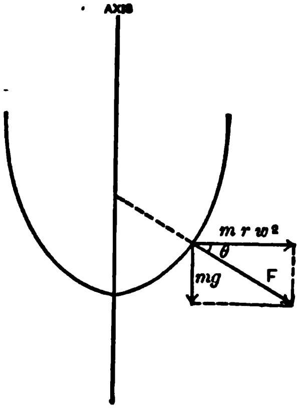

Fig. 81.

The Principle of Archimedes

150. The modern marine engineer, in designing a boat, computes the weight of all the material which enters into its construction. Often the iron used in the hull is actually weighed. Knowing the dimensions of the boat, the designer is then able to say, generally within less than an inch, and before the boat is launched, just how deep she will lie in the water. The simple manner in which this prediction is made will be evident from the following: —

Whether a body floats or sinks when placed in a liquid, one fact is patent to everybody, — the body apparently loses weight. But just *how much* weight a body loses under these circumstances was first determined by Archimedes (287–212 B.C.), the creator of our present system of hydrostatics.

It is evident that, since the pressure increases with the depth, the bottom of an immersed body will be subjected to a pressure upward which is greater than the pressure downward to which the top of the body is subjected. The pressure of the water on a body will, therefore, always exert a buoyant force.

Archimedes saw very clearly, by a simple deduction from experiment, that if from any vessel of liquid (Fig. 82) we imagine a portion of the liquid to be removed, the position of the remainder being unchanged, the upward force exerted by the pressure on the surface so exposed would just be able to support the weight of the liquid removed. Not only so, but it is evident that this upward force acts through the center of mass of the displaced fluid; since, when the fluid is replaced, there is no moment of force tending to rotate it. This point is called the **center of buoyancy.**

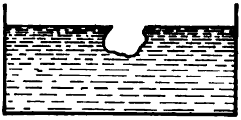

FIG. 82.

By such reasoning he established the following theorem, which is known as the Principle of Archimedes.

Proposition VII. **The resultant force of a fluid on a body immersed in it acts vertically upward through the center of mass of the displaced fluid; and is equal to the weight of the displaced fluid.**

STABILITY OF FLOATING BODIES

151. No one can read Archimedes' two papers "On Floating Bodies" — translated by Heath — without feeling that the author of those papers thoroughly understood the conditions of equilibrium. But the Dutch mathematician Stevin (1548–1620) appears first to have given these conditions a simple statement, which is essentially as follows: —

(i) The sum of all the forces (of translation) acting upon the floating body is zero.

$$\Sigma F = 0.$$

(ii) The sum of all the moments of force acting upon it is zero.

$$\Sigma L = 0.$$

When a body which will float is placed in the water, it will therefore sink to such a depth that the water displaced is equal to its own weight. The first of the two conditions is then satisfied. But when the body is first put into the water its center of gravity *G* will not, in general, lie in the same vertical line with the center of buoyancy *B*. The result is a couple, as indicated in Fig. 83, which tends to rotate the body until *B* and *G* *do* lie in the same vertical line. Then both of the above conditions for equilibrium will be satisfied.

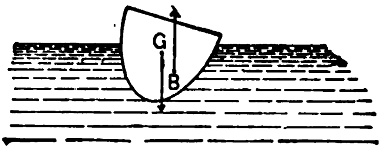

FIG. 83.

When a body "rolls" in the water, as indicated in Fig. 84, the center of buoyancy moves in the body, and thus describes a curve BB', Fig. 84 (*c*). **The center of curvature of this curve described by the center of buoyancy in its motion through the body is known as the metacenter.** The metacenter may, therefore, be considered as the point in a floating body at which the upward thrust of the water is applied. **The condition for stability in a vessel is, therefore, that its metacenter shall be higher than its center of gravity.** For then if a boat heels over, there will be called into play a moment of force tending to right it; but if the metacenter lies *below* the center of gravity, then when a boat rolls ever so little there will be a torque tending to roll it still farther.

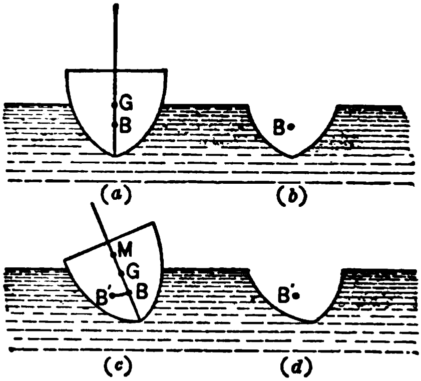

FIG. 84.

For a vessel of given displacement it is clear that the stability varies directly as the metacentric height. The capsizing of the

British gunboat *Captain* in the Bay of Biscay with great loss of life in 1870 was due to insufficient metacentric height at large angles of heel. The overturning of the *Eastland* in the Chicago River, July, 1915, appears also to have been a consequence of low metacentric height. Whether the metacenter was lowered by overloading or by letting out water ballast, is quite another question.

A ship of great initial stability is said to be "stiff"; one of small stability is described as "crank." Dynamics and experience alike indicate that crank boats are much easier and more comfortable in a rolling sea. Hence the Atlantic liners are built with a metacentric height as small as is consistent with safety.

II. Hydraulics

152. The hydrostatic problems considered thus far are only those in which the forces are in equilibrium because the pressures, at each point, are equal in all directions. But now let us consider what happens when two bottles are connected as in Fig. 85. At C there is a stopcock which retains the water at a height h in the vessel A. So long as the stopcock is closed, the pressure on any element of the liquid in the small tube is the same on the right as on the left; but when the cock is opened, the pressure on the right is only that of the atmosphere, while that on the left is considerably greater. The result is that the water flows from the left to the right until it stands at the same level in each bottle.

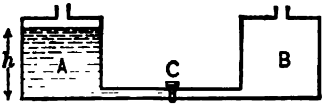

Fig. 85.

How does this happen? One reply — and a perfectly valid one — is that this is merely a special case of Newton's second law. But in order to look at the matter from another point of view, let us compute the potential energy of a mass of water m: —

1. When it is all confined in the bottle A.
2. When equally divided between the bottles A and B.

Imagine the bottles each of the same diameter and of uniform diameter throughout.

Let h denote the height of the liquid in A before the stopcock C is opened. If, now, we neglect the very small amount of liquid in the connecting tube, the center of mass of the liquid when it is all in A will lie at a height $h/2$; and the potential energy of the water with reference to the bottom of the bottle is therefore $mg \cdot h/2$.

When, however, the stopcock C has been opened, the water is shared equally by the two bottles. The center of mass in each bottle stands at a height $h/4$; the potential energy of the water in each bottle is, therefore $(mg/2)(h/4)$; and the total potential energy of the water now is $2(mg/2)\ h/4$ or $mgh/4$ instead of $mgh/2$ as in the preceding case. While neither the *mass* nor the *weight* nor the *volume* of the liquid has changed, we observe that the potential energy has diminished by 50 per cent. The motion of the liquid — the fact that " water seeks its level " — is, therefore, simply a special case of the general dynamical principle that the potential energy of a system at rest tends to become as small as possible. See § 112.

What becomes of the 50 per cent of energy which, in this particular case, ceases to be potential?

The laws of fluid friction — very different from those for solids — must be postponed for later study.

SPEED OF EFFLUX. TORRICELLI'S THEOREM

153. When a liquid is flowing through a pipe, such, for example, as that which connects the two bottles in Fig. 85, one needs frequently to know not only to what height the water will rise, but also with what speed the liquid will flow under given conditions. Owing to the difficulty of handling fluid friction, a general answer to this question has never been obtained.

But there are many cases in which friction becomes negligible; these were first solved by Daniel Bernoulli (1700–1782), the solution being described in a theorem which bears his name. We can here consider but a single special case, which was solved experimentally in 1643 by Torricelli, the lifelong friend and pupil of Galileo, two centuries before the law of the conservation of energy had been established.

Imagine a tank of water, in the side of which is placed an aperture A (Fig. 86) at a distance h below the upper surface. With what speed will the water flow from the aperture? Neglecting friction, it is permissible to apply the law of the conservation of energy to the system.

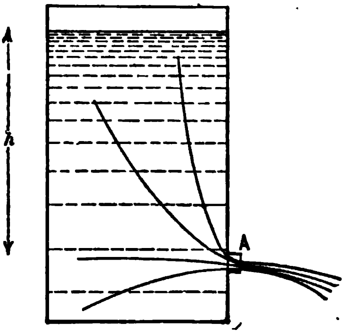

FIG. 86.

Let m denote the mass and v the speed of liquid which passes the aperture during any small interval of time, such that the upper level of the liquid will not be appreciably altered. The kinetic energy of this mass as it leaves the aperture is $\frac{1}{2}mv^2$. This energy has been gained at the expense of work done by gravity. The loss in potential energy is measured by the amount of work done in raising the mass m from the nozzle to the upper surface of the liquid against gravity. This is mgh. Since the increase of kinetic energy must just equal the diminution in potential, we have

$$\frac{1}{2}mv^2 = mgh,$$

or

$$v = \sqrt{2gh}. \qquad \text{Eq. 68}$$

This is the theorem of Torricelli, which may also be derived as a special case of the elegant theorem of Bernoulli given below.

154. It has been known since early Roman times that the quantity of liquid passing through an orifice in a given time may be materially increased by fitting to this orifice a spout which shall conform as nearly as possible to the paths of the water particles. Otherwise these paths crowd together and interfere, so to speak, and give a diminished cross section to the jet. This smallest cross section is called the *vena contracta*. It is the area of the *vena contracta* which must be used in computing the output of any orifice.

Bernoulli's Theorem

155. In general the motion at any point in a fluid is varying from instant to instant; but there are many cases of great practical importance in which the velocity of the fluid varies from point to point, but remains constant at any given fixed point.

A motion of this kind which may vary with position, but which does *not* vary with time, is said to be **steady**. The path which any particle of fluid describes in steady motion is called **a stream line**. From the definition, it is evident that one stream line never intersects another; so that we may consider a tube of the fluid bounded by these stream lines as an isolated pipe through which the fluid is flowing without friction. This is sometimes called **a tube of flow**. Five tubes of flow are represented in Fig. 87.

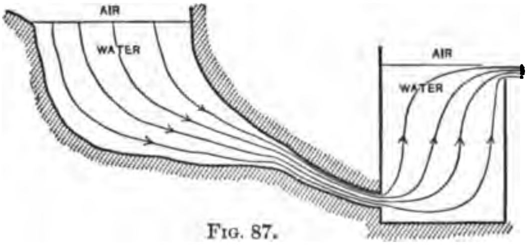

FIG. 87.

Let us now consider the still more special case in which the fluid is incompressible. Water is, for all practical purposes, such a fluid; for an increase of pressure amounting to one atmosphere will alter its volume by only one part in 20,000. If the pipe be large so that friction is negligible, we may apply to any given mass of this incompressible fluid, under steady motion, the principle of the conservation of energy; in other words, we may equate to a constant the total energy of a given mass of the fluid as it moves along through its tube of flow.

Let us consider the mass of unit volume which is numerically the same as the density and is indicated by ρ. Denote the velocity of the fluid by v; then the *kinetic* energy of unit volume is $\frac{1}{2}\rho v^2$. The *potential* energy of unit volume due to *position* is clearly ρgh, where h is its elevation above the horizontal plane of reference. In addition to this, the unit volume has potential energy due to the fact that it is under pressure. This last statement will be clear when one considers that the work required to force a volume, v', of incompressible fluid into a tank under constant pressure, p, is pv'. The area of the piston multiplied by p is the force; the product of the force by the length of stroke of the piston is pv', and is also the work done.

Equating the total energy per unit volume to a constant, we have then for any particular tube of flow,

$$\tfrac{1}{2}\rho v^2 + \rho g h + p = H = \textit{constant}, \qquad \text{Eq. 69}$$

which is the theorem of Bernoulli (1738). Upon this fundamental equation is built the entire science of hydraulics.

Variation of Pressure with Speed

156. We can consider here only a single illustration of this theorem — a very special case; namely, that in which the fluid flows through a horizontal pipe, a case in which, by proper choice of axes, we may write $h = 0$. Bernoulli's Theorem then becomes

$$\tfrac{1}{2}\rho v^2 + p = \textit{constant}, \qquad \text{Eq. 70}$$

which means that **when particles flowing along a stream line reach a portion of the tube where the speed is greater, they at the same time reach a point where the pressure is less, and *vice versa***; for otherwise the right-hand member of the equation could not remain constant.

Accordingly if a fluid be forced through a pipe with a constriction in it, such as shown in Fig. 88, the speed at *b* will be greater than either at *a* or at *c*. Now it has been shown experimentally by attaching upright tubes at these three points that the pressure is least where the speed is greatest, namely, right at the constriction. This is the experimental fact which is described by Eq. 70; and is exactly what one would expect when he remembers that it is the larger pressure at *a* which imparts the extra speed at *b*. This particular case was first described by the Italian Venturi in 1797 and is the principle used in the "Venturi water meter," invented by the American engineer, Clemens Herschel, in 1887.

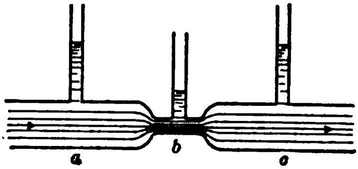

Fig. 88.

157. The various other applications of this principle which are encountered in everyday life are numerous and interesting.

Following is a short list, each of which is to be explained in terms of Eq. 69.

(i) The ordinary atomizer; an air jet produced by squeezing the bulb lowers the pressure sufficiently to raise the liquid from the bottle to the nozzle.

(ii) The Bunsen filter pump or aspirator.

(iii) The forced draught in a locomotive furnished by exhaust steam.

(iv) The ball nozzle, shown in Fig. 89, where a jet of fluid is blown so rapidly past the ball that the atmospheric pressure on the lower side of the ball holds it up snug against the socket.

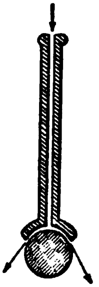

Fig. 89.

(v) The curved baseball, where the pitcher, by giving the ball the proper twist, can make it deviate either to the right or the left from the vertical plane in which the ball leaves his hand. If for the moment we neglect gravity, then the forces which act upon a rotating ball translated through still air at a given speed are precisely the same as those which act upon a rotating ball with its center at rest in a current of air moving over the ball with the same speed which the ball had in the first case. Let us therefore imagine a spinning ball tethered in a current of air, such as that represented in Fig. 90. The stream lines in air will be approximately those shown in the diagram. The sense of rotation of the ball is indicated by the arrow R. The passage of the air over the upper side of the ball is greatly facilitated by the fact that the top of the ball is moving in the same direction as the air. Consequently the speed of the passing air is greater on the upper than on the lower side of the ball; and

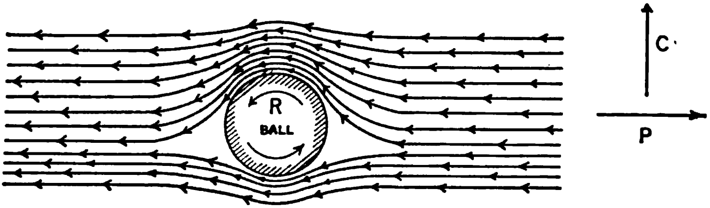

Fig. 90.

with greater speed there goes less pressure, according to Bernoulli's Theorem.

Hence if the ball leaves the pitcher's hand in the direction indicated by the arrow P, and rotates in the sense of the arrow R, it will curve toward the side indicated by the arrow C; in other words, a ball always "follows its nose." A golf ball behaves in exactly the same way. A "slice" is produced by giving the ball a clockwise rotation as seen from above; a "pull" is produced by rotation in the opposite sense; and Tait has written an essay to prove that "underspin," in which the nose of the ball is moving vertically upwards, is essential to the long carry. All these facts were thoroughly established by Magnus more than half a century ago.

Problems

208. A solid cube 12 cm. on each edge is made of a material whose density is 13.5. What is its weight in a vacuum? Find its apparent weight in water. Density of water = 1. *Ans.* 23,328 g. 21,600 g.

209. A body whose volume is 100 cm.^{3} has a density of 1.4. Find its apparent weight in alcohol whose density is 0.8. *Ans.* 60 g.

210. A piece of wood whose density is 0.6 floats on water. The volume of the wood is 40 cm.^{3} What is the volume of the water displaced? *Ans.* 24 cm.^{3}

211. What variation in pressure would be produced by descending to a depth of 76 cm. of mercury? Density of mercury is 13.546 at 20° C.

212. Find the speed of efflux from the bottom of a standpipe which is filled with water to a height of 30 ft.

213. A wire 120 cm. long weighs 40 g. in air; when immersed in water, it weighs 30 g. Find the mean diameter of the wire.

214. If a submarine boat weighs 50 tons and displaces 3000 cu. ft. when immersed, how much water will it have to take in to sink?
Duff, *Mechanics*, p. 258.

215. A rectangular vessel partly filled with water is given an acceleration of 50 cm./sec.^{2} in a horizontal direction. Find the inclination of the free surface of the liquid. *Shearer*, 461 a.

216. How much mercury will be required to fill a cylindrical tube of 5 mm. internal radius and 100 cm. length?

217. What is the specific gravity of a substance such that 3 cu. in. of it weigh a pound?

218. What is the pressure at a point 100 m. below the surface of the sea, the density of the sea water being 1.024?

219. A piece of quartz whose density is 2.7 weighs 50 g. in vacuo. What will be its apparent weight in water?

220. Show that when a cubical box is filled with water the total force exerted on the sides and bottom of the box is three times the weight of the water.

221. A town on the prairie is supplied with water from a standpipe 110 ft. high. What pressure will be available on the first floor of a house in that town?

222. From one end of a balance beam is suspended in water a 50-gram brass mass. From the other end of the beam hangs a piece of quartz, also suspended in water. Given the density of brass as 7.0, the density of quartz 2.7, and the fact that their two weights are in equilibrium, find the weight of the quartz.

223. A silver coin having a density of 9 is dropped into a lake 20 ft. deep. How long will it take the coin to reach the bottom, neglecting friction?

III. SURFACE TENSION. CAPILLARITY

158. When two liquids are brought together, either they mix and diffuse the one into the other, or they do not mix, and consequently exhibit what are known as the phenomena of surface tension. In these capillary phenomena, as they are sometimes called, the very laws of hydrostatics which we have just been studying *appear* to be contradicted.

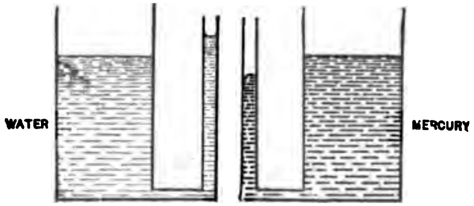

FIG. 91.

Arrange two U-tubes (as in Fig. 91), the smaller arm having an inner diameter of from one to two millimeters.

Partly fill one U-tube with mercury and the other with water: the mercury in the small arm does not rise so high as in the large; while with the water the reverse is true. Either of these phenomena is sufficient to show that, in communicating vessels, liquids do *not always* stand at the same height. But, as noted above (§ 142), this is only the fact when the communicating vessels have the same diameter, or when each is large.

Any one who has ever placed a small drop of mercury on a plate of glass knows that its center of mass stands at some distance, perhaps a millimeter, above the surface of the glass (see Fig. 92). This phenomenon, *on its face*, would appear to be an exception to the general dynamical principle that the potential energy of a system tends always to a minimum. Why does not the drop flatten out? Why does not its center of mass assume a lower position? Our task now is to show that these contradictions are only apparent, and that the phenomena of the liquids in the U-tube and the phenomenon of the drop of mercury are in perfect accord with general principles already known. **In the discovery of the harmony between any one truth and all other truths lies the essential feature of an "explanation."**

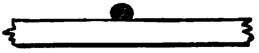

FIG. 92. — Mercury on glass.

159. No one can try the following qualitative experiments without being impressed with the fact that liquids **in air** behave very much **as if** they were contained in a sac or thin membrane, continually holding them in position.

1. A glass rod dipped into a tumbler of water picks up a drop and carries it on the end of the rod **as if** a little rubber bag were tied about the end of the rod and filled with water.

2. A falling raindrop nearly always assumes a spherical shape **as if** it were inclosed in a skin of some sort, and this skin were continually compressing it into the smallest possible volume.

3. In a camel's-hair brush, when dry, the individual hairs stand apart to some extent, giving it a rather bushy appearance. Dip the camel's-hair brush into a tumbler of water; the same bushy appearance remains. Remove the wet brush from the water to the air; the film of adhering water at once binds the individual hairs together as a band does a sheaf of oats. The liquid **in air** acts **as if** it were covered by a film always tending to contract.

4. A piece of clean, fine wire gauze may be bent into the shape

of a small open box. Water may be poured into this box to a considerable depth without its running through the meshes.

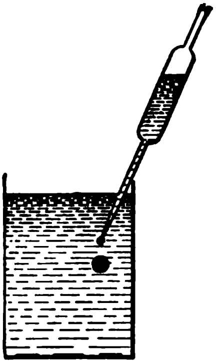

FIG. 93.—A drop of oil freed from gravity.

5. One may make a mixture* of alcohol and water that will have very nearly the same density as olive oil — a combination known as Plateau's Mixture. A drop of oil placed in this liquid, as indicated in Fig. 93, will not mix with it, nor will it be affected by gravity. How do you explain the perfectly spherical shape which the oil assumes?

6. One of the simplest methods of observing this phenomenon of liquid surfaces is the following, due to Van der Mensbrugge: —

Make a strong solution of castile soap. Bend a piece of aluminum wire (about No. 20 or No. 25) into the shape shown in Fig. 94, making the diameter of the ring about two inches. At some point *A* of this ring tie a loop of thread *B*. If, now, the ring be dipped into the soap solution, a film will spread over it. This film has two surfaces inclosing a thin layer of the soap solution between them. The thread floats on the film; now with the point of a lead pencil puncture the film at some point within the loop of thread *B*.

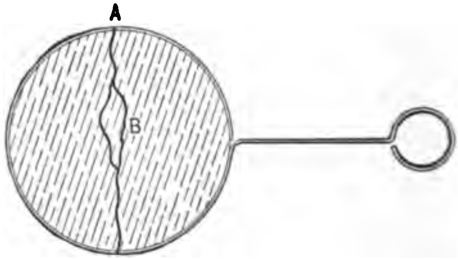

FIG. 94.

Each surface of the *film* between the wire and the thread now *tends to become as small as possible,* as is

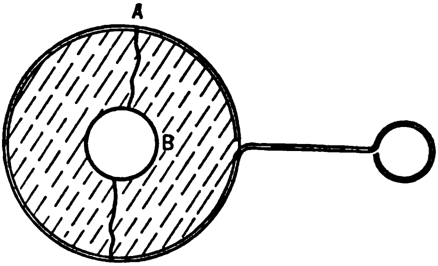

FIG. 95.—Soap film assuming smallest area.

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* About 60 per cent of alcohol (density 0.806) and 40 per cent of water will give a mixture which is nearly right for ordinary olive oil (density 0.917).

indicated by the fact that the area within the loop tends to become as large as possible, *i.e.* the loop assumes a circular form, as shown in Fig. 95.

160. Such evidence as this proves that if we enlarge a liquid surface we shall have to do work upon it just as truly as if we stretch a rubber band or inflate a bicycle tire. On the other hand, if a surface contracts, it exerts a force in so doing, and thus does work for us.

A wire is easily bent into the shape indicated in Fig. 96. Dip this wire into a soap solution to cover it with a film. If, now, a small, straight wire, *CD*, is laid across the frame, the film on the *B* side may be broken out with the finger. The two surfaces on the *A* side will exert a certain definite force and draw the bar *CD* quickly to the end of the frame. The total force thus exerted upon the bar, divided by twice the width of the film, will give the **force per centimeter which a single liquid surface exerts** upon the cross bar. This quantity is naturally measured in dynes per centimeter, and is called the **surface tension** of the liquid. Surface tension is denoted by *T*.

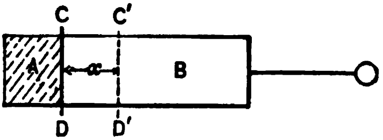


FIG. 96.—Work done by a contracting film.

Suppose the bar $\overline{CD}$ is displaced a distance x, as indicated in the figure. The *force* which we must use to do this is $2\ T \cdot \overline{CD}$ dynes. The *work* which one must perform is $(2\ T \cdot \overline{CD})x$ ergs. The increase in area is $2\,x \cdot \overline{CD}$ square centimeters. For the film has two surfaces.

Consequently, the amount of energy *in unit area* is

$$2\,x \cdot T \cdot \overline{CD}/2\,x \cdot \overline{CD} = T.$$

Accordingly we may define the surface tension of any liquid as the amount of potential energy in unit area of its surface, a definition which is strictly equivalent to the one given above.

And now we see that, when a drop of mercury gathers itself up into a little round globule, lifting its center of gravity above the plate on which it rests, it does so in order to make its surface as small as possible, and thus reduce its potential energy to a minimum. So far, therefore, from violating our previous experience, this behavior of liquid surfaces is quite in accord with it.

161. Before examining more closely into the nature of this energy which we have found resident in the surface of a liquid, it will be necessary to show that liquid particles attract their immediate neighbors just as solid particles do. A crayon of chalk is more or less difficult to break, because across any section of it the particles on one side are attracting the particles on the opposite side with considerable force. That this same phenomenon happens in liquids may be conveniently demonstrated by a tube partially filled with water and then freed from air and sealed — what is ordinarily called a "water hammer." The liquid in a tube of this kind may, without difficulty, be made to assume the position indicated in Fig. 97, an experimental fact which it is difficult to explain except by assuming that the glass walls of the tube attract the adjacent particles of water, and these particles in turn attract the neighboring particles of water; and so on, throughout the entire mass of liquid. These forces — which are imperfectly understood — are sufficiently great to sustain the excess of weight of one column over the other.

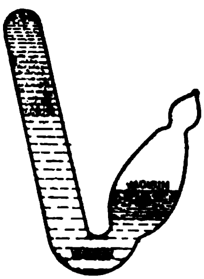


FIG. 97. — Water hammer, illustrating cohesion in liquids.

162. Another fact which is very certain is that there is nothing of the nature of an actual membrane stretched over the free surface of a liquid; for actual ordinary membranes (i) are not perfectly extensible and contractile, and (ii) do not, in general, show the same tensile force in all directions; nor (iii) is this tensile force independent of the thickness of the film.

How, then, is this apparent membrane and this surface energy to be explained? The following answer, for the sake of its admirable clearness and comprehensiveness, is taken from Professor W. F. Magie's *Lectures on Physics*, Sec. 67 (Princeton, 1904): —

"The general laws of hydrostatics depend upon the principle that a liquid subject to the attraction of gravity will be in equilibrium only when its configuration is such that the action of gravity on it introduces no shearing stresses. Now gravity is not the only force which acts on the liquid. Its parts also exert forces of cohesion on each other, and true equilibrium will not be reached until the liquid assumes such a position that these forces of cohesion so act, together with the weights of the parts of the liquid, as to introduce no shearing stresses. We can explain all the phenomena which are treated under capillarity by taking these forces of cohesion into account. It is not necessary for us to know, and in fact we do not know, the way in which the cohesion depends upon the masses of the interacting parts and the distances between them. It is necessary, however, to assume this much, that the force of cohesion exerted by a small part or element of the body acts only on those elements which are in its immediate neighborhood. That is, we assume, as the law of the force of cohesion between elements, that the force between contiguous elements is very great, and diminishes very rapidly when they are separated, so as to become imperceptible, even when the distance between them is still very small. This general law of cohesive forces is illustrated by the behavior of an iron bar when we break it. It requires a very great force to break it; but after it is broken, even though the two surfaces at the break are fitted together again with the utmost nicety, the two parts can be separated with no perceptible effort.

"Because of the short distance within which an element of the liquid acts on its neighbors, those elements which lie below the surface by a depth equal to this distance, which we call the range of action, are in equilibrium under the action of their neighboring elements. It is only those elements which lie in or very near to the surface which are attracted unequally in different directions. These unequal attractions, acting on the elements of the surface film, will produce a peculiar condition in it. This will be the same for all parts of the surface, owing to the minuteness of the range of action, so long as the radius of curvature of the surface is not very small, that is, is not of the same order of magnitude as the range of action. Thomas Young (1773–1829) suggested that the special action of the cohesive forces in the surface film may be represented by supposing the film to be under tension, similar in general to that in a stretched membrane. This tension should be the same, in the case of any given liquid, for all parts of its surface. It is called the surface tension, and its numerical value, when determined for the surface of separation between any two bodies, is a characteristic number for those bodies. The position of the liquid column in the capillary tube, or any of the other phenomena ascribed to capillary action, are, on this view, due to equilibrium between the weights of the parts of the liquid and the forces due to the tensions acting in curved portions of the surface film.

" Another useful concept for the study of capillary phenomena was introduced by Gauss (1777–1855). It is now called surface energy. It is plain from the consideration of the action of the cohesive forces, that when the surface of a liquid mass is enlarged, the potential energy of the liquid is increased on that account. For the surface can only be enlarged by the passage of elements of the liquid from the interior mass into the surface film. Each of the elements in the film is drawn inward toward the interior by a force of cohesion, and hence negative work is done on all the elements which pass out from the interior into the surface film. The negative work thus done is equivalent to an increase in the potential energy of the liquid. The surface, therefore, possesses an energy peculiar to itself, proportional to the extent of surface and characteristic of the bodies separated by the surface. For a given surface its numerical value is the same as that of the surface tension for the same surface.

" By the aid of this concept of surface energy all the phenomena of capillarity may be explained as illustrations of the general principle that the potential energy of a system of bodies at rest tends to become the least possible."

LIQUID JETS

163. When a continuous stream of liquid flows from a tap or garden hose and breaks up into small drops, the phenomena occur in such rapid succession that the eye cannot follow them with any certainty. In order to reduce the rapidity with which these events occur Plateau arranged a large drop of olive oil between two rings — one capable of moving up and down above the other — as shown in Fig. 98 — the whole immersed in a mixture of alcohol and water of the same density as oil.

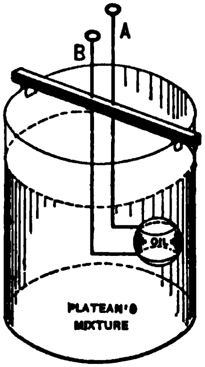


Fig. 98.

By sliding the rod A up, the globule of oil elongates itself in the same way that it would if it were falling freely under gravity from a nozzle. It is found that the column of oil becomes unstable when its length exceeds its circumference, and that the surface tension proceeds first to form a sort of waist and then to pinch the column in two, leaving between the two parts a small drop such as that pictured in Fig. 99. The first three of these figures represent the behavior of the drop of oil in Plateau's Mixture; the right-hand figure refers to the jet of water.

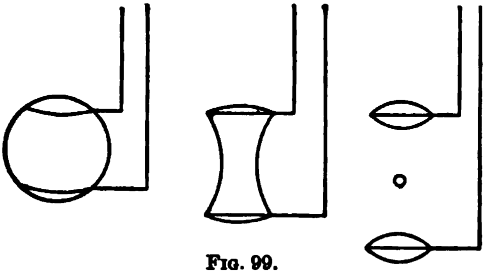
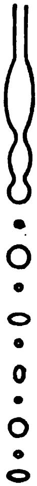


FIG. 99.



Instantaneous photographs of jets of water, falling freely, show that they are pinched off by surface tension in precisely this manner.

Capillary Phenomena

164. When a solid is partially immersed in a liquid, either one of two things may happen; namely, the particles of the solid may attract those of the liquid more strongly than the particles of liquid attract each other, in which case the liquid is said to "wet" the solid; or the particles of the liquid may cohere with a greater force than that with which the particles of the solid attract them, in which case the liquid "does not wet" the solid.

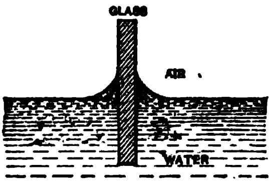


FIG. 100.—A solid which is wet by the liquid.

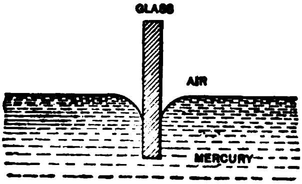


FIG. 101.—A solid which is not wet by the liquid.

In the former case the liquid appears to run up the side of the solid, as shown in Fig. 100; while in the latter case the liquid is depressed in the neighborhood of the solid, as indicated in Fig. 101.

The angle at which the liquid surface meets the immersed portion of the solid is perfectly definite and is known as the angle of contact.

165. If the immersed solid be in the form of a fine glass tube as shown in horizontal and vertical section in Fig. 102, it is observed that water will rise to a much greater height on the inside than on the outside.

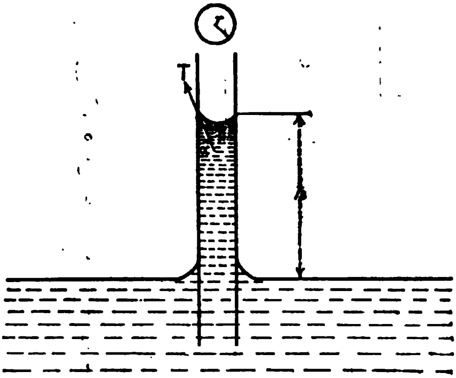


Fig. 102.

Let T denote the surface tension of water, r the radius of the tube, α the angle of contact, D the density of water; then h, the excess of height, may be computed as follows: —

The upward component of the force per unit length acting at each point where the liquid surface touches the inside wall of the tube is $T \cos \alpha$. Hence the total force along the direction of the axis of the tube is $2\pi r T \cos \alpha$. The total weight of the column of liquid raised above the level of the outside liquid is $\pi r^2 hDg$. Since these two are the only forces acting upon the liquid column, their sum must be zero. Hence we have

$$2\,T \cos \alpha - rhDg = 0,$$

a single equation which serves to determine h, and which shows that, other things remaining the same, $h \propto \frac{1}{r}$, a fact which is known as **Jurin's Law.**

Attraction and Repulsion of Floating Bodies

166. Another curious consequence of the elevation and depression of liquids on the surface of immersed solids is the attraction of floating bodies such as may be observed about the edge of any quiet lake, where all the little twigs that have fallen into the water will be found attached either to each other or to the shore as if tied up to a wharf.

An after-dinner method of observing this phenomenon is to take two English walnut shells, and float them in a finger bowl. It will be found that one of these shells will tow the other about with perfect ease, the only connection between the two being the liquid film which rises between them.

This state of affairs is diagrammatically represented in Fig. 103, and may be completely explained by reference (§ 148) to the following general principle; namely, *Over any portion of the free surface of a liquid which is plane, and at all points in the liquid which are at this same level, the pressure is constant and is equal to that of the atmosphere. As one ascends in the liquid from this level the pressure diminishes; as one descends in the liquid, the pressure increases.*

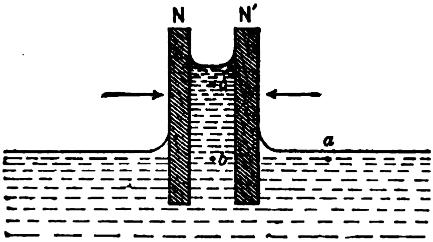


FIG. 103. — Attraction of two bodies, each wet by the liquid.

Thus in Fig. 103, the pressure at *b* is the same as that at *a*; and the pressure at each of these points is that of the outside atmosphere. As one ascends to the point *c* between the two floating nut shells, the pressure diminishes, while outside the liquid the pressure remains that of the atmosphere. The result is a force in the direction from the greater to the less pressure, which holds the two bodies together, so that if one shall be pulled about in the finger bowl it will tow the other behind it.

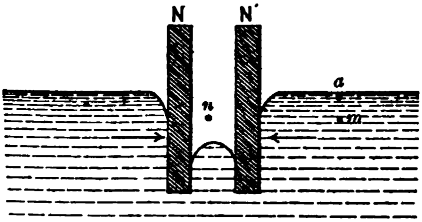


FIG. 104. — Attraction of two bodies, neither of which is wet by the liquid.

167. Oddly enough the same thing happens when *neither* of the two floating bodies is wet by the liquid, as for instance two pieces of paraffine or two pieces of beeswax, a case which is diagrammed in Fig. 104. Here the pressure at *a* is that of the atmosphere; the pressure at *m* is considerably greater; while at *n*, on the same level with *m*, but in the open air, the pressure is only that of the atmosphère.

The result is, therefore, two forces in the direction of the arrows tending to hold the two bodies together.

168. Only one other possibility remains, namely, one of the bodies may be wet and the other one not. When two such bodies float near each other, it will be observed that, in the region *between* them (Fig. 105) the liquid does not rise quite so high against the wetted one as it does on the *outside* of the wetted one. Note also that in the region between the bodies the liquid rises a little higher against the body which is not wet than it does on the outside of this same body.

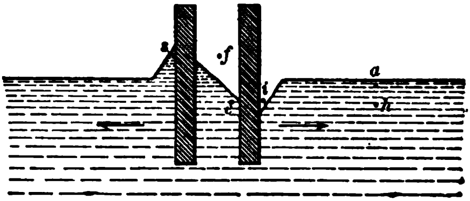


FIG. 105. — Repulsion of two bodies, one of which is wet, the other not.

Following the ordinary laws of dynamics, as in the preceding cases, the pressure at *a*, at *i*, and at *f* is that of the atmosphere; pressure at *h*, and at *e*, which is on the same level, is greater than that of the atmosphere. Hence there is a force from *e* towards *i* as indicated by the arrow.

The pressure at *s*, which is higher in the liquid than *a*, is less than that of the atmosphere, resulting in a force from *f* towards *s*, as shown by the arrow. Accordingly the two bodies appear to repel each other.

Many floating bodies, by being slightly elevated or depressed in the water, may be made to act either as bodies which are wet or not wet by the liquid. It is often convenient thus to illustrate all three cases by the use of only two different bodies.

169. There is a host of other beautiful phenomena which fall under the head of capillarity, such as those of thin soap films, the variation of surface tension with impurities, electric field, etc.; but these must be left for outside reading and observation.

Two remarks, however, before passing to a new chapter. The spreading of a drop of oil over the surface of water or of iron is a phenomenon of especial interest to the engineer; for on this property depends the lubricating effect of oil upon surfaces which would not otherwise be reached by the lubricant or would not retain the lubricant, *e.g.* the pallet of a clock.

The surface tension of water is diminished by almost any substance — even soap — which one may dissolve in it. The tenacity of a soap solution, making possible the exquisite soap bubbles of childhood, arises from the increase of viscosity — temporary rigidity, so to speak — conferred by the soap.

Problems on Capillarity

224. Two mercury globules, of radii 2 mm. and 1 mm., respectively, are brought into contact. They unite and form one globule. Compare the surface of this globule with the sum of the surfaces of the two globules before union. Compare the sum of the surface energies before union with that after union.

225. How is it that you can round the end of a glass rod by holding it in a Bunsen flame?

226. Chemists often use a solid glass rod, instead of a funnel, in pouring a liquid from one vessel to another. Explain this.

227. Find the pressure, due to surface tension, inside a spherical drop of pure water whose radius is 2 millimeters. The value of the surface tension for pure water in air is 75 dynes per centimeter.

228. How high will pure water rise in a glass tube whose internal diameter is $\frac{1}{10}$ mm.? Assume the angle of contact to be zero.

229. Find the pressure in a spherical soap bubble which is 8 centimeters in diameter, assuming the surface tension as 30 dynes per centimeter.

230. Explain by a diagram on the blackboard just how it happens that a jet of water flowing from a tap breaks into drops at a short distance below the nozzle.

231. An inverted U-tube supports two soap bubbles, one from each arm of the tube. The diameter of one bubble is 3 cm., the diameter of the other 1 cm. If the tube be left to itself, which bubble will increase in size, and why?

232. How does surface tension determine the size of the drop of liquid which may be held on the end of a glass rod?

233. When a soap bubble breaks, it is observed that the water is thrown violently in many directions. Explain.

234. A test tube is filled with dry sand at the temperature of 20° C. If this sand be moistened by pouring on a little water at 20° C., is there any reason for thinking that the temperature of the mixture will be different from 20° C.?

PROPERTIES OF GASES

170. Most of the theorems which we have just been studying under the subject of Hydrostatics are quite as true for fluids in general as for liquids; all of them, in fact, except those which refer to the "free surface" of a liquid. For this "free surface" is precisely the thing which we do not find in gases. A gas might, indeed, be defined as a body which has no free surface.

A sailing vessel under way, a tree bending to a breeze, a bank of sand raised by the wind, is each an evidence that this air, upon which we are so dependent for life, is something which can hand over momentum to other bodies. Any one who has ever pedaled a wheel against a head wind knows that air exerts pressure when in motion, and that air, therefore, probably has inertia. The boy who has blown up a paper bag for the purpose of bursting it between his hands knows at least that the inclosed air is a body.

The lad who has lifted a brick with a piece of wet leather and a string (Fig. 106) knows that he must keep the edges of the leather down snug on the brick; and that, when he does so, the earth's atmosphere, without difficulty, keeps the brick up snug against the leather, except in the middle. Here the leather has been lifted, and has in turn lifted the whole column of the earth's atmosphere, leaving, on top of the brick, a space in which there is practically no air.

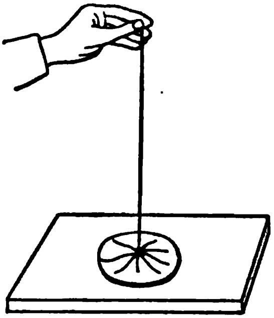


FIG. 106. — The "sucker."

The ancients were, perhaps, essentially as familiar with these facts as we are; at any rate, they were familiar with several kinds of pumps dependent upon these principles. Of the connection between these various facts it appears that they had little or no conception. But since the essential feature of scientific knowledge consists not only in knowing the facts, but also in knowing the connection between them, we proceed to the consideration of some of the properties of gases and of their connection one with another.

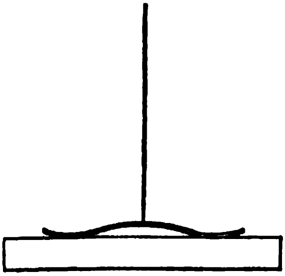


FIG. 107.—Section of sucker shown in Fig. 106.

171. Before the time of Galileo nearly all the phenomena of pumps were explained by saying that "nature abhors a vacuum." And this *horror vacui* is a fairly good explanation; it will, in any event, explain a large part of the more familiar phenomena, such as those of air pumps and barometers. In addition to this "horror of a vacuum," it was proved by Galileo that air had weight. This he accomplished by weighing a bottle into which he had pumped two or three times its volume of air, and afterward, having allowed the excess of air to escape, again weighing the bottle. For Galileo's own account of this experiment see his *Two New Sciences*, pp. 78–80 (Macmillan, 1914).

These were two capital discoveries: (1) that nature abhors a vacuum, and (2) that air has weight.

172. Torricelli (1608–1647) showed, by a justly celebrated experiment, the connection between these two facts; namely, nature abhors a vacuum *because* the air has weight. Torricelli's experiment was as follows: Taking a tube more than 76 centimeters long and closed at one end (Fig. 108), he filled it with mercury, and placing his finger over the open end, inverted the tube in a dish of mercury. The column of mercury fell a short distance, but remained standing in the tube approximately at the height of 76 centimeters above the surface of the mercury in the dish.

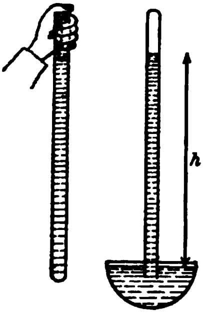


FIG. 108.—Torricelli's experiment.

Torricelli ascribed the support of this column of mercury to the pressure of the atmosphere, and explained the pressure in turn as due to the weight of the atmosphere. He showed, indeed, that the weight of the earth's atmosphere is very approximately that of an ocean of mercury covering the earth to the depth of 76 centimeters.

These views of Torricelli were verified in 1648 by Pascal, who carried the inverted tube to the top of the Tour St. Jacques in Paris and found that the mercury fell as he ascended. Later Pascal had the inverted tube carried to the top of Puy de Dôme, a mountain in France; here the mercury fell seven or eight centimeters during the ascent. Robert Boyle in 1664 gave to this inverted tube the name **barometer.**

The evidence for thinking that we reside at the bottom of an ocean of air, exerting its pressure equally in all directions, was completed by Otto von Guericke, who devised, in 1650, a pump exactly like a single-acting air pump, only the valves were not automatic. This enabled him to remove the greater part of the air from a glass vessel attached to the pump. In the nearly vacant space thus formed he found that the pressure was much less than that of the outer air, and on opening the valve, air would immediately rush in until the pressure within became equal to that without.

Scientists have agreed to adopt 76 centimeters as the standard height of the barometer at sea level; and the pressure required to support a column of mercury of this height is known as the normal atmospheric pressure. See § 144.

From the observed height of the barometer at any time, we can readily calculate the pressure of the atmosphere. For in Fig. 108 the pressure B over the free surface of the mercury in the basin is that of the atmosphere; while the pressure at the same level inside the tube is that due to a column of mercury whose height is h; and since these two pressures are equal we have, Eq. 67,

$$B = Dgh \text{ dynes per sq. cm.,} \qquad \text{Eq. 71}$$

where D at 0° C. is 13.596, $g = 980.69$, and h, at the surface of the earth, never very far from 76 centimeters. The pressure which the atmosphere exerts upon a barometer is mainly dependent upon two factors; namely, (1) the height of the atmosphere, and (2) the average density of the atmosphere. Anything which changes either one of these changes the "reading of the barometer," *i.e.* the difference in level between the two horizontal surfaces of the mercury in the barometer.

Water vapor, when under the same pressure as air, has a density which is less than that of air. If, then, there be much water vapor in any portion of the atmosphere, its density will be diminished, consequently its pressure will be diminished, and the mercury column which it supports will be shorter. The barometer is said to "fall." But the same thing happens when the height of the atmosphere is lowered or its pressure is changed by a cyclonic motion. The barometer is, therefore, *not* an instrument for telling whether or not it is about to rain, but for measuring the pressure of the earth's atmosphere. Nevertheless, barometric observations are of great aid to the Weather Bureau in predicting atmospheric changes.

Composition of the Earth's Atmosphere

173. More than a century elapsed between the time of Torricelli and the discovery of the principal elements in the air which we breathe. These were first shown by Lavoisier (1743–1794), one of the founders of modern chemistry, to be nitrogen and oxygen, in the approximate proportion four of nitrogen to one of oxygen. In addition to these there is always present a variable quantity of water vapor, $\frac{3}{100}$ per cent of carbonic acid gas, nearly 1 per cent of argon, together with traces of the recently discovered gases, neon, crypton, xenon, and helium. Dust is another constituent of the air whose importance has not long been appreciated. It is indeed since 1880 that J. Aitken has proved that water vapor in the air will condense only upon some solid or liquid nucleus. If there were no dust in the atmosphere, this condensation would take place upon our clothes, furniture, etc. The purest sea air contains about 100 dust particles per cubic centimeter; while in large cities a single cubic centimeter of air contains often more than 100,000 particles.

When we remember that this ocean of air in which we live is invisible, odorless, tasteless, apparently devoid of weight, and almost intangible, it is little wonder that the discovery of its properties came rather late in the history of science.

The principal facts in regard to the composition of the earth's atmosphere have been admirably summarized by Professor W. J. Humphreys of the U. S. Weather Bureau in the following diagram.

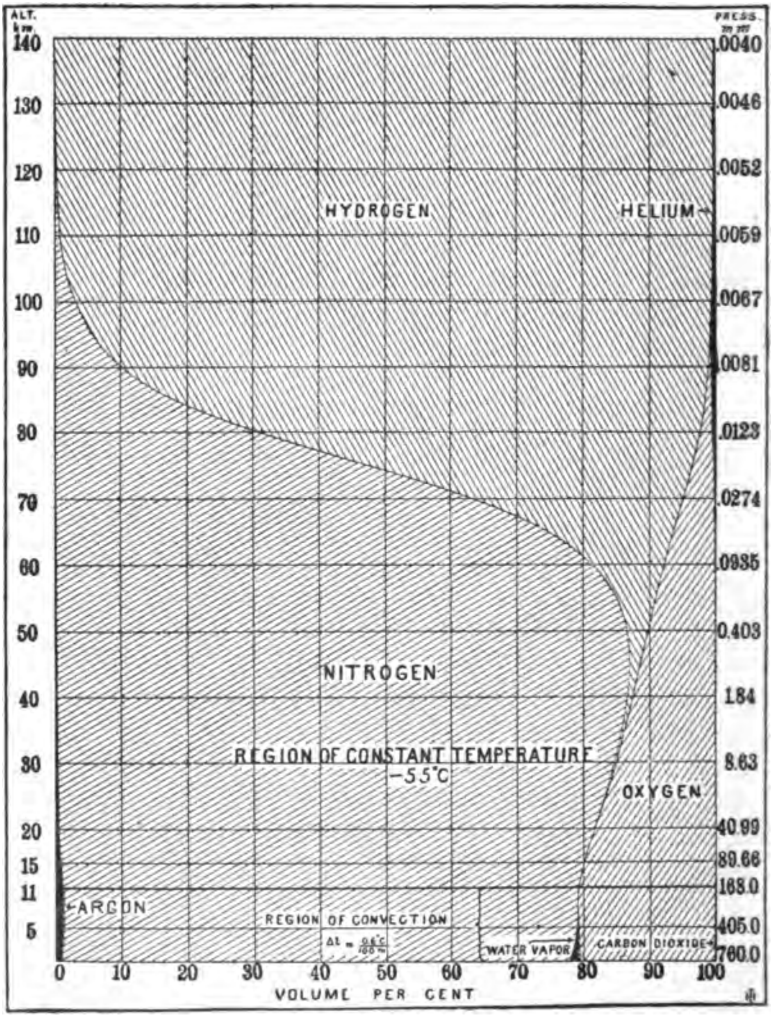


Fig. 108 *bis*. — Composition of the atmosphere.

Professor Humphreys bases this diagram (Fig. 108 *bis*) upon the following assumptions: —

" 1. That at the surface of the earth the principal gases of the atmosphere and their respective volume percentages in dry air are: —

Table I

Nitrogen	78.03	Hydrogen	0.01
Oxygen	20.99	Neon	0.0015
Argon	0.94	Helium	0.00015
Carbon dioxide	0.03		

" 2. That water vapor at the surface of the earth amounts to 1.2 per cent of the total quantity of gas present.

" 3. That the water vapor rapidly decreases, under the influence of lower temperatures, with increase of elevation, to a negligibly small amount at or below the level of 10 kilometres.

" 4. That the temperature decreases uniformly at the rate of 6° C. per kilometre from 11° C. at sea level to – 55° at an elevation of 11 kilometres.

" 5. That beyond 11 kilometres above sea level the temperature remains constant at – 55° C.

" 6. That up to the level of 11 kilometres the relative percentages of the several gases, excepting water vapor, remain constant — a result, of course, of vertical convection.

" 7. That above 11 kilometres, where the temperature changes but little with elevation, and where vertical convection, therefore, is practically absent, the several gases are distributed according to their respective molecular weights.

"A number of atmospheric gases — neon, krypton, xenon, ozone, etc. — are omitted both from Table I and from its accompanying figure. This is because all these occur — in the lower atmosphere, at any rate — in quantities too small for graphical illustration in the same diagram and to the same scale as are the principal gases.

" In using this diagram it should be distinctly remembered that it is supported by direct experimental observations only from the surface of the earth up to a level of about 30 kilometres, and that while the extrapolated values are based upon apparently sound logic and not upon mere surmises, still they necessarily become less and less certain with increase of elevation."

Jour. Franklin Institute, March, 1913.

CIRCULATION OF THE EARTH'S ATMOSPHERE

174. Few motions have more significance for the human race than those of our atmosphere. Of these there are many; but the trade winds and cyclones are of especial importance and interest. It is assumed that the student has already made the acquaintance of trade winds in his study of physical geography. Accordingly we pass at once to the explanation of cyclones, a name which has been given to those great circular rotating storms which sweep over both our northern and southern hemispheres, and which are responsible for most of what we call "our weather." But since these immense whirls, ranging from 500 to 1000 miles in diameter, acquire their peculiar twist entirely from the earth's rotation, it will be necessary first to consider just how the motion of any body over the surface of the earth is affected by the daily motion of our planet.

Deflection to the Right

175. We are living on the surface of a spinning top whose angular speed is $\frac{2\pi}{86400}$ radians per second. The angular velocity of this sphere, ω, may be represented by the vector $\overline{OP}$ in Fig. 109, where O is the center of the earth and P the north pole. Let E represent a point on the equator and C any locality in latitude λ. Then the vector ω may be resolved along the direction OC. The value of this component $\overline{OH}$ is $\omega \sin \lambda$. The effect of the rotation of the earth, then, at any point C is precisely the same as if the point C were situated at the pole P and the earth had an angular speed of $\omega \sin \lambda$ instead of ω. Let us consider, therefore, how the motion of, say, a bullet would be affected by the earth's rotation (i) when the gun is directed along any meridian at a target situated exactly at the north pole. Imagine the distance of the target from the gun to be 400 yards.

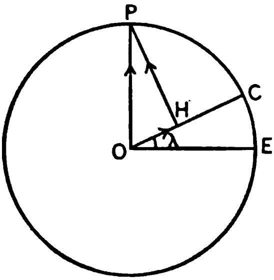


FIG. 109.

The speed of the gun toward the east will then be $400\left(\frac{2\pi}{86400}\right)$ yards per second, or approximately 1 inch per second. If the muzzle speed is 1800 feet (600 yards) per second, it will require $\frac{2}{3}$ of a second for the shot to reach the target. But during this $\frac{2}{3}$ of a second the bullet has drifted some $\frac{2}{3}$ of an inch towards the east; and, hence, if the gun is accurately aimed at the target, the shot will strike $\frac{2}{3}$ of an inch *to the right* of the bulls'-eye. (ii) Next suppose the muzzle of the gun placed exactly at the north pole and the target placed on a parallel of latitude 400 yards away. The bullet as it leaves the gun now has no easterly velocity impressed upon it; but the target *has*. During the $\frac{2}{3}$ of a second that the projectile is in the air, the target has moved $\frac{2}{3}$ of an inch to the left, the result being that the bullet strikes *to the right* of the bull's-eye as before. (iii) Next let us suppose that both gun and target are situated 400 yards apart on the same parallel of latitude. Let M (Fig. 110) denote the muzzle of the rifle and T_1 the position of the target at the instant when the shot is fired. With the north pole P as center, draw a circle through M and T_1. It is then clear that, during the $\frac{2}{3}$ of a second that the shot is in the air, the target will move through the angle θ, amounting to $\frac{2}{3} \times 15''$, that is, $10''$ of arc; so that when the bullet strikes the target, the latter is in the position marked T_2. Hence the bullet moving straight ahead according to Newton's first law hits the target *at the right* of the bull's-eye.

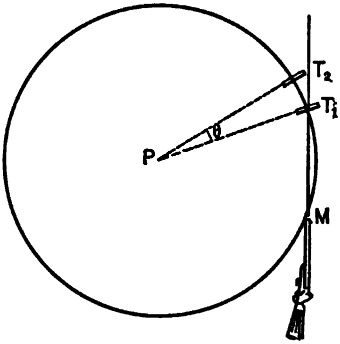


FIG. 110. — Illustrating deflection to the right.

The student will find it interesting to prove that if the gun had been directed exactly west instead of east as shown in Fig. 110, the deflection would still have been to the right. Prove also that at the *south* pole the deflection will be always to the *left*.

(iv) Next suppose we leave the polar region and pass to a locality C whose latitude is λ. The only change is that the angular speed, *about a vertical line,* has now been reduced in the ratio of sin λ to unity. Thus in latitude 30° N. the drift is always to the right, but just half as large as at the north pole.

Cyclones

176. We are now prepared to consider the typical storm which takes its origin in a region of low atmospheric pressure. This low barometer may occur in consequence of various changes — let us imagine it due to excessive heating of the earth's surface by sunshine in some particular locality. As the heated air rises and flows off, the surrounding air rushes from all sides into this region of low barometer, as shown in Fig. 111. But from whichever of the four corners of the earth the air moves, observe that it will have impressed upon it the same deflection to the right that we found in the case of the rifle ball. The result is that **all storms in the northern hemisphere having a "low" center rotate in a counter-clockwise sense.** Hence also the rule of Buys-Ballot: To find the storm center stand with your back to the wind and then the center of low barometer will be on your left.

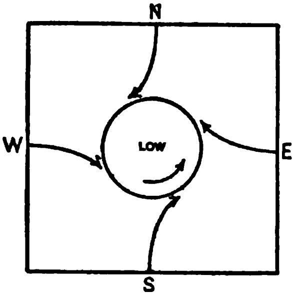


Fig. 111. — A typical cyclone of the northern hemisphere.

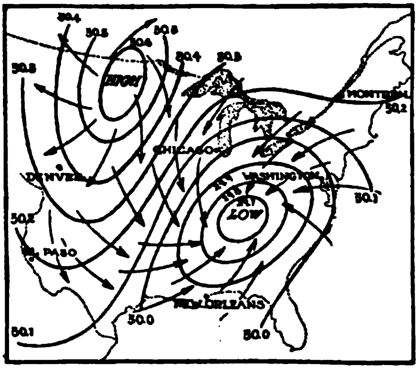


Fig. 112. — Typical cyclone. This chart shows lines of equal barometric pressure in inches of mercury, with resulting winds, at 8 A.M. January 2, 1904. The arrows show the tendency of the wind to blow from a High towards a Low, and also the spiral motion caused by the earth's rotation, outward from a High, inward towards a Low.

But some disturbances, generally accompanied by clear weather in summer and cold in winter, take their origin in regions of *high* barometer. These are called **anticyclones.** Here the air is moving away from the center and toward the periphery; and since the deflection is still to the right, the circulation of the air is now in the clockwise sense. The accompanying chart, Fig. 112, shows the circulation of the atmosphere in the United States for January 2, 1904. The fundamental phenomenon of meteorology is, of course, the transmission of solar radiation through the earth's atmosphere. But so far as circulation of the air is concerned the dominant feature is the great west wind which prevails through the temperate regions and represents the returning trades.

177. Another point of view, and perhaps a better one, from which the deflection to the right may be regarded, is the following, given by Professor Cleveland Abbe in terms of centrifugal force: —

> "The difference between the deflection to the right in the Northern Hemisphere and that to the left in the Southern Hemisphere results from the nature of the forces that produce these deflections, and not from the way in which the observer looks at the weather map. The deflections are true natural phenomena, not mere optical delusions.
>
> "When a body rests quietly on the earth's surface the centrifugal force, *cd* (Fig. 113), due to the diurnal rotation of the earth, gives the body a slight tendency to move toward the equator, which tendency is counterbalanced by the fact that the surface of the earth, and especially of the ocean, is an oblate spheroid; the attraction of gravity, *ac*, is not perpendicular to this spheroidal surface, but is directed toward the center of the earth and its action on any body at the surface must be resolved into two components; the principal one, *ab*, is vertical or normal to the spheroidal surface and constitutes the greater part of what we call weight, the other component, *bc*, is a feeble horizontal sliding force directed toward the pole (the North Pole in the Northern Hemisphere and the South Pole in the Southern). The centrifugal force, *cd*, is directed outward in the plane of the small circle of latitude and is also to be resolved into two parts, one of which, *ed*, is normal to the surface of the spheroidal globe; it acts upward, and therefore partly counteracts the force of attraction; the difference between it and the attraction is called *apparent gravity*,

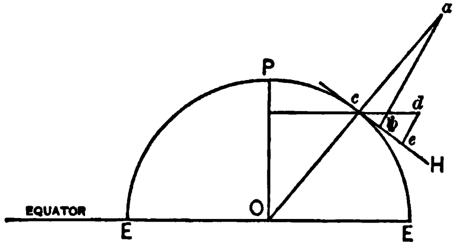


FIG. 113. — Deflection to the right.

and gives rise to what is ordinarily known as the weight of a body. The other component of the centrifugal force, namely, *ce*, is parallel to the surface of the globe and is a horizontal sliding force directed toward the equator. But the earth's surface represents a state of equilibrium; therefore, the two horizontal components, respectively pushing northward and southward, just counterbalance each other, or *bc* is equal and opposite to *ce*. If the earth should rotate faster or slower, then the curvature of the spheroid would change so as to always maintain this balance between *bc* and *ce* so that bodies would have no tendency to slide either north or south.

"Now a body or a mass of water or air that is in motion east or west relative to the earth's surface is rotating around the earth's axis respectively faster or slower than the earth itself. If it has a greater velocity than the earth, it must therefore have a greater tendency to slide toward the equator; if it moves westward, as does an easterly wind, then it presses from the equator. These laws are true for both hemispheres; in both cases a west wind moving eastward presses toward the equator, which is toward the right hand for west winds in the Northern Hemisphere but toward the left hand for the Southern Hemisphere."

Monthly Weather Review, October, 1905.

APPLICATION OF PRECEDING PRINCIPLES TO PUMPS

178. Pumps of all kinds, whether for gases or liquids, whether to exhaust a vessel of the fluid it contains, or to force fluid into a vessel, are all modifications of the typical form shown in Fig. 114.

S is a hollow tube dipping into a vessel of fluid W.

P is a piston fitting snugly into the cylinder C.

V_1 is a valve opening out of C.

V_2 is a valve opening into C.

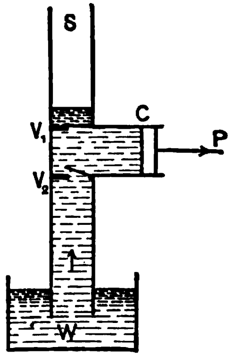


FIG. 114. — A typical pump.

1. **Force Pump.** — Fig. 115 as it stands represents the common force pump, such as that employed in the ordinary fire engine and in pumping water into standpipes. When the piston P is moved in the direction opposite to the arrow, the pressure in the region between the valves is diminished. The effect of this is to open the lower valve and close the upper one.

Not only so, but the pressure of the atmosphere on the free surface of the water in the well begins to raise the water in the tube. And if the lower valve be not too high above the surface of the well, the water will pour through the valve V_2, and partly fill the space between the valves.

If the motion of the piston be now reversed (Fig. 115), the water inclosed between the valves will be forced through the upper valve into the hose or vessel in which it is desired.

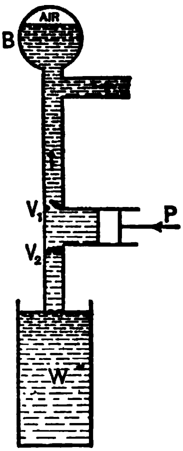


FIG. 115. — The force pump.

Such a stream, however, would be intermittent; for in the region above the upper valve the pressure would cease, during the out stroke of the piston. To avoid this difficulty, most force pumps are furnished with an air chamber (*B*, Fig. 115). When the pump first begins, this chamber is filled with air. When the water is forced through the upper valve, this air is compressed; energy is stored up in it; and, when the out stroke of the piston occurs, this compressed air supplies enough energy to drive the water partially out of the bulb *B*, and thus keep the stream flowing until the piston begins its inward stroke. This large copper bulb on the fire engine is familiar to every one.

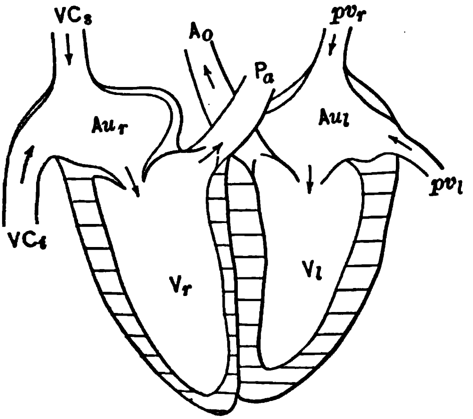


FIG. 116. — The human heart considered as a force pump.

2. **The Human Heart.** — The human heart is exactly such a force pump as that diagrammed in Fig. 115, only here the space between the valves is no longer furnished with a piston, but has elastic walls, which, by alternate contraction and expansion effect the same result as the

piston. When the pressure in the right ventricle of the heart (V_r, Fig. 116) is relieved, the blood from the veins rushes in by way of the right auricle Au_r; when the walls of the heart contract, this blood is driven into the pulmonary arteries P_a on its way to the lungs. In like manner, the left side of the heart acts as a force pump, receiving fresh blood from the lungs and driving it to the extremities of the body. The elasticity of the arteries and veins takes the place of the air chamber in the force pump.

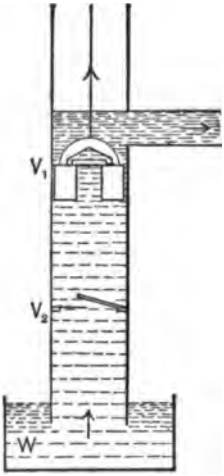


FIG. 117. — Lift pump; on up stroke.

3. **The Ordinary Lift Pump.** — Here the upper valve is placed in the piston itself, as indicated in Fig. 117. Since the pump has no air chamber, the stream is of course intermittent. On the down stroke the upper valve opens; on the up stroke it closes; the pressure between the valves is diminished; the lower valve opens and the water rushes through. On the next down stroke of the piston, it dips into the water, which is held up by the lower valve; and, on the succeeding up stroke, lifts its load of water to the spout.

4. **The Bicycle Pump.** — This is a force pump, in which air is the fluid transferred. The piston here also contains the valve; is the valve, in fact. The down stroke of an ordinary foot pump is represented in Fig. 118. On the up stroke the air rushes into the cylinder all around the edge of the piston; while the valve V_2, next the tire, closes by the back pressure of the air in the tube.

Why is such a pump so hard to operate when the tire is fairly well inflated?

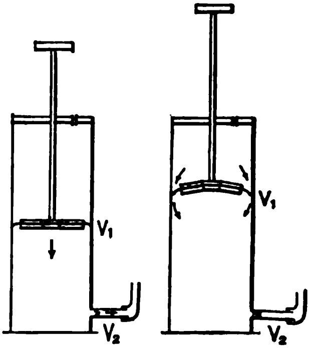


FIG. 118. — Single-acting bicycle pump.

5. **The Vacuum Pump.** — If we reverse the valves indicated in Fig. 118, we shall have a pump which, instead of pumping air into a tire, will empty the tire.

This is only the general principle of the ordinary air pump, which is one of the most important instruments of the physical laboratory. During the last half century it has received many radical modifications and improvements. Of these perhaps the most valuable is the substitution of mercury in glass for the solid piston in a brass cylinder. Mercury does not wet or soil glass, does not appreciably evaporate at ordinary temperatures, moves through a glass tube easily; and is, hence, excellently adapted to this use.

The action of the Sprengel air pump will be clear from the accompanying figure. The falling drops of mercury from the nozzle *N* entangle the air in the chamber *C* and carry it down the narrow glass tube. The drops of mercury act as pistons. More air rushes in from the bulb *B*, and the whole vessel is thus gradually exhausted.

The reservoir *R* is kept filled with mercury; the pressure of the atmosphere forces this over to the nozzle *N*. It is collected again in a glass vessel *G*, and poured back into the reservoir *R*. It can thus be used over and over many times.

The vertical heights of the columns *A* and *D* of mercury must each exceed the barometric height. Why? Such pumps were long used in the manufacture of incandescent electric lamps and "X-ray" bulbs.

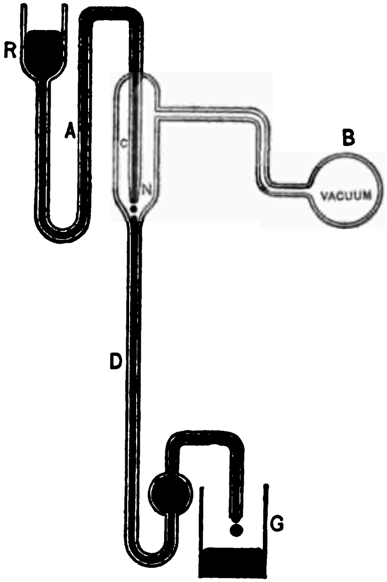


FIG. 119. — Sprengel air pump.

6. **The Geryk Pump.** — A mechanical pump of high perfection has recently been devised in which oil is used as a sort of packing for the piston, thus leaving practically no "clearance" and producing a very high vacuum in a very few strokes. A diagram of this instrument is shown in Fig. 120. When the piston

comes down upon the oil in the bottom of the cylinder, the air is all forced up through the valve in the piston, into the region above the piston.

On the upward stroke the valve not only closes, but is hermetically sealed by the layer of oil on the upper side of the piston. A pump of this type will in a few minutes produce a vacuum of 0.001 millimeter of mercury. That is to say, out of every million particles originally in the vessel only one will remain at the end of, say, ten minutes. These pumps are frequently used in tandem, the intake *I* of the second pump being directly connected with the exhaust *E* of the first pump.

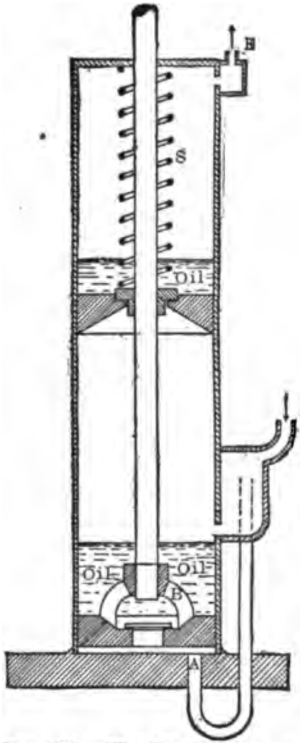


Fig. 120.—The Geryk air pump.

7. A pump still more effective than any of the preceding has recently (1905) been invented by W. Gaede of Freiburg. The principle of the pump is as follows. The gas meter ordinarily used in residences consists of an iron box in which is housed a drum made up of a shaft and several buckets attached to this shaft. The iron box is about half filled with water. The gas enters from the high pressure side, fills a bucket and by its pressure makes this bucket and shaft rotate. The bucket while being filled is mouth-down in the water. The number of rotations of the drum in any given time interval, say one month, is proportional to the amount of gas which has passed through the meter during the month. Now the Gaede pump, shown in cross section in Fig. 121, is merely an instrument of this kind worked backwards, so that the gas (generally air) is taken in on the low-pressure side and delivered on the high-pressure side. The three buckets which are indicated in cross-section by the heavy black lines are carried on a shaft, not shown, which

is perpendicular to the plane of the figure. The drum made up of these three buckets instead of being driven by the gas is turned either by hand or motor. The cylindrical iron box which houses the drum is shown in cross section at *G*.

The buckets instead of being made of metal, as in the gas meter, are here made of porcelain — a difficult and beautiful piece of work. The liquid through which the drum rotates is mercury (shown by dark shading) instead of water. The sense of rotation is indicated by the arrow on the left.

The operation of the pump is as follows. The air from the vessel to be exhausted enters the bucket W_1 through the opening *f*. A little before the bucket has completely filled itself, by suction, the opening *f* has been carried along until it has been submerged in the mercury. Thus in the bucket W_2 the opening is already submerged, as shown by the small circle. The connection with the vessel to be exhausted is now cut off. As the rotation proceeds, the air (indicated by absence of shading) is forced into the passage between the walls, Z_1 and Z_2, of the buckets W_1 and W_2. The bucket full of air thus finds its way into the clearance space between the drum and the inner wall of the containing box, thence up the steel tube *r* and out through s_2 into an auxiliary pump. The auxiliary pump keeps the pressure in the iron box down to a few centimeters of mercury, since otherwise the atmospheric air would bubble back through the mercury.

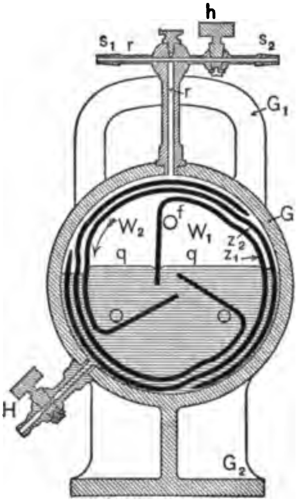


FIG. 121.

As the reader may imagine, many important details have been omitted from this hasty sketch. Such a pump will in 15 minutes exhaust a gallon flask to a pressure of only 0.00001 millimeter of mercury. These pumps are in very wide use.

8. Still more recently Dr. Gaede has invented what he calls a "molecular pump," which is at present (1916) the most effective

one known. It rests upon a principle entirely different from that of any mercury pump, a principle from the molecular theory of gases. This must be left for the more advanced student. The device is mentioned here merely to remark that the molecular pump will give in a shorter time and with less trouble a vacuum 5 times better than the Gaede mercury pump.

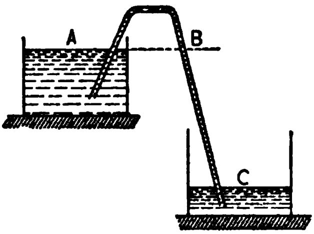


FIG. 122. — Siphon.

9. **The Siphon.** — When it is desired to transfer liquid from a vessel *A* at one level to another vessel at a lower level, a bent tube something like that shown in Fig. 122 is very convenient and is known as a siphon. To start the siphon it is first filled with liquid, then inverted and placed with the upper arm under the surface *A* as shown. The operation then becomes simple if we understand (i) that the pressure is the same in both arms of the tube at the level *AB*, and (ii) that the pressure increases as we go down in the liquid from *B* to *C*. But the pressure on the free surface at *C* is just one atmosphere. Hence the liquid will be continually forced out of the lower end of the tube until the free surface in each vessel stands at the same level. Evidently the height of the bend in the tube above the surface *AB* cannot exceed the barometric height of the liquid in question. There are many other useful and clever forms of this instrument, all dependent upon this same principle.

BOYLE'S LAW

179. When Torricelli discovered the proper method of measuring the pressure of the earth's atmosphere, he furnished us also a method by which gaseous pressures, in general, may be measured. The method is as follows: —

Connect the vessel containing the gas to one end of a U-tube containing mercury. The pressure due to the excess of height h (Fig. 123) of the mercury in the open arm over that in the closed arm will be measured by Dgh. If to this we add the pressure of the atmosphere B, we obtain the pressure P in the vessel.

$$P = B + Dgh, \qquad \text{Eq. 67}$$

where

B = barometric pressure;
D = density of mercury;
g = acceleration of gravity;
h = difference of level between the two mercury surfaces.

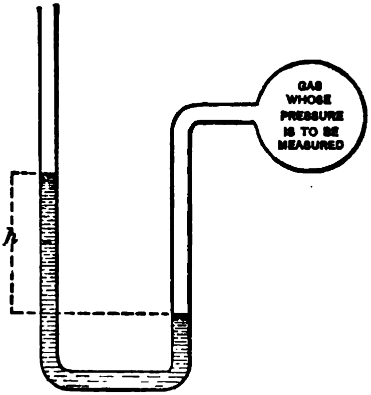


FIG. 123.

If the mercury in the open arm of the U-tube stands at a lower level than in the closed arm, h becomes negative, and we have

$$P = B - Dgh.$$

Shortly after the time of Torricelli's experiment, Robert Boyle, a distinguished English chemist, invented an air pump, and became greatly interested in the following question:—

Gases are easily compressed. A bladder filled with air and closed at ordinary pressure can be reduced in volume without much difficulty, *i.e.* without greatly increasing the pressure. But, "Just *how* does the volume of a given body of gas vary when the pressure is varied?"

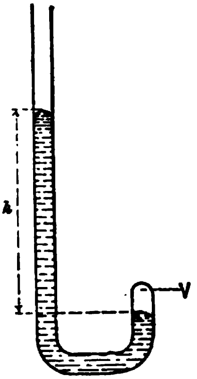


FIG. 124.—Boyle's experiment.

To discover how one quantity varies with another, the general method is to let one quantity vary, measure its successive values, and also measure the corresponding values of the other quantity. This Boyle did. He inclosed a definite mass of air in the closed arm of a U-tube, as indicated in Fig. 124. He varied the volume V of this inclosed gas by pouring in more and more mercury, and measured the pressures corresponding.

The method employed for measuring the pressure is that which we have just described.

$$P = B + Dgh = Dg \cdot (H + h),$$

where H = height of barometer.

Now Boyle found that (i) *so long as the mass of air remained* *constant*, and (ii) *so long as its temperature remained constant*, **the product of the pressure by the volume of the gas was very approximately a constant.** This law was published in 1660. It may be stated algebraically as follows: —

$$PV = C = \text{constant.} \qquad \text{Eq. 72}$$

When the mass of gas inclosed in the short arm of the tube was doubled, it was found that, for any given pressure, the volume was also doubled; or to obtain any given volume it was now necessary to double the pressure. And in general it has been found that the product PV is proportional to the mass M of the inclosed gas, a result which may be stated as follows: For constant temperatures,

$$PV = MK, \qquad \text{Eq. 73}$$

where K is a constant.

Since, however,

$$M = DV, \qquad \text{Eq. 28}$$

we may eliminate M, and obtain a result which is entirely independent of the mass of gas inclosed, namely: —

$$\frac{P}{D} = K = \text{constant.} \qquad \text{Eq. 74}$$

The interpretation of this equation is evidently the following: **At any given temperature, the pressure in a gas varies directly as its density, and is independent of the mass.**

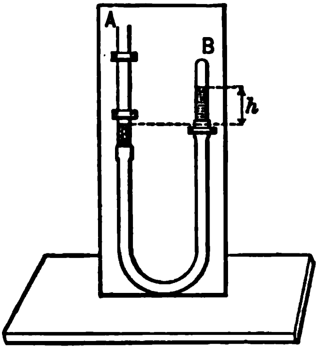


Fig. 125. — Mariotte's experiment.

180. Boyle's experiments were mostly performed with pressures greater than one atmosphere (76 centimeters of mercury); but Mariotte (1620–1684), a French physicist, shortly afterward showed that Boyle's Law holds when the air in the containing vessel is rarefied; that is, when the pressure is less than one atmosphere. This he demonstrated as follows: —

Two pieces of straight glass tubing are connected by a rubber tube, as indicated in Fig. 125. A portion of air is sealed off by mercury in the closed arm B. On lowering the arm A, the mercury in A falls below the level of that in B. Let this depression be indicated by h, then

$$P = Dg \cdot (H - h),$$

where, as before, H is the height of the barometer and D the density of mercury. The tube B is graduated so that the volume of the gas V can be read off directly.

In this case also the product PV was found to be invariable. Quantities such as PV are known as *invariants*. The factors which enter the product are each variable, but the product itself is a constant.

181. The usefulness of this law of Boyle's can scarcely be overestimated. For, if we know the pressure corresponding to any one known volume, we know at once the value of the constant K, and can hence *compute* the pressure corresponding to any other volume; or, if it be desired, one can compute the volume corresponding to any other pressure. For example, suppose that a bottle whose capacity is 100 cubic centimeters contains compressed air at a pressure equal to that of 180 centimeters of mercury. Let us compute the volume which this air, at the same temperature, would occupy if allowed to escape into the atmosphere at normal pressure. Let P_1, V_1 represent the pressure and volume, respectively, of the air while in the bottle, and P_2, V_2 its pressure and volume after being released. It is required to find V_2. According to Boyle's Law, the product of pressure and volume remains constant. Hence Eq. 72 becomes

$$P_1V_1 = C = P_2V_2.$$

But $P_1 = 180$; $V_1 = 100$; and $P_2 = 76$. Hence, substituting these values, and using one centimeter of mercury as the unit of pressure, we obtain the following value for C in Eq. 72: —

$$P_1V_1 = 180 \times 100 = 18{,}000 = C.$$

And, therefore,

$$V_2 = \frac{C}{P_2} = \frac{18000}{76} = 236.8 \text{ cm.}^3$$

Hence the 100 cubic centimeters of compressed air, if allowed to escape, would occupy 236.8 cubic centimeters.

182. A **perfect gas** is defined as one which obeys Boyle's Law exactly; but it has been found by many observers that all gases deviate more or less from the law; in other words, there are no perfect gases in nature. These deviations have been especially studied by the French physicist, Amagat, whose description of the facts, together with Boyle's original memoir, will be found in *The Laws of Gases*, edited by Professor Carl Barus (New York, 1899).

Boyle's Law has been modified by the Dutch physicist, van der Waals, so as to describe the behavior of actual gases with much greater accuracy. Using the same notation as above, and denoting by a and b two quantities which are constant for any one gas, he writes

$$\left(P + \frac{a}{V^2}\right)(V - b) = \text{constant}. \qquad \text{Eq. 75}$$

For the changes in volumes which occur in gases under a wide range of pressures, this description of van der Waals is much more adequate than that of Boyle. The interpretation of his constants a and b will be of interest in a later chapter.

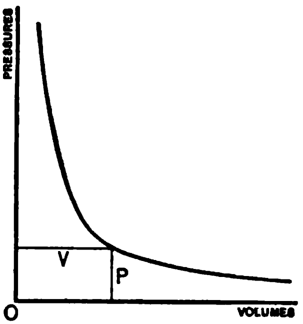


FIG. 126. — Graph of $PV =$ constant.

183. The graphical interpretation of Boyle's Law leads at once to the equilateral hyperbola shown in Fig. 126; or, if one employs Eq. 74 and plots pressures as a function of densities, he is then, of course, led to the straight line.

184. Before leaving the subject of this chapter, it is necessary to warn the beginner that there are many fundamental phenomena, such as osmosis of liquids, diffusion of gases, absorption of gases by liquids and solids, which we have not been able here to discuss.

Problems

235. What is the height of a water barometer when the mercurial barometer reads 760 mm.? Density of mercury = 13.6.

Ans. 10.336 m.

236. An atmospheric pressure of 15 lb. to the square inch is equivalent to a pressure of how many kilograms to the square centimeter? *Ans.* 1.055.

237. Twenty-four cubic centimeters of gas at a pressure of 71 cm. of mercury would have what volume under a pressure of 76 cm.? *Ans.* 22.4.

238. A cylinder 16 in. long is closed at one end by a piston. If the cylinder is filled with air at atmospheric pressure (76 cm.), and the piston is thrust down within 1 in. of the other end, what will be the pressure of the inclosed air? (It is here assumed that the temperature is the same in each case.) *Ans.* 1216 cm. of mercury.

239. Two hollow spheres of radii 8 and 10 contain equal masses of hydrogen. Compare the pressures. *Ans.* 1000 : 512.

240. Find the lifting power of a balloon, filled with coal gas, from the following data: volume of balloon = 1000 m.^{3}; weight of balloon and car = 60 kg. A liter of coal gas weighs 1.193 g.; a liter of air weighs 1.293 g. *Ans.* 40 k.

241. One wall of a room is 9 × 24 ft. If the room is closed tightly at a time when the height of the barometer is 30 in., what pressure must the wall sustain when the barometer suddenly drops to 29 in.? *Shearer*, 527.

242. If 1000 cm.^{3} of hydrogen under a pressure of 20 cm. of mercury is subjected to a pressure of 84 cm. of mercury, what will be the resulting volume? *Ans.* 238.1 cm.^{3}

243. Compute the weight of air contained in a moderate-sized living room, say, one which is 5 × 5 × 3 m., at the temperature of 0° C.

244. When a barometer stands at 75 cm., the volume of the empty space above the mercury is 8 cm.^{3} and its length is 13 cm. How far would the column fall if you introduced 5 cm.^{3} of air into the tube? *Ans.* 19.$^+$ cm.

Solution of Problem 244. — Let s denote the cross section of the tube. Then $s = 8/13$ cm.^{2}

Before the admission of the air,

$$\text{(i)}\quad p_1v_1 = mk, \text{ where } \begin{cases} m = \text{mass of 5 c.c. of air.} \\ v_1 = 5 \text{ c.c.} \\ p_1 = 75 \text{ cm. of mercury.} \end{cases}$$

After the admission of the air,

$$\text{(ii)}\quad p_2v_2 = mk, \text{ where } \begin{cases} v_2 = \text{volume assumed by air after admission.} \\ p_2 = \text{pressure, above the mercury.} \end{cases}$$

(iii) $v_2 = s\,[13 + (75 - p_2)]$.

Eliminate k and v_2; solve for $75 - p_2$ cm.

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ARCHIMEDES. *Works.* Translated by T. L. Heath. (Camb. Univ. Press. 1897.) Especially the two books "On Floating Bodies."

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SHAW, W. N. "Air Currents and the Laws of Ventilation." (Camb. Univ. Press.)

BOYS, C. V. "Soap Bubbles and the Forces which Mould Them." (Soc. Prom. Chris. Knowledge.) Clear and beautiful treatment of surface tension; simple and experimental.

"Be sure of it: give me the ocular proof."

SHAKESPEARE, *Othello*, III, 3.

CHAPTER V

WAVES

185. The word at the head of this chapter will recall to most minds one or more of the following phenomena: the surface of a lake swept by a wind, the constantly enlarging rings produced by a stone falling into quiet water, the "swell" that follows a storm at sea, the waves which always accompany a moving boat. Most people have seen waves run along a clothesline, or some stretched rope, when struck by a walking stick. Experience shows us that these and all similar cases have two features in common. **First, there is a disturbance at some point in a certain medium. Second, this disturbance is propagated to other points of the medium.**

At a given instant the stone is dropped into quiet water; at a later instant the disturbance may have reached points several meters away. At a given instant the stretched clothesline is at rest in its position of equilibrium. When one end is suddenly depressed, this depression runs along the line at a definite rate; or if the disturbance be an elevation, it is an elevation that runs along the line. Concerning any wave motion the two important questions are (i) concerning the origin of the disturbance, and (ii) concerning the nature of the propagation.

But what is meant by "a disturbance"? Simply this, **a body is said to be disturbed when it is thrown out of equilibrium.**

But when is a body or system of bodies in equilibrium? Not necessarily when it is at rest, but when its potential energy is a minimum. A pendulum is at rest at the end of its swing, but is by no means in equilibrium. A pendulum moves most rapidly at the lowest point in its path; but precisely there is the one point where it is in equilibrium.

A disturbance, then, implies the addition of energy to a system. And as this disturbance is propagated through the medium, some energy is in general distributed through the medium.

DEFINITION OF WAVE MOTION

186. We may, therefore, define a wave motion as a state of disturbance being propagated from one part of a medium to another.

Note that in this definition nothing is said about any up-and-down motion, such as is seen in the disturbed clothesline. One has only to replace the clothesline by a stretched spiral spring to see that waves may travel without any such up-and-down motion. Such a spring (Fig. 127) should be from two to three meters long, made of brass wire, No. 18 or 20, wound in a spiral of approximately eight millimeters diameter. Fix the ends of the spring to two rigid points *A* and *B*, so that the spring will be stretched to nearly twice its unstretched length. If this spring be grasped at any point *C*, moved in the direction of the arrow, then quickly and gently released, the disturbance thus introduced will run to and fro along the line; but there will be no appreciable up-and-down motion. This very important experiment should be carefully observed.

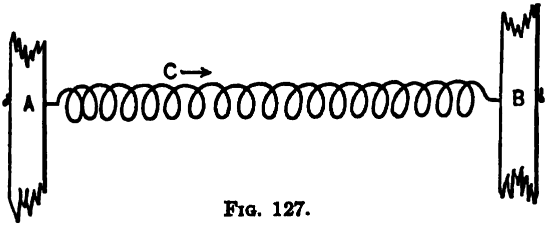

FIG. 127.

Note further that, in the definition of wave motion, nothing is said about any transfer of matter. For although in each case experiment shows that the particles of the medium oscillate through a small range, it is evidently not the medium, but the energy, which is transferred. For clearly exhibiting the distinction between the motion of the wave and the motion of the particles of the medium, there is perhaps nothing superior to the wave

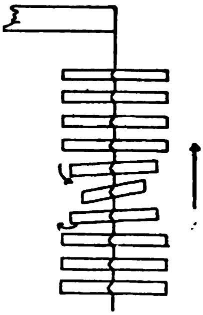

FIG. 128. — **Kelvin's wave model.**

model used by Lord Kelvin in his *Baltimore Lectures* in 1884. This device consists of, say, 20 or 30 equal wooden bars suspended by a piano wire, as shown in Fig. 128. Since these bars are fastened so that when one of them is twisted the wire is also twisted, it is clear that a single rotation given to the lowest bar will be propagated throughout the entire series, the wave moving vertically upwards, while the particles (the bars) move in a horizontal plane only.

SOME SPECIAL KINDS OF WAVES

I. *Water Waves*

187. Of these there are several kinds, and most of them are studied to best advantage in the laboratory or in a canal.

The genesis of water waves by wind is very imperfectly understood; but we may consider that the source of these waves is anything which elevates or depresses the water at any point. Imagine OX to represent the level surface of the undisturbed water. If by any means the surface of the water be given the form indicated in Fig. 129, the operation will be equivalent to taking the water from the "trough" of the wave BC, and lifting it up into the "crest" of the wave AB. Let m be the mass of water which will just fill the trough BC; the amount of water in the crest will then also be m. Let h denote the difference in height between the center of mass of the trough and the center of mass of the crest. The operation of producing a single wave in water previously quiet will require that one lift a mass m through a height h against gravity. The potential energy stored in the wave will, therefore, be mgh. This potential energy tends to a minimum, *i.e.* in this case the center of mass tends to fall, and, since water has inertia, the crest in falling goes on beyond its position of equilibrium, and thus we have a continual oscillation due to gravitation.

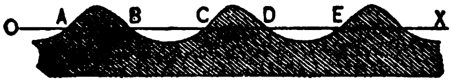

FIG. 129. — Typical sea wave.

In addition to this potential energy, it must not be forgotten that waves have considerable kinetic energy. Indeed, it may be shown that in wave motion practically one half the energy is kinetic and one half potential. The disturbance at any point in the water over which a wave is passing is then very similar, but not identical, with that in one arm of a U-tube partially filled with mercury and the mercury in oscillation.

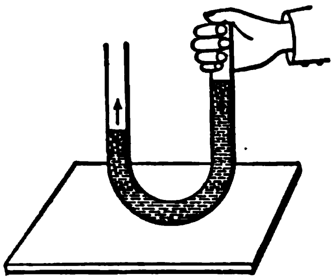

Fig. 130.

Take a U-tube of a diameter not less than one centimeter. Pour it half full of clean mercury — water will answer nearly as well. Incline it and close one end with the thumb, as indicated in Fig. 130. Bring the tube into a vertical position and suddenly raise the thumb. A dozen or more oscillations follow. They are, so far as vertical motion is concerned, such as would occur at any point in an ocean of mercury, or of water traversed by waves.

Why do these oscillations so quickly stop? Would this same cause be at work in a large lake or at sea to stop the waves?

Waves or oscillations which are made to diminish gradually in amplitude are said to be **damped**.

188. The actual motion of the water particles in sea waves seems to have been first accurately guessed by Franz Gerstner (1802), who assumed that these particles move in circular orbits, as shown in Fig. 131, where the line of centers *O* represents the undisturbed surface of water. When a train of waves passes over this surface, the particles *P*, which, in equilibrium, are situated at *O*, are now set into uniform circular motion. The curve drawn through their positions at any particular instant is called the "locus" of these particles. This locus is the wave form at any instant. To find the wave form at any *later* instant, say when each particle has moved 90° ahead in its orbit, one has only to

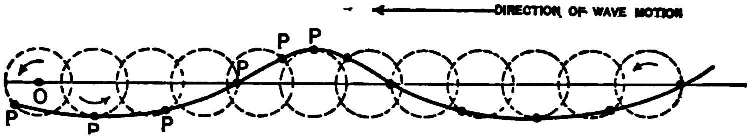

Fig. 131. — Gerstner's explanation of the form of sea waves.

mark in each circle a point 90° ahead of the present position of P and then draw the locus of these new points.

In *deep* water, the motion of the particles under the surface is identical with that at the surface, except that the circular orbits become smaller and smaller as one leaves the surface. In *shallow* water, the particles move in ellipses which become flatter and flatter as one approaches the bottom. The justification for this view of Gerstner's lies in the fact that these elliptical motions can be observed on small particles scattered through the water, and also in the fact that the actual wave form is seen to be very nearly what his theory — called the "trochoidal theory" — predicts. Many of the properties of trochoidal waves are beautifully illustrated by an apparatus devised for this purpose by Professor C. S. Lyman, an excellent modification of which is made by Dr. Forbes of Columbia University.

Definition of Wave Length

189. A series of waves, such as one can start in water by rocking a boat, or by raising and lowering a floating block, is called a **train of waves.** In any train of water waves the distance between two successive crests is called a "wave length." The distance between two successive troughs is also a wave length. **But the general definition of a wave length is the shortest distance between any two particles whose motion is in the same phase.**

What would correspond to "two successive crests" in the waves produced by compression in the spiral spring?

Speed of Water Waves

190. A train of waves is traveling *over a point;* how far does this train travel during the period of one complete oscillation of the water at this point? Evidently one wave length. Let us call this wave length l, and the period of one oscillation T.

If, by any means, direct observation or computation, we can obtain l and T, we have at once the speed of the wave; for, denoting speed by V and frequency by n, we have

$$V = \frac{l}{T} = nl. \qquad \text{Eq. 76}$$

This may be called the fundamental equation of wave motion; for, since it depends simply upon the defining equation for speed, it is necessarily true for a wave motion of any type. There enters into the equation no dynamical quantity, nor, indeed anything characteristic of the medium; it is merely a kinematic definition of wave velocity in general.

Regarding the speed of water waves *as a function of the body of water upon which they travel,* time permits us only to say that: —

(i) In shallow water, *i.e.* where the depth h is rather small compared with the wave length, the speed V is

$$V = \sqrt{gh}, \qquad \text{Eq. 77}$$

where g, as usual, indicates the acceleration of gravity. Observe that in this case V is independent of the wave length.

(ii) In deep water, *i.e.* where the depth is rather large compared with the wave length l, the speed is

$$V = \sqrt{\frac{gl}{2\pi}}. \qquad \text{Eq. 78}$$

This equation describes those great waves which overtake the swiftest Atlantic liners; a simple computation shows, for instance, that a deep-sea wave which is 400 feet in length travels with a speed of 26 knots.

Refraction of Water Waves

191. The variation of speed with depth, and the independence of speed and wave length indicated in Eq. 77, may be easily tested in the laboratory by means of a wave trough. The equation for shallow water waves also describes very perfectly the curious fact, easily observed on the shore of nearly any lake or pond, or at the sea coast, that, whenever waves approach the shore, the crest of the wave is always more or less parallel to the shore line. In Fig. 132 let the heavy line represent the shore near the entrance to a small bay. A train of waves, A, B, C, D, etc., approaches the shore. When the waves leave the deep water, the crests have the direction indicated by $\overline{GG'}$. The nearer they approach the shore, the more they are turned about. Such a process is called **refraction.** The end A has traveled faster than the end A'. The dotted lines are called **contour** lines, and

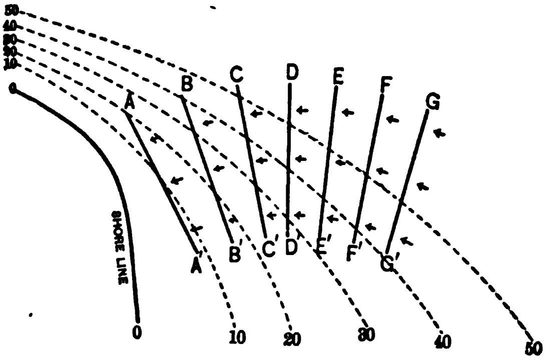

FIG. 132. — Refraction of water waves.

indicate the depth of the water. Every point on the contour line marked "10" has a depth of 10 centimeters; every point on the line marked "20" has a depth of 20 centimeters, etc.

Explain why the crests of the waves are rotated as they approach the shore. Since they start in parallel to each other, why do they not remain so?

Reflection of Water Waves

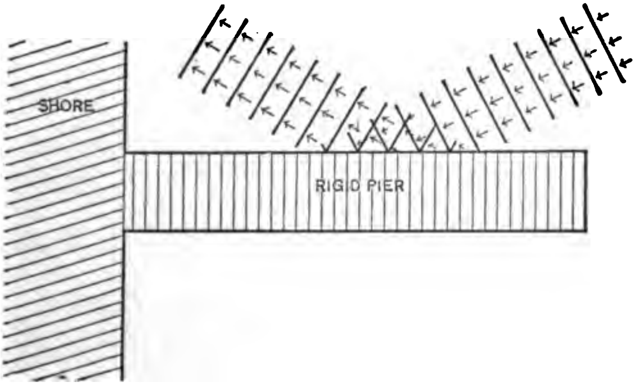

FIG. 133. — Reflection of water waves.

192. If a breakwater or solid pier (Fig. 133) is built out into a body of water, one has there an opportunity to see what happens when water waves strike a rigid body.

In such circumstances it is not difficult to see that the direction of the reflected waves is very different from that of the incident waves.

From our experience in the laboratory, we know that when a train of waves is started at one end of a wave trough, they no sooner reach the other end than they are reflected back.

These two trains of waves, the incident and reflected, may combine with each other to produce a series of waves which appear to have no motion of translation, the so-called **stationary waves.** In order that these two trains of waves should produce stationary waves, it is necessary that all the waves should be of one length. A moment's practice with a wooden paddle in a trough of water enables one to produce such waves.

A more detailed study of stationary waves will be made under the head of Waves in Strings.

Summary concerning Water Waves

1. **Medium of propagation** — bounding surface between air and water.

2. **Motion of particles** — small, in paths, nearly circular; through a part of the path the motion of the particle is in the same direction as the motion of the wave; in part of the path it is at right angles to the motion of the wave.

3. **Speed** — in deep water, depends upon length of wave, but in shallow water, it depends upon the depth.

4. Water waves **are reflected,** and may thus produce **stationary waves.**

II. *Ripples*

193. In our study of liquids we found that work was required to enlarge the surface of any given body of liquid. We found that equilibrium was reached only when the liquid surface became as small as possible.

Consider any quiet body of liquid, such as a cup of tea. Its surface is in equilibrium when level, because then it has a minimum of potential energy. Any disturbance of this surface, such as is produced by drawing a spoon through it, will increase the area, and will, therefore, impart to the surface some energy.

Suppose a surface disturbed as shown in Fig. 134, and then left to itself. Not only will *gravity* tend to lower the liquid crests and fill the troughs, but the *surface tension* of the liquid will also do exactly the same thing. **Surface tension is, therefore, competent to produce waves in a liquid just as gravity is.** And when the lengths of the waves become very small, the surface tension is even more effective than gravity. So that in the case of very small waves, we may consider them as propagated by surface tension alone.

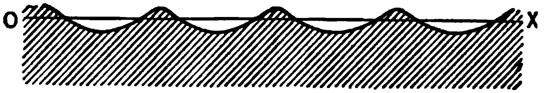

Fig. 134.—Showing that a surface disturbed by ripples is larger than a level surface.

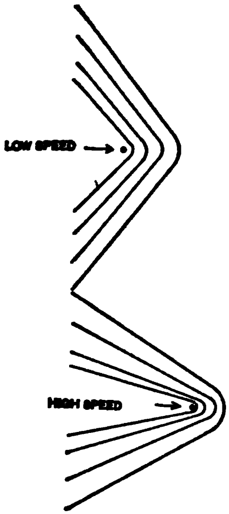

Fig. 135.—Showing that short ripples travel faster than long ones.

These ripples are known to every one who has watched the bow of a rowboat as it moves through quiet water. They are beautifully seen in the case of a duck swimming on a smooth pond. And what is a very curious thing, these ripples always manage to keep just a trifle ahead of the duck, however fast or slowly he swims.

We may see the same phenomenon in the laboratory. When a piece of wire is drawn through a dish of water (Fig. 135), at a moderate rate, the ripples which precede it have a certain length; but **when the wire is drawn more rapidly through the water the ripples become finer,** *i.e.* ripples of a shorter wave length are produced. **But in each case the ripples precede the moving body. It follows, therefore, that ripples of short wave length travel faster than those of long wave length.** This is just the reverse of what happens in ordinary water waves; for there the longer the waves the faster they travel.

194. Neglecting the effect of gravity upon the speed of these short waves and considering only the surface tension T, it may be shown, by ordinary dynamics, that their speed is

$$V^2 = \frac{2\pi T}{lD}, \qquad \text{Eq. 79}$$

where D is the density of the liquid and l the length of the wave.

If now one takes account of gravity also, remembering Eq. 78, and that from this cause the square of speed will be increased by the amount $\frac{gl}{2\pi}$, he has the general expression for deep-water waves, namely: —

$$V^2 = \frac{2\pi T}{lD} + \frac{gl}{2\pi}. \qquad \text{Eq. 80}$$

It follows, therefore, that the slowest waves are not the very short ones, nor the very long ones. The slowest of all water waves have been found to have a length of about 16 millimeters. Waves longer than this are spoken of as **water waves**, while waves shorter than this are called **ripples**.

In order to discover whether or not rippples are reflected, as water waves are, one has merely to produce ripples in a shallow dish of water and let the ripples proceed until they strike the edge of the dish.

Summary concerning Ripples

1. **Medium of propagation** — free surface of liquids.
2. **Speed** — increases as the wave length diminishes.
3. **Reflected** — in same manner as water waves.
4. **Motion of particles** — same as that of water waves.

III. *Tidal Waves*

195. To one who goes to the seashore for the first time, few things are more striking than the two great changes in the height of the water which occur every twenty-four hours. The water recedes until at a certain hour it reaches what is called **low tide**; not many minutes are required to see that the water is again rising. The tide is coming in; at the end of six hours after low tide, the maximum height, called **high tide**, is reached; the tide immediately begins to recede; and so on in endless procession. So gigantic is this phenomenon that one would never suspect its being a wave motion until he has formed a mental picture of the whole disturbance as reported by observers from all parts of the world.

Approximate Explanation of Oceanic Tides

196. Those who have rightly understood Newton's capital discovery of universal gravitation, know that the moon attracts each part of the earth with a force which is greater as the distance between that part of the earth and the moon is less. The larger part of the earth's surface is covered by its oceans. Imagine the whole of it to be under water. In Fig. 136, consider three particles *A*, *B*, and *C*, lying on a diameter of the earth, drawn in the direction joining the moon and the earth. *A* is on the side of the earth next the moon; *B* at the center of the earth; *C* on the side of the earth away from the moon. *A* will be more strongly attracted than *B*, that is, it will be attracted away from *B*; while *B*, in turn, will be more strongly attracted than *C*, that is, *B* will be attracted away from *C*. The difference between the actual attraction of the moon at any point on the earth's surface and the average attraction is called the "tide-generating force" at that point. The result of this tide-producing force is that the water at *A*, attempting to get as near the moon as possible, is heaped up, is drawn away from the solid body of the earth.

FIG. 136. — Lunar tides. Arrow at *A* indicates direction of tidal wave in passing about the earth.

To draw *B* away from *C* is equivalent to repelling *C* from *B*. So that in case the earth always presented the same side to the moon, we should have at the points *A* and *C* high tides. On the great circle which lies 90° away from these we should have low tide.

197. The above is a brief sketch of what is known as the equilibrium theory of the tides, and is a fundamental picture which each student should clearly grasp. But as a matter of fact it comes very far from accurately representing the time at which high water would occur on a rotating earth, even if we assume, as in the preceding section, that the ocean covers it to a uniform depth. For as the solid nucleus B is rotating on its axis, carrying the towns and cities of the earth with it, the result is evidently that every twenty-four hours each seacoast town must pass through these two antipodal tide crests, that is, through two high tides and two low tides.

This great heap of water, which is dragged over the earth's surface every day, consists, *at any one point of the earth's surface*, of a rise and fall of sea level. It is periodic as waves are; that is, the tides recur at equal intervals, — the disturbance is propagated from one part to another of the medium (the oceans); and these features show that what we have here to deal with is a wave, on a magnificent scale, traveling at a stupendous speed. At the equator the length of this wave is 12,500 miles. Its period is 12 hours and 25 minutes, half the apparent diurnal period of the moon.

198. But in order to understand the actual behavior of this great wave it is necessary at this point to make a brief digression upon the subject of

Forced Vibrations

For since this tidal wave is compelled by the moon to travel about the earth at the same rate as the moon itself, the disturbance is called a **forced vibration.** The water waves, which have been considered in previous pages, are called **free vibrations.**

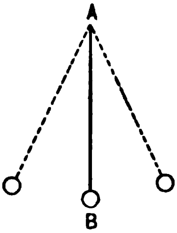

FIG. 137.

In general, a free vibration is one in which the period is determined by the properties of the vibrating body itself; while a forced vibration is one whose period is determined by some outside body acting upon it, and supplying energy to it. Let us consider a simple gravitational pendulum, which has a point of support at A, a bob at B, and a length l, as shown in Fig. 137. If this pendulum be once set in motion and then left to itself, it will vibrate with a period T, where

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

This is known as the **free period** of the pendulum.

(i) Suppose, however, that the point of support A is moved to and fro in a horizontal direction *with a motion which has a period exactly equal to* T. The result is familiar to every one: the amplitude of the pendulum will increase until as much energy is given by the pendulum to the air as is given to the pendulum by the agent which moves the point A. This phenomenon is usually spoken of as **sympathetic resonance or sympathetic communication of vibrations.**

(ii) Let us next suppose that the point of support A is urged by a periodic force *whose period is longer than that of the free pendulum.* Here it will be observed that, as soon as the motion becomes steady, the pendulum has assumed a new period, namely, the period of the force; not only so but the phase of the force and the phase of the pendulum are the same. Thus when the force acts toward the right, as indicated in Fig. 138, so also does the pendulum bob move toward the right. The effect may be stated by saying that the pendulum acts as if it were vibrating freely but with an increased length OB.

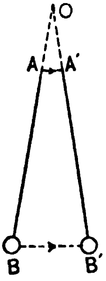

FIG. 138. Period of force greater than period of pendulum.

(iii) If, however, we make the *period of the force at A shorter than the free period of the pendulum,* we shall again find that the actual period of the pendulum soon becomes identical with that of the force; but, unlike the preceding case, the phase of the pendulum is now just opposite that of the force, as indicated in Fig. 139. The effect is indeed that of a free pendulum with a shortened length OB.

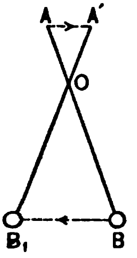

FIG. 139. Period of force shorter than period of pendulum.

199. Returning now to the subject of the tides, it will be seen that we have a tide-generating force whose period is 12 hours 25 minutes acting upon a body of water whose depth, say 3 miles, is very small compared with the length of the tidal wave. The problem is then one of waves in shallow water where the speed of the wave varies with the depth. Referring to Eq. 77, it will be seen that the oceans would have to be 13 miles deep in order that a *free* wave would sweep around the earth at the equator once in 24 hours 50 minutes. And since the actual oceans are not nearly so deep as this, it is clear that a *free* wave, even in the Pacific Ocean, would have a period much longer than that of the apparent motion of the moon.

The tides then fall under the third case which we met in the preceding section, the case where the period of the force is less than the period of the free vibration. This being so, it follows that the tides will be in opposite phase with the tide-generating force; that is, in opposite phase with the tides predicted on the equilibrium theory.

This is known as the dynamical theory of the tides, and shows that under the circumstances described, high water would actually occur where low water would be expected on the equilibrium theory.

The presence of large continents of land, the varying motion of the moon, the superposition of solar tides, and many other circumstances render the actual facts tremendously complex when compared with the simple outline of lunar tides here given. Every one interested should read Sir George Darwin's popular lectures on *The Tides* (Boston, 1898).

Summary concerning Tides

1. They satisfy the conditions of wave motion.
2. They are forced vibrations.
3. Wave length — at equator — 12,500 miles.
4. Period of lunar tides 12 hours 25 minutes.
5. Period of solar tides 12 hours 0 minutes.
6. Velocity at equator 1042 miles per hour.

IV. *Waves in Strings*

200. An easy way to study these roughly is to fasten together two lengths of ordinary rubber tubing, and thus make one piece about 20 feet long. By the aid of a heavy mass, or of some more convenient means, clamp one end of the tube to the edge of a table, or

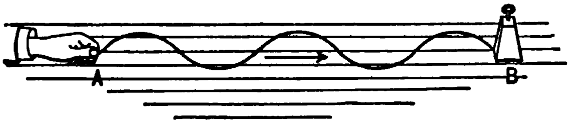

FIG. 140. — A simple method of studying waves in strings.

even to the floor, as shown at B in Fig. 140. Taking the free end A in the hand, one can produce in the tube any tension he desires, and can send trains of waves down the tube in the direction given by the arrow.

Transverse and Longitudinal Waves

Consider the motion of any one point in the rubber tube. As a train of waves passes through this point, it oscillates, to and fro, in a direction perpendicular to that of the arrow. The motion of the particles in the medium is then across the direction in which the waves travel. Such waves are said to be **transverse.**

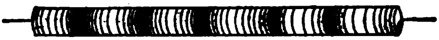

FIG. 141. — Longitudinal waves in a spiral spring.

In the case of the spiral spring, alternately compressed and elongated (Fig. 141), any one small part of the spring is displaced in the *same* direction as that in which the wave is traveling. The displacement of the particles is lengthwise. Such waves are accordingly called **longitudinal waves.**

In the short time at our disposal the only type of waves in strings which we can consider are the transverse ones.

Returning now to the rubber tube, stretched upon the floor, observe the following: —

1. That you can vary the *length* of the waves you send along the string by varying the *rapidity* with which you move your hand to and fro. Let us denote by T the time occupied by your hand in one complete vibration. During this interval the disturbance travels just one wave length l along the tube.

If we had any means of measuring T and l, we could determine the speed of the waves V; for (§ 190): —

$$V = \frac{l}{T}. \qquad \text{Eq. 76}$$

2. Observe also that you can vary the *length* of the waves by varying the force with which you stretch the rubber tube, while your hand moves to and fro at the same rate as before. From this fact it follows that the speed of the wave depends upon the tension in the string.

3. Notice that a single disturbance sent down the tube does not in general cease when the wave strikes the fixed end (*B*, Fig. 140). If it does, the wave has been damped out by friction on the floor; stretch the tube a little more, holding it up so as just to clear the floor. The reflected wave can now be easily seen running back in a direction just opposite that of the incident wave. Such a disturbance is called a *solitary* wave.

4. With a moment's practice one can obtain a reflected *train* of waves which interfere with the incident train in such a way as to produce stationary waves, exactly similar to the stationary water waves obtained in the water trough.

Speed of Transverse Waves in Strings

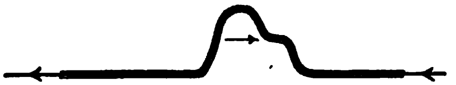

FIG. 142. — Illustrating how centrifugal force balances stretching force when a transverse wave traverses a string.

201. Imagine a perfectly flexible and uniform cord drawn through a brass tube and stretched as shown in section in Fig. 142. It is an astonishing fact that there is a perfectly definite speed with which any such cord may be drawn through the tube without exerting any pressure upon the tube. This is true whatever the curvature of the tube may be, provided only that the curvature is continuous, *i.e.* provided there are no sharp angles in the tube. And what is still more curious is the fact that this particular speed is exactly that with which a free wave will travel in this same cord. Our problem now is to determine from simple dynamics this critical velocity.

(i) First one must compute the pressure (force per unit length) which the cord would exert upon the tube, if the entire system were at rest. Let *A*, *B*, and *C* (Fig. 143) represent three consecutive points on the tube, and *R* its radius of curvature at *B*.

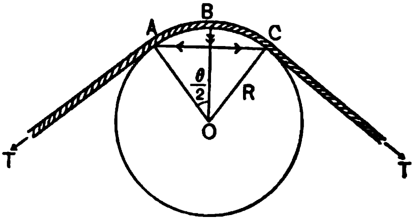

FIG. 143. — Resolution of forces at any point *B* on the tube.

Suppose that the arc ABC subtends the angle θ at the center of curvature O; then, if the tension in the string is T, the normal component of this tension at B will be $2\ T \sin\frac{\theta}{2}$; and hence

$$\text{Pressure at } B = \frac{2\ T \sin\frac{\theta}{2}}{R\theta} = \frac{T}{R}, \text{ approximately.}$$

This is the force with which each unit length of string is pressed against the tube when the cord and the tube are at rest.

(ii) Next one must compute the centrifugal force with which each unit length of the string is thrust away from the center of curvature during its motion through the tube.

Let μ denote the mass of unit length (linear density) of the string and V the speed with which it is drawn through the tube. Then when the string passes the point B (Fig. 143) where the radius of curvature is R, it is evident that the outward pressure on each unit length of the cord will be $\mu\frac{V^2}{R}$.

It follows, therefore, that the total pressure will be zero when these two opposite pressures are equal, that is, when

$$\frac{T}{R} - \frac{\mu V^2}{R} = 0. \qquad \text{Eq. 81}$$

The value of the speed given by this expression,

$$V = \sqrt{\frac{T}{\mu}}, \qquad \text{Eq. 82}$$

is then the critical speed with which the string must be pulled through the tube. This solution is due to P. G. Tait.

But since, with this speed, there is no pressure on the walls of the tube, one may entirely remove the tube, and there remains a solitary wave traveling along the string with a definite speed, which is independent of the curvature and is a function of only the tension and the linear density.

Interference of Waves in Strings

202. Returning for a moment to the stretched rubber tube, it is easily shown that a solitary wave sent down from the hand to the fixed end is reflected and travels back to the hand. If a second wave be sent from the hand so as to meet the reflected wave, the two are said to "interfere," and the resultant disturbance at any point on the tube where they pass is the sum of the disturbances due to the outgoing and incoming wave.

The following experiment, due to John Wylie, illustrates very simply the manner in which two *trains* of waves interfere.

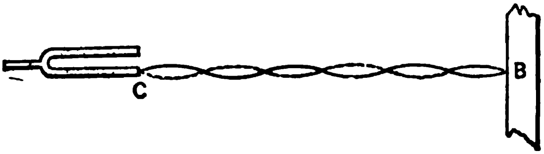

Fig. 144. — A single train of waves excited by one prong of a fork.

To one of the prongs of an electrically driven tuning fork attach a light string — silk floss is excellent — and make the other end fast to some rigid object, as shown in Fig. 144. When the fork starts to vibrate, a train of waves will run down the whole length of the string from C to B; and if the tension of the string is properly adjusted, stationary waves will be produced *throughout the entire length of the string.*

At some point A (Fig. 145) now tie a second string of length equal to CA; attach the other end to prong D and make the tension the same in each string. Each prong of the fork will now send a wave down the cord. But the prongs of the fork, at any instant, are moving in exactly opposite directions; hence at A the trains of waves in DA and CA are in exactly opposite phases. And since they each tend to displace the point A by equal amounts in opposite directions, the result is that the point A is not displaced at all. The waves DA and CA are said to **interfere completely** at A. No disturbance, therefore, can now enter that portion of the string lying between A and B.

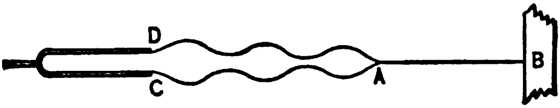

Fig. 145. — Wylie's experiment, showing two trains of waves interfering at *A*.

Interference is often observed also in trains of *water* waves. Thus when the solar and lunar tides reach any point on the earth's surface in the *same* phase, they interfere and thus produce the **spring tides.** When, however, they reach any point in *opposite* phases, they interfere and thus produce the **neap tides.**

It has been suggested that the so-called tidal waves which occasionally sweep the decks of the Atlantic liners are due to the interference of two trains of running waves.

Formation of Stationary Waves in a String

203. Referring to Fig. 144, where the string is fixed at one end *B*, we have first to discover how the wave reflected from *B* is related to the wave incident at *B*. The condition under which reflection here occurs is that the end is fixed; hence *the reflected wave must be of such a character that when compounded with the incident wave the result will be zero displacement of the string at B.*

The simplest method for obtaining the form of the reflected wave from this principle is by use of the terms *inversion* and *perversion.* Hold a mirror perpendicular to the direction of the stretched string; the image of a wave (in the mirror) is called its **inversion.** But if the mirror is held parallel to the cord, the image of the wave is called its **perversion.**

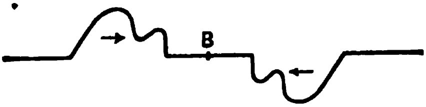

FIG. 146. — Condition of reflection from fixed end.

Let the upper wave in Fig. 146 represent the incident wave; then the wave reflected from *B* will be exactly the same as one which is both the inversion and perversion of the incident wave. For if we imagine the cord to be infinitely long, instead of stopping at *B*, and if an inversion *and* perversion approaches *B* from the rear, — as far behind *B* as the incident wave is in front, — then the two waves will evidently interfere at *B* in such a way as to leave that point unmoved; in other words the boundary condition will be satisfied. **The reflected wave may therefore be simply described as the perversion *and* inversion of the incident wave.**

But suppose now that a *train* of waves is reflected at B; then each particular reflected wave will, *at certain points in its return path,* meet an incident wave which is in exactly opposite phase; call this point N_1; furthermore, if at any particular instant N_1 is a point of exactly opposite phases, it will *always* be such a point for waves of this length. When the reflected wave has traveled half a length farther, it will again meet an incident wave in precisely opposite phase; this will also be a point of no motion; call it N_2. **All such points where the interference is such as to produce no motion are called nodes.** Midway between these are other points where the two trains of waves — incident and reflected — meet in exactly the same phase: these are called **antinodes.**

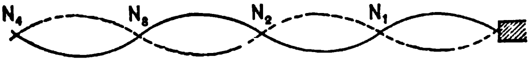

Fig. 147. — Production of nodes in a string.

With a little practice one can make the rubber tube vibrate in such a way as to show either one, two, or three, perhaps more, of these nodes. The fixed point B is of course always a node. Any portion of the vibrating string which lies between two consecutive nodes is called a **loop** or **ventral segment.**

The chief characteristic of stationary waves is that, *at each point* in the medium, the two interfering trains differ in phase by a constant amount, ranging from 0° to 180°. A string in which stationary vibrations have been set up may therefore be clamped at any two nodes — even two consecutive nodes — without changing the mode of vibration.

One convenient feature of stationary vibrations is that the nodes are easily observed by the eye; and since they are just half a wave apart, one can easily measure the length of the wave. Then, after eliminating V between the following equations: —

$$V = nl, \qquad \text{Eq. 76}$$

$$V = \sqrt{\frac{T}{\mu}}, \qquad \text{Eq. 82}$$

the frequency (§ 48) of the wave motion is obtainable from the directly measurable quantities, l, μ, and T.

Summary concerning Transverse Waves in Strings

1. Medium is linear.

2. Motion of particles is at right angles to direction in which wave travels.

3. Speed of propagation is independent of the length of the string, the material of the string, and the shape of the wave.

Speed depends only upon (i) the stretching force T, and (ii) the linear density μ.

4. Wave reflected from fixed point is perversion and inversion of incident wave.

5. In stationary waves the distance between two successive nodes is half a wave length.

V. Resonance

204. When one attempts to ring a large church bell, he soon learns that he must pull the rope at certain intervals if he wishes to ring the bell with ease. Any heavy object, such as a stone suspended from a crane, can be set swinging if one's pushes are properly timed. A heavy rowboat or small sailboat, moored in quiet water, can without difficulty be set rocking, if one throws his weight first on one side, then on the other, at correct intervals. The order, "rout step," given soldiers about to cross a bridge, is to prevent dangerous vibrations in the bridge.

What is more, every one knows how to discover the correct interval in each of these cases. By watching the amplitude of vibration, he at once times his efforts so that this amplitude will increase. These are all instances of the mechanical principle known as **sympathetic resonance.**

205. The following experiment should be carefully studied: —

Mount two heavy pendulums of any kind upon one common wall bracket or frame, as shown in Fig. 148. Heavy lead disks on $\frac{1}{4}$-inch rod, suspended by knife-edges at the top, do very well. The upper end of the rod should be furnished with an eye through which a piece of a three-cornered file may be thrust for a knife-edge.

The lower end of the rod should be threaded for three inches, so that the bob of the pendulum may be held up by a nut, and so that the period of either pendulum may be slightly varied at will. Paint one pendulum red, the other blue.

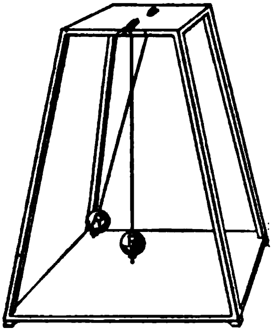

Fig. 148.—Illustrating transfer of energy by resonance.

1. Adjust the pendulums so that they have very nearly the same period. Set the blue one in vibration, leaving the red one at rest. In less than one minute, the red one will pick up sufficient kinetic energy to set itself in vibration with an amplitude not greatly inferior to that of the blue one.

Where did this energy come from? How did it get from the blue pendulum into the red pendulum?

2. Now reverse the order. Set the red pendulum in motion, allowing the blue one to be initially at rest.

3. Take a wooden mallet and tap one of these heavy pendulums in a most irregular way. The result of a dozen such blows is to disturb the pendulum very little.

Now give the same pendulum one half dozen gentle taps, delivering each tap *just as the pendulum passes its position of equilibrium,* going away from you. How do you explain the large amplitude which these six taps produce in the pendulum?

4. Adjust the pendulums so that one is from 8 to 10 per cent longer than the other. Set the *blue* one in vibration. What is the history of the *red* one for the next five minutes? How do you explain the small amplitude and the complete stoppages of the red one?

206. A supremely beautiful example of resonance is obtained in the following simple manner described by Professor Wilberforce of Liverpool (*Phil. Mag.*, October, 1894): —

At the lower end of a spiral spring suspend a mass whose moment of inertia is variable, and which can be so adjusted that its period of rotational vibration about a vertical axis is very nearly equal to its period of translational vibration along its vertical axis.

Now it may be easily shown that a spiral of wire — such as a watch spring, or even a paper ribbon — cannot be elongated without at the same time being twisted, nor can it be shortened without being at the same time untwisted.

Accordingly, if such a spring as that shown in Fig. 149 be set to vibrating up and down, the suspended cylinder will at each vibration be given a slight torque. If these torques are timed, as we have supposed, so as to coincide exactly with the free period of rotation, the cylinder will be gradually set into rotation, until finally practically all the energy of translation has been transformed into energy of rotation. But it is equally true that a spiral spring cannot be twisted without at the same time introducing more or less of a compressional stress along the axis; nor can it be untwisted without producing a stress which tends to elongate the spring.

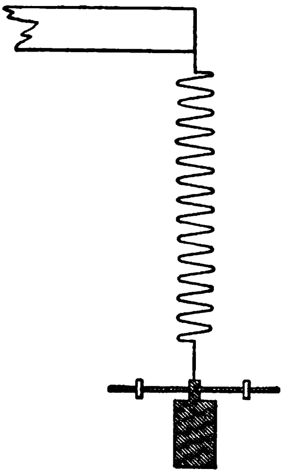
FIG. 149. — Wilberforce's spiral spring.

Accordingly, when the kinetic energy of the spring has become entirely rotational, and the difference of phase between the two vibrations has changed sign (for the periods are not *exactly* equal and even if they were at some one amplitude, they would not be so at others), each oscillation of the cylinder will produce a slight up-and-down motion, and the energy will now be transferred in the opposite direction, until it is again practically all translational.

What we have here, then, is resonance between two degrees of freedom in the same body, each motion being simple harmonic.

The following dimensions for Wilberforce's spring have been found convenient: —

Height of brass cylinder (Fig. 149)	$1\frac{39}{40}$	inches
Diameter of brass cylinder (Fig. 149)	$1\frac{1}{2}$	inches
Length of brass screw	$6\frac{1}{2}$	inches
Diameter of brass screw	$\frac{1}{7}$	inch
Weight of brass traveling nuts, each	$\frac{1}{5}$	ounce
Diameter of steel wire in spiral	$\frac{1}{16}$	inch
Diameter of spiral	$1\frac{1}{4}$	inches
Length of spiral (unstretched)	18	inches

Change of Wave Length due to Motion. Doppler's Principle

207. So far in our study of wave motion we have considered both the source of the waves and the observer of the waves to be relatively at rest. But if either the point at which the waves originate or the point at which they are observed is in motion, one with respect to the other, then the frequency of the train of waves will either be changed or appear to be changed in a definite ratio.

This idea may be most easily grasped perhaps from the following analogy, which is quantitatively correct for any ordinary train of waves.

Imagine two boys seated some distance apart upon the bank of a creek. Let the boy who is upstream have a basket of corks from which he throws into the creek n corks each second. So long as both boys remain seated the corks will float by the lower boy at the rate of n per second, *i.e.* with a frequency of n. And if the creek flows with a uniform speed V, the distance l between the successive corks will be given by the equation

$$l = \frac{V}{n}. \qquad \text{Eq. 76}$$

But let us now suppose that the upper boy starts downstream still emptying his basket at the same rate, the corks will now pass the lower boy *more* frequently than n per second: while if the basket of corks be carried upstream, the individual corks will float by the lower boy *less* frequently than n per second.

When both boys were at rest, there were n corks distributed over the distance traversed by the stream in one second; *i.e.* over a distance numerically equal to V. When the upper boy moves downstream with the speed v, there are n corks distributed over a shorter distance, namely, a distance numerically equal to $V - v$. Hence the wave length (distance apart of the corks) is shortened in the ratio $\frac{V - v}{V}$.

If the boy with the basket had moved upstream, instead of down, the wave lengths would have been increased, of course, in the ratio $\frac{V + v}{V}$.

The student will find it an interesting exercise to prove that if the boy with the basket remains seated, while the other boy moves upstream with a speed u, the frequency of the train will appear to be increased in the ratio $\frac{V+u}{V}$; and hence the apparent wave length is altered in the inverse ratio $\frac{V}{V+u}$.

When the boys both move up or down stream with the same speed, the problem becomes more complicated, and its discussion must be postponed. But when v is very small compared with V, it will be observed that $\frac{V-v}{V}$ is very approximately the same as $\frac{V}{V+v}$; so that in this case the effect is practically the same whether the source or the observer be in motion.

This general notion appears to have been first distinctly enunciated by Christian Doppler in 1843, and is therefore known as "Doppler's Principle." It may be stated as follows: —

When a source of waves, such as a tuning fork, is approaching or receding from an observer, the length of the waves emitted is decreased or increased respectively. When, on the other hand, an observer is approaching or receding from a source of waves, the frequency of the waves appears to be increased or diminished respectively.

ON THE TRANSFER OF MECHANICAL ENERGY. WAVES AND CURRENTS

208. The effects of waves on the seacoast are so well known as to need no description. The energy of waves is here exhibited on a gigantic scale, — in some places undermining cliffs, in others transporting large quantities of sand or tearing away sea walls. **Wave motion is, indeed, one of nature's general methods for transferring energy.**

Every child who has played with a garden hose knows that the jet of water can be used to dig a considerable hole in the ground. In the far West these jets are employed on a large scale for mining. The process is known as **hydraulic mining.** Currents of air are employed for running windmills, thus transferring the energy of the wind to the water, which is pumped into a tank in the attic of a house. One may transfer energy to a body by throwing a stone at it or by firing a shot at it. All of these are simply special kinds of currents.

Currents of matter may be called the second general method which nature employs for transferring energy. The moving matter, in these currents, may be air, water, stones, shot, etc.

Our study of the subjects which follow, Sound, Heat, Electricity, Light, we shall find to be largely a study of the transfer of energy by one or the other of these two general methods, *Currents* and *Waves*.

Problems

245. A clothesline which weighs 1 kg. is stretched between two posts which are 20 m. apart. The tension on the line is 3 million dynes. How long will it take for a transverse disturbance started at one end to travel the entire length of the rope?

246. A stretched cord is set into stationary transverse vibration so that its frequency is 120. The nodes are 40 cm. apart. With what velocity does a wave travel in this string?

247. Some long waves coming from the deep water of a lake strike upon the edge of a shoal which is covered with a uniform depth of 8 in. How fast will the waves travel through this shallow water?

248. A train of water waves each of which is 20 ft. long passes a buoy which is practically at rest. If at the instant under consideration a crest is passing the buoy, what is the condition of the water at a point 85 ft. back of the buoy?

249. Compute the speed of a wave 10 ft. long when traveling over the surface of deep water.

250. How long must a wave in the Atlantic Ocean be to have the same speed as the steamer *Lusitania* had, namely, 24 knots?

251. The water in a wave tank diminishes in depth from 9 in. to 1 in. How will the speed of long waves in this tank be affected?

252. Will a train of very short ripples travel more rapidly in mercury or in water? The surface tension of water in contact with air is 75; that of mercury in air is 513.

253. A whistle emitting 1200 vibrations per second is approached by an observer at a speed of 60 ft. per second. Assume the speed of sound 1100 ft. per second. Find what number of vibrations the observer will *appear* to hear in one second.

254. How much faster will a transverse wave travel in an aluminum wire than in an iron wire of the same cross section stretched by the same force?

255. Explain with diagrams on the blackboard the equilibrium theory of the tides.

256. Derive by Tait's method the expression for the speed of a transverse wave in a stretched string. Illustrate by diagram on blackboard.

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> "More important than any particular truth is the love of truth."
>
> CHARLES JOSEPH LITTLE.
> *Essay on Galileo.*

CHAPTER VI

SOUND

209. The study of Sound is largely a study of waves in air. Most of the motions involved in sound can, however, be made evident to the eye by proper experimental devices. The subject is, therefore, an important one, forming as it does a natural connecting link between those visible motions which we have just been considering in Mechanics and those invisible motions which we are shortly to consider under the heads of Light, Heat, and Electricity.

210. In taking up the study of any group of phenomena which are perceived by one of our senses, we are always confronted by three problems: —

(i) The first of these is the origin and transmission of the disturbance, or stimulus, which is detected by the sense in question; a purely physical problem.

(ii) Next there is the structure and function of the sense organ which detects the disturbance; a purely physiological problem.

(iii) Then there is the purely psychological problem involved in the translation of physiological stimulation into perception.

The first of these problems, which is also the simplest, is of course the only one considered in this chapter. Any good text-book on Physiology, such as Huxley's, will discuss the second problem; and any good text book on Psychology, such as James', will do the same for the third.

ANALYSIS

211. The disturbances which give rise to the sensation of sound will be considered in the following order: —

(i) Sound a wave motion.
(ii) Stationary waves in air.

(iii) Graphical representation of sound waves.
(iv) Reflection of sound waves.
(v) Speed of sound in { Solids. Gases.
(vi) Distinction between a noise and a musical sound.
(vii) Three features by which musical tones are distinguished from each other { Loudness. Pitch. Quality.
(viii) Relation of tones to each other. The musical scale.
(ix) Application of preceding principles to musical instruments { Wind instruments. Stringed instruments. Vibrating membranes.

I. Sound a Wave Motion

212. Of the multitudinous sounds which reach our ears during any single day, there are very few which we are not able to "locate" at once. And when we find the source of any sound, we find ourselves in the presence of some vibrating body.

When one string of a guitar is emitting a note, there is no difficulty in telling which one; we have only to look at the strings. A piece of thin steel — say part of the blade of a hack saw or piece of clock spring — clamped in a vise gives a note only when set in vibration. The sound disappears as the vibrations "die down." The edges of a tuning fork become hazy in appearance as it begins to emit a note.

Even when the sounds are not so distinct and definite as those mentioned, we have no difficulty in finding the vibrating body. A carriage rolling over a stony street emits many sounds; but a carriage maker will quickly detect the vibrating parts, and remove them or tighten them up, or interpose rubber in such a way that these vibrating parts no longer receive blows. The carriage then runs much more quietly.

In addition to the vibrating body, there is one other essential feature in the production of every sound; namely, an elastic medium — practically always air — between the vibrating body and the hearer.

The sound of a watch, a bell, or a clock, placed under the receiver of an air pump, is much diminished when the air is even partially removed. It is very difficult to make the sounding body quite inaudible in a vacuum, for the reason that the body must rest on some kind of support; and this support, being necessarily somewhat elastic, will transmit some vibration to the receiver, and hence to the outside air. But we are perfectly justified in inferring that, if no elastic medium whatever lay between the source of vibration and the listening ear, no sound would be heard.

For the hearing of a sound, then, two things are always required: —

(i) A vibrating body, rapidly disturbing its surrounding medium.

(ii) An elastic medium connecting the vibrating body and the listening ear.

II. STATIONARY WAVES IN AIR

213. One of the shortest cuts to a clear and accurate conception of sound waves is through the following series of experiments. Indeed, the whole philosophy of air waves and the propagation of sound is wrapped up in them. But the first step in understanding stationary waves in air is to understand the nature of the vibrating body which produces them. Hence we first consider the

Longitudinal Vibrations of a Rod

These may be illustrated by hanging a ball so as to just touch one end of a long wooden table, as in Fig. 150. A sharp blow with a hammer given to the opposite end of the table will send a wave of momentum through the entire length of the table. This momentum will in turn arrive at the ball and drive it out.

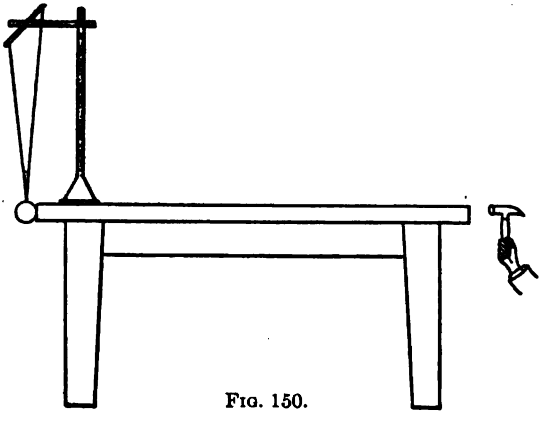


FIG. 150.

Let us now substitute for the table

top an ordinary round brass rod *AB* from 8 to 10 millimeters in diameter and from 80 to 100 centimeters in length, clamped to a table in an ordinary vise or by means of an iron clamp, as shown in the figure. A small ivory ball or glass marble or hollow sphere is suspended by two strings, as indicated, so as just to touch the end of the rod *B*. When rubbed with a dry cloth upon which a little powdered rosin has been sprinkled, the rod will emit a very high musical note. It is not the easiest matter in the world to get this note at first, but every one succeeds after a few trials; a fairly firm grasp and a quick but steady pull will generally accomplish the end.

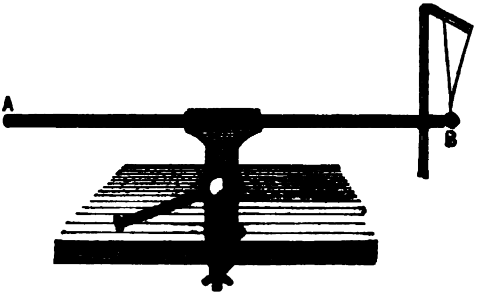


FIG. 151.—Showing longitudinal vibrations of brass rod.

When this note sounds, it is observed that the ivory ball rebounds, often several inches, *showing that it has received a blow from the direction* $\overline{AB}$.

The rosin on the cloth has evidently taken hold of the rod and elongated it, then let it go, taken hold of it again, suddenly let it go again, and so on. The vibrating body here is evidently the brass rod. The motion of the particles in the rod is evidently in the direction $\overline{AB}$. This is shown by the fact that they are originally displaced along that line by the rosin and by the fact that the momentum which they communicate to the ball is in the direction $\overline{AB}$. Such vibrations we have (§ 200) called longitudinal.

When the brass rod is elongated by a pull and suddenly let go, would you expect, from analogy with other bodies, that it would go back to its position of equilibrium and stop there, or would you expect its inertia to carry it beyond? Would you consider it necessary, for the production of free vibrations, that the vibrating body have *inertia* as well as *elasticity?* When the length of a rod is altered, as in this case, by stretching, what four factors determine the force which restores it to its original length? The present is an excellent occasion for reviewing the whole subject of elasticity.

Are the ends of the vibrating rods nodes or antinodes? The middle point must be a node because it is held fast by the iron clamp.

Character of Disturbance in Air

214. Let us now suppose that the nature of the vibrations in the rod is clearly understood. What sort of disturbance does the rod produce in the air? The displacement of the particles in the rod was so small and rapid as to be invisible; we were obliged to get our evidence by means of the ivory ball. In air, not only is the disturbance small and rapid, but the whole vibrating mass is invisible; so that, even when the disturbances are large and slow, we do not ordinarily see them.

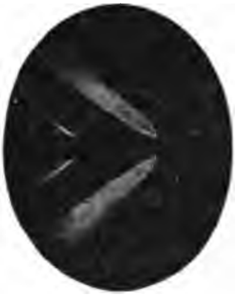


FIG. 152. — Showing the compressions and rarefactions which accompany a rifle ball in its flight.

However, Töpler of Dresden has shown that by a certain disposition of apparatus which he describes as the *schlieren Methode* it is quite possible to see these waves; while Mach of Vienna has succeeded in photographing the stationary waves which accompany a Mauser rifle ball, as shown in Fig. 152, and Wood of Baltimore has photographed sound waves under many interesting circumstances.

Professor A. L. Foley of the University of Indiana has again greatly improved the photographic method. His method is described in the *Physical Review*, Vol. 35, pp. 373–386 (1912).

Kundt's Resonance Tube

215. Nevertheless, the simplest manner of putting air waves into objective evidence is probably the following, which was devised in 1865 by the late Professor Kundt of Berlin: —

To one end of the brass rod described in the preceding section attach a disk of cork *B* which fits loosely into a glass tube about 30 millimeters in diameter and a meter in length.

By the aid of a cork and a file you can, in a moment, make enough cork dust to lightly cover the bottom of the tube.

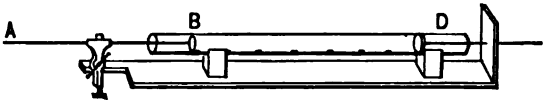


FIG. 153. — Kundt's device for studying waves in air.

At the end *D* of the glass tube, fit another easily moving piston, so that the distance between the two pistons *D* and *B* can be varied. A large glass tube is more easily excited into longitudinal vibrations than the solid brass rod.

Such an arrangement is called a **Kundt's resonance tube.**

Set the brass rod into longitudinal vibration as before. If the cork piston at *B* is free and does not rub on the walls of the glass tube, the elastic air will be disturbed very much as the ivory ball was. As the rod elongates, the air between the two pistons must be suddenly compressed, for it has not time to escape around the edge of the disk — the compression is too quick for that; when the rod contracts, the inclosed air is suddenly rarefied.

This rod thus starts a train of waves down the tube exactly analogous to those which we produced in the long spiral spring (§ 200). When this train of waves strikes the fixed piston (Fig. 154), we have all the conditions necessary for reflection. And if reflection occurs, we shall have a train of waves starting back in the direction from *D* to *B*. We should then expect this train of reflected waves to interfere with the incident train and produce stationary waves with their characteristic nodes and loops. The nodes would be those points in the tube at which the cork dust is not disturbed; the loops would be clearly marked throughout their length by the motion of the dust.

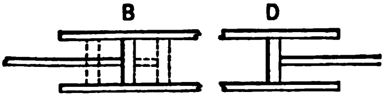


FIG. 154. — Showing action of piston in Kundt's tube.

216. The evidence for thinking that the phenomena which we have just predicted are the phenomena which actually occur is then the following: —

1. Air is a fluid. It has, therefore, no elasticity of shape. It cannot, therefore, transmit transverse vibrations.

2. There are certain equidistant points in the tube where the cork dust is not at all disturbed. (If these are not sharply marked on first rubbing the brass rod, the piston at D should be shifted a little.) These correspond to the nodes predicted.

3. Between these points of rest there are certain regions of very great disturbance. These correspond to the predicted loops.

4. By slightly adjusting the fixed piston, the nodes and antinodes become much more sharply marked. And this result is exactly what one might expect if the air waves had a definite wave length determined by the vibrating rod and *not* by the glass tube.

5. To these reasons for thinking sound waves longitudinal, must be added the very cogent evidence furnished by the manometric flame, which will be considered later, § 227.

From testimony of this kind one is led to think that sound consists of a wave motion in the air, and that the motion of the particles of air is to and fro in the same direction as that in which the sound is traveling, that this longitudinal wave motion can be reflected, and that two trains of sound waves may interfere to produce stationary sound waves.

What really travels through the air is, then, a series of compressions, alternating with a series of rarefactions. This is fairly represented in Fig. 155.

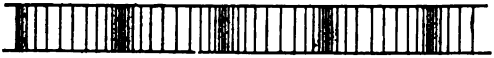


Fig. 155. — Showing the compressions and rarefactions in Kundt's tube.

Just how far apart these successive rarefactions and compressions are depends entirely upon how fast the piston works. Every time the piston moves forward, a compression occurs; this compression starts out on its journey traveling at a definite rate. When the piston goes back, a rarefaction starts out. These disturbances travel in the same direction and with the same speed. The distance through which a compression (or a rarefaction) travels while the piston makes one complete oscillation to *and* fro is called a wave length. And this corresponds exactly with the definition of **wave length** given above (§ 189); viz., **the distance between any particle and its nearest neighbor that moves in the same phase.**

III. GRAPHICAL REPRESENTATION OF SOUND WAVES

217. It is not possible to go much farther with the discussion of air waves without an exact graphical or analytical mode of describing them. The former of these two methods is the only one here available.

As to the proper mode of representing *water* waves there can be no question. We have only to draw a vertical section through the body of water at right angles to the crest of the wave. Such a section is shown in Fig. 129. The distance of any particle above or below the level surface OX measures the displacement of the particle.

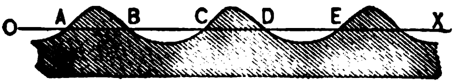


FIG. 129.

Nor can there be any hesitation as to the proper way of representing transverse waves in strings. Imagine the vibrating string at any instant to lose its flexibility; let it instantaneously become a rigid body, and we have at once a picture of the wave. Such a diagram is shown in Fig. 140.

The vertical distance of any particle from the position of equilibrium $\overline{OX}$ in Fig. 129 gives the displacement of that particle. This is the distance which is called the ordinate of the particle. From such a curve we can instantly point out those particles which are in the same phase, or in opposite phase. Be able to go to the blackboard and point out on such a curve a series of points which differ in phase by 180°; and another series which differ in phase by 90°.

But suppose that we are dealing with a compression running along a spiral spring. Here the displacements of the particles all lie in the direction of the stretched spring, *i.e.* along the line $\overline{OX}$, which represents the position of equilibrium. How then shall we draw these displacements? And the same question arises when we attempt to picture a sound wave.

The whole question is this: How shall we represent the state of disturbance at any and every point in the air when a train of waves is passing through it?

The method is as follows: We agree to represent all waves in one way, namely, after the manner of the transverse wave in Figs. 140 or 147. To every particle of air in the line along which the wave travels corresponds a point on the curve. But when the wave is *longitudinal,* the ordinate of any point on the curve represents the displacement of the corresponding particle *along* the line OX. **Displacements then are represented by lines drawn at right angles to the actual displacement.** This convention leads naturally to the following rules, for any point in the medium: —

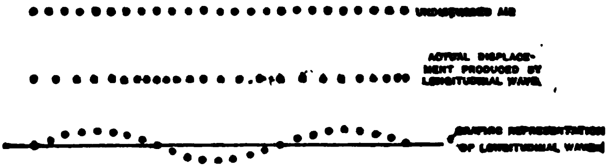

UNDISTURBED AIR
ACTUAL DISPLACEMENT PRODUCED BY LONGITUDINAL WAVE
GRAPHIC REPRESENTATION OF LONGITUDINAL WAVE

FIG. 156. — Displacement curve of sound wave.

(i) $\left.\begin{matrix}\text{Positive}\\ \text{Negative}\end{matrix}\right\}$ ordinates indicate displacement of air particles to $\left\{\begin{matrix}\text{right.}\\ \text{left.}\end{matrix}\right.$

(ii) Where the tangent to the wave is $\left\{\begin{matrix}\text{positive}\\ \text{negative}\end{matrix}\right\}$, there is $\left\{\begin{matrix}\text{rarefaction.}\\ \text{compression.}\end{matrix}\right.$

Thus Fig. 157 represents a wave in which the displacements are all to the right, and in which the progression of the wave is also to the right. Any point *c* on the front of the wave indicates a compression. Any point *r* on the

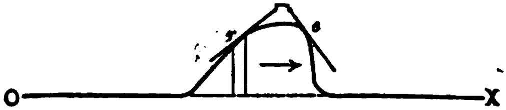


FIG. 157. — A solitary wave in air, compression leading.

rear side of the wave indicates a rarefaction. A point exactly at the top of the wave represents that point in the medium where there is neither compression nor rarefaction, but ordinary atmospheric pressure. Such a wave might have been produced by giving the piston of a Kundt's tube a single short quick displacement to the right.

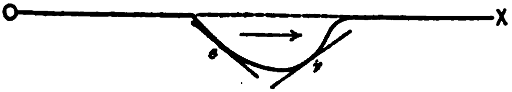


FIG. 158. — A solitary wave in air, rarefaction leading.

Figure 158 represents a wave in air which might have been produced by displacing the piston suddenly to the left. Here the rarefaction leads and the displacement of the particles is in a sense just opposite that of the wave motion. A solitary wave of the type of either Fig. 157 or Fig. 158 is generally called **a pulse**; so also is the blow which produces it.

IV. REFLECTION OF SOUND WAVES

218. The echo from a hillside or from a large building is familiar to every one. But reflection of sound is more conveniently shown by placing a watch in the focus of a parabolic mirror, as indicated in Fig. 159. The mirror thus reflects a beam of sound, so to speak, which may be directed to any portion of the room. This also illustrates the fact that the direction of the reflected beam is determined by the same rule as in the case of a billiard ball reflected from the cushion of the table or as in the case of water waves (Fig. 133) striking a pier; namely, **the direction of the incident sound and the direction of reflected sound make equal angles with the normal to the reflecting surface.**

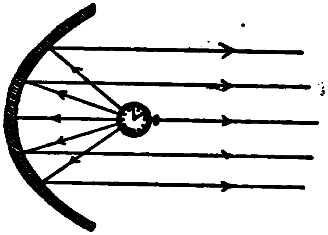


FIG. 159.

219. But in many cases

The Phase of the Reflected Wave

is also a matter of great importance — especially in Kundt's tube. Here reflection occurs at the face of a solid piston; hence *the condition of reflection is that there shall be no motion of the air particles* which lie immediately against the wall. The graphical method studied above tells us at once that the reflected wave must be both the perversion and inversion of the incident wave as shown in Fig. 146. For this is the only wave form which will compound with the incident wave and travel in the opposite direction, yet always give a zero displacement at B.

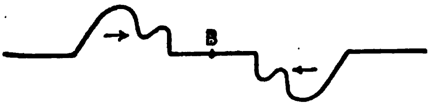


FIG. 146.—Condition for reflection from closed end.

Observe, however, that the slope of the resultant at B is far from zero. The value of the tangent is sometimes positive, sometimes negative, sometimes zero, showing that while the *motion* at B is zero, the *pressure* there is continually varying.

Curiously enough the waves in the Kundt's tube are reflected also when the fixed piston is removed, and the end of the tube is left open. Here the boundary condition is changed; for now the air at the end of the tube is perfectly free to move to and fro, but the pressure is invariable, namely, that of the atmosphere.

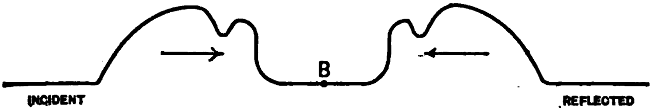


FIG. 160.—Condition of reflection from open end.

The graphical method tells us now that the reflected wave must be such as to keep the tangent of the resultant wave aways zero. From which it follows that the reflected wave is the inversion of the incident, as shown in the accompanying figure, 160.

220. One of the most highly instructive cases of reflection is that which occurs when a single sharp disturbance of the air is produced in front of a long flight of steps or near a fence constructed of vertical bars. From each step or bar is reflected a little wavelet; but since the distances of the steps from the ear increase in arithmetical progression, it follows that these wavelets will reach the ear as a succession of pulses — a train of waves in fact. The result is then that a single incident disturbance is reflected as a definite musical tone, whose frequency is determined by the distance apart of the bars.

This phenomenon has been appropriately called a **musical echo.** It is well shown by a paling fence or by one built of overlapping boards or by an iron fence in which are used square iron bars. It is most distinct on a day when the sky is overcast or at night when the air is most likely to be acoustically clear.

This phenomenon, which seems to have been first recorded by Thomas Young in 1813, has a very important bearing upon the explanation of what we call " white light," a difficult inquiry which must be reserved for much later study.

V. SPEED OF SOUND

221. The interval which elapses between the fall of a distant tree or stone and the noise which reaches the ear must have been observed in primitive times. But the first accurate measure of speed of sound was made by a commission appointed by the French Academy of Science in 1738. Some five or six stations near Paris were selected; a cannon was placed at each station; observers at each of the other stations recorded the interval between seeing the fire and hearing the report of one of the guns. By combining observations made in diametrically opposite directions from the gun, they eliminated the effect of the wind. The results obtained were, when reduced to a temperature of 0° C., 332.9 meters per second. These experiments were suggested largely by the fact that Newton (*Principia,* Book II, Proposition 50, Scholium) had some years previously computed the speed of sound from purely dynamical principles.

By methods whose details would here occupy too much time he concluded that the speed with which a compression or rarefaction travels through the air is given by the expression

$$V = \sqrt{\frac{E}{D}}, \qquad \text{Eq. 85}$$

where V = speed of the disturbance,
E = voluminal elasticity of the air,
D = voluminal density of the air.

In Newton's time the pressure of the air (approximately 1.014 million dynes per square centimeter) was thought to represent its elasticity. For the density of air, Newton employed the approximately correct value $\frac{1}{870}$ gram per cubic centimeter. These data gave him a value of V which is nearly 20 per cent smaller than the observed values. In the meantime, 1816, Laplace had explained this discrepancy by pointing out that the rarefactions and condensations which occur in sound waves are so rapid that there is not sufficient time for equalization of temperature between them, and that the value of E to be employed in Eq. 85 is that which represents the elasticity of air when no heat is admitted to it or leaves it. This elasticity was measured and was found to be 1.41 times greater than when the temperature of the air is maintained constant. Introducing this correction factor, which is generally indicated by γ, and denoting pressure by p, Laplace's expression for the speed of sound becomes

$$V = \sqrt{\frac{\gamma p}{D}} = \sqrt{\frac{1.41\,p}{D}}. \qquad \text{Eq. 86}$$

At this point Laplace requested the French government to make some new measures of this quantity, which they did in 1825, by a more refined application of the same method as employed by the French Academy. Their result (331.36 meters per second at 0° C.) compared very favorably with that predicted by Laplace. Concerning the speed of sound the best that can be said at present is that its value is not accurately known but in all probability lies between 331. and 332 *m./sec.* at 0° C.

Laplace's expression carried with it two interesting consequences. (i) From Boyle's Law (Eq. 74) we know that $\frac{p}{D}$ does not vary at any one temperature. *Hence as the barometer rises and falls one does not expect the speed of sound to change;* for the density is altered in the same ratio as the pressure.

(ii) But when the barometer remains constant, and the temperature changes, we should expect the density of the heated air to be less than that of the cooler, and consequently *the speed of sound to be greater in warm air than in cool.* And, indeed, the fact has been well established that, for atmospheric temperatures, the increase of velocity is about 60 centimeters per second per degree centigrade.

Speed of Sound in Solids

222. When the speed of sound in air is known, Kundt's tube offers us a beautiful method of finding the speed in the rubbed brass rod. When the brass rod is set into longitudinal vibrations we know that, whatever the length of the wave which travels through the brass rod, the two ends of the rod are antinodes; for they are free, and when compared with other parts of the rod they are places of great motion. The middle point likewise must be a node, because it is clamped, and is hence a place of no motion. Not only so, but it is the *only* node in the rod. The ends of the rod are then successive antinodes; accordingly the distance between them is one half the wave length of the disturbance in the brass.

Let us denote by N the frequency of the brass rod, and by L its length; then, employing the fundamental equation of wave motion, § 190, —

$$\text{Speed in brass} = 2\,NL.$$

Whatever the frequency of the vibrations in the column of air, we know *it is the same as the frequency of the brass rod.* Let us call the distance between successive dust heaps d; then

$$\text{Speed in air} = 2\,Nd.$$

Hence
$$\frac{\text{Speed in brass}}{\text{Speed in air}} = \frac{2\,NL}{2\,Nd} = \frac{L}{d}.$$

$$\text{Speed in brass} = (L/d)\ 332 \text{ meters per second.}$$

Accordingly, we have only to measure the distance between the dust heaps and the distance between the ends of the brass rod.

Evidently the same method may be employed with steel, glass, wood, and other substances.

Speed of Sound in Gases

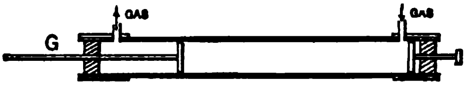


FIG. 161.—A form of Kundt's tube for use with various gases.

223. Most useful of all is Kundt's tube in determining the speed of waves in various gases. For this purpose a glass rod (*G*, Fig. 161), held rigidly at its middle point by being corked and waxed into the outer tube, is better than the brass rod.

The movable piston rod at the other end must be reasonably gastight. Into each end is sealed a tube through which various gases can be introduced and removed. Measure the distance between the dust heaps when the tube is filled with air. Denote this distance by l. Now fill the tube with ordinary coal gas. The dust heaps are much farther apart than before. Call that distance L. Let N be the frequency of the glass rod. Then, since the fundamental equation of wave motion always holds, we have

$$\begin{aligned} \text{Speed in coal gas} &= 2\,NL, \\ \text{Speed in air} &= 2\,Nl, \end{aligned}$$

and hence, eliminating N, the numerical value of which we do not know,

$$\frac{\text{Speed in coal gas}}{\text{Speed in air}} = \frac{L}{l}, \qquad \text{Eq. 87}$$

where both quantities in the right-hand member are easily measurable. The process for other gases is essentially the same.

When once the speed of sound has been measured for any particular gas, Laplace's equation (86) furnishes a convenient means of determining the value of γ, a constant of great importance in the theory of heat.

Sound Waves in Three-dimensional Space

224. Hitherto we have been considering waves in air as they travel through tubes, say through a speaking tube; but in ordinary conversation we stand in an open space, surrounded by air on all sides; the voice is heard by those at our back, also by those in front, and by those at our side. The waves produced by our voice evidently travel away from us in all directions. As the pebble dropped into smooth water sends out rings of disturbance, the voice sends out spheres of disturbance. These expanding spheres are propagated in the same manner as waves in tubes, they travel at the same rate; the displacement of air particles is longitudinal. These spherical waves are therefore fairly represented in Fig. 162.

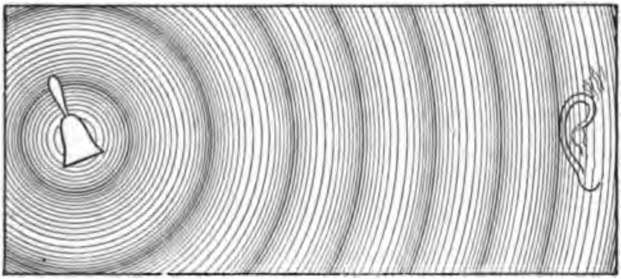


Fig. 162. — Sound waves in air.

As an illustration of a wave motion propagated through a volume (*i.e.* a three-dimensional wave) one may take the remarkable explosion which occurred at Krakatoa on the 26th and 27th of August, 1883.

In this volcanic eruption a great column of gas was shot up to a height of perhaps 20,000 feet. When this mountain of air and other gases settled down, it sent out an air wave through the rest of the earth's atmosphere. This air wave expanded in circles. It girdled the whole earth, converging at the antipode of Krakatoa. Passing over any point of the earth's surface, it compressed the underlying air; the height of the barometer was accordingly increased; the wave of depression following had just the opposite effect. In this way all the recording barometers on the earth were affected. And by such means this great disturbance was followed through three trips from Krakatoa, near the island of Sumatra, to the antipodes. The entire circuit of the globe was performed in the almost incredibly short interval of 36 hours. In other words, the speed of this great wave was

700 miles an hour. The speed of sound at 18° C. below zero is 723 miles an hour.

225. Following is a table containing the speed of sound in a few of the more important substances.

SPEED OF SOUND

SUBSTANCE	TEMP. C	SPEED, METERS PER SECOND	SPEED, FEET PER SECOND	AUTHORITY
Air	0°	332.5	1091	Kayser
Air	− 45.6	305.6	1002	Greely
Alcohol	8.4	1264	4148	Martini
Aluminium		5104	16740	Masson
Brass		3500	11480	Various
Carbon dioxide	0	261.6	858	Dulong
Copper	20	3560	11670	Wertheim
Glass { from		5000	16410	Various
Glass { to		6000	19690	
Hydrogen	0	1286.4	4221	Zoch
Illuminating gas	0	490.4	1609	Zoch
Iron and soft steel		5000	16410	Various
Oxygen	0	317.2	1041	Dulong
Water	13.7	1437	4714	Martini
Wood:				
Oak along fiber		3850	12620	Wertheim
Pine along fiber		3320	10900	Wertheim

Smithsonian Tables, pp. 100, 101.

VI. DISTINCTION BETWEEN A NOISE AND MUSICAL SOUND

226. It is not a violent assumption to suppose that every student recognizes at once the difference between a musical tone and a noise — even though individuals may classify differently the performance of a bagpipe or of a hurdy-gurdy. But the problem which confronts us here is to distinguish between the character of two mechanical disturbances, one of which produces music, the other noise. How do the vibrations of a wagon rumbling over cobblestones, of the splash of water, or of footsteps on a pavement, differ from those of a wire in a piano? To answer this question intelligently one must first discover some means for accurately describing the vibrations emitted by various bodies.

The Manometric Flame

227. So far as concerns a picture of sound waves in the air, nothing can surpass in simplicity the "manometric flame," which was invented and perfected by Rudolph König, of Paris. The construction of the instrument will be clear on consideration of Fig. 163.

Two blocks of wood, *R* and *S*, are bored out. Between them is clamped a diaphragm of very thin goldbeater's skin. This diaphragm divides the whole apparatus into two parts. On the left-hand side of the diaphragm illuminating gas is admitted, as indicated by the arrows, and so long as the diaphragm is undisturbed, the flame is apparently undisturbed. But if the disturbance of the flame is very rapid, as is the case with acoustical disturbances generally, we might not be able to detect by the eye any changes in the flame. Accordingly a rotating mirror is placed near the flame. A crank enables the observer to give this mirror a moderate angular speed, say two or three turns a second. For every new position which the mirror assumes, the image of the flame takes a new position. The effect of rotation on the mirror is, then, to separate the images of the flame at successive instants.

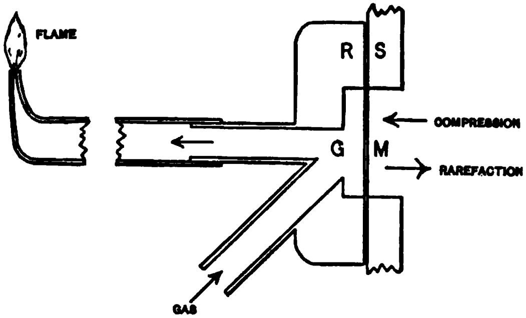


Fig. 163.—König's device to show variation of pressure produced by sound waves.

The space on the right of the diaphragm *M* is one end of a speaking tube, a sort of *cul de sac*, into which all kinds of sound waves can be sent for examination and study.

The effect of a sound wave upon the diaphragm will be (if our previously formed conception of sound waves be correct) to drive it alternately forward and backward. When a compression arrives at the diaphragm, the membrane will be thrust slightly to the left. The gas space *G* will suddenly be made smaller in volume; the pressure of the gas will as suddenly rise; the flame will immediately become higher — just as if more pressure had been put on at the gas works.

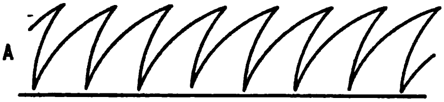
But this compression in the speaking tube will be followed by a rarefaction. What is the effect of this rarefaction on the membrane? The pressure on the right diminishes, the membrane moves to the right, the pressure in the gas chamber *G* suddenly diminishes; the flame as suddenly diminishes in height.

The essential features of the instrument are, then: —

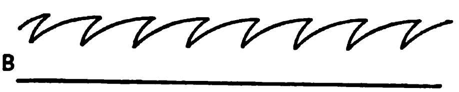
1. A membrane which responds at once to any displacement of the air particles adjoining it; *i.e.* to any wave of compression or rarefaction.

2. A gas flame so arranged as to respond to every motion of the membrane.

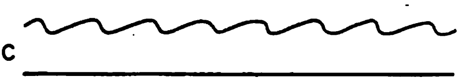
3. A mirror that can be rotated so as to separate the successive images of the flame, and thus allow us to detect the variations of pressure at the membrane due to any disturbance of the air.



A VIOLIN STRING STRONGLY BOWED.



A VIOLIN STRING GENTLY BOWED.



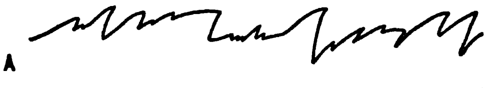
A TUNING FORK YIELDING THE SAME NOTE.

FIG. 166.

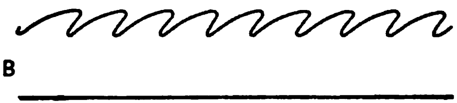
Since now the height of the flame measures, in a general way, the displacement of the particles adjoining the membrane, we need only draw an outline of the image of the flame in the rotating mirror, and we have at once a graphical representation of a sound wave. Such are the following diagrams (see Fig. 166). They merely record the variations of pressure produced by three different trains of waves at the membrane of the manometric flame. As the name indicates, such a flame is simply a quick-acting barometer, responding to changes of pressure which occur at rates even as high as several hundred per second.

Within the last few years Professor D. C. Miller, of Cleveland, has succeeded, by use of a delicate membrane and mirror, in photographing the displacement curves of various notes. The accuracy with which he is able to record the motion of the air particles during the passage of a sound wave far surpasses anything else accomplished along this line.

228. If now one examines with this instrument a number of noises and musical tones, he finds that corresponding to these two different sensations, the manometric flame shows two different kinds of disturbances in the air. These are exhibited fairly well in Fig. 167. The noise is made up of many small parts of trains of waves, all jumbled together. But the flame shows that a musical note is produced by *one* train of uniform waves.



GRAPHICAL REPRESENTATION OF A NOISE.



GRAPHICAL REPRESENTATION OF A MUSICAL NOTE.

FIG. 167.

It shows also that vowels, as employed by singers, are musical tones, while consonants are essentially brief noises.

"**The sensation of a musical tone is due to rapid periodic motion of a sonorous body; the sensation of a noise to non-periodic motion.**" — Helmholtz, *Sensations of Tone*, p. 8.

VII. THREE FEATURES BY WHICH MUSICAL TONES ARE DISTINGUISHED FROM EACH OTHER

A. Loudness

229. When the key of a piano is struck gently, and again with a hard blow, we recognize that the two notes thus produced are alike except that the second is *louder* than the first. The vibrations of a guitar string when first struck are sufficiently large for us to see them; but as the vibrations diminish in amplitude, we observe that the note diminishes in *loudness*.

When a note is sung into the tube of a manometric flame, it is at once evident that the indentations in the ribbon of flame are deeper in proportion as the note is louder (see Fig. 168). That is, the louder the sound, the greater the displacement of the air particles against the membrane.

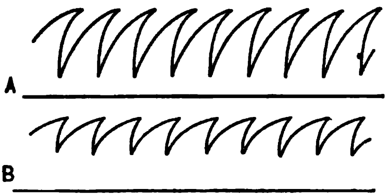


FIG. 168.—*A* and *B* are two notes alike in all respects, except that *A* is louder than *B*.

From these, and similar experiments, it is concluded therefore that **the loudness, or intensity, of a sound increases or diminishes as the amplitude of the vibrating air particles at the ear increases or diminishes.**

Since the energy of a vibrating body is, other things being equal, proportional to the square of its amplitude, loudness evidently increases with energy of vibration in the source. The variation of loudness with distance is an immediate consequence of this fact. For when a sound wave spreads out as indicated in Fig. 162, § 224, the energy emitted by the source during any minute interval of time is, at any later instant, spread out over a spherical surface which has the source at its center. Now the area of this spherical surface must increase as the square of its radius and hence the quantity of energy which is incident upon the ear at any given instant must vary inversely as the square of this distance, a fact with which every one is familiar. Few facts in nature are more surprising than the minuteness of the amplitude of ordinary sound waves.

Digression on the Effect of Wind upon the Intensity of Sound

230. When a spherical wave is propagated in the manner just described, it is customary to describe the continuous locus of all air particles having the same phase as the **wave front**. When an observer is at some distance from the origin of the sound, the wave front which strikes his ear is practically plane. Suppose that a wind is blowing in the same direction in which the sound is proceeding. One might expect the apparent speed of sound to be slightly increased; or if the wind be traveling against the sound, he would find the speed slightly diminished. But it is a matter of common observation that the audibility or loudness of sound coming from any considerable distance is also dependent upon the direction of the wind. A sound is said to "carry" better with the wind than against it.

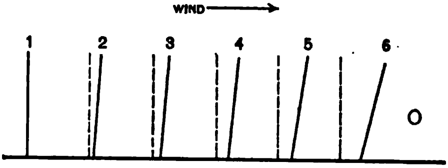


FIG. 169. — Sound traveling with wind. Dotted lines show successive positions of wave front in still air.

The explanation of this phenomenon was first given in 1857, by Stokes of Cambridge. He argued that since the upper layers of the air are always carried along more rapidly by the wind than those next the surface of the earth, a vertical plane wave front traveling with the wind will, as it advances, become more and more inclined toward the surface of the earth, as shown in Fig. 169. In this manner the sound is, so to speak, continually brought down to the ear of the observer. But if the sound be traveling against the wind, the upper portions of the advancing plane wave front will be bent backwards so that the sound is directed somewhat upwards and above the listening ear as shown in Fig. 170.

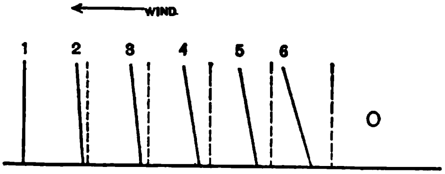


FIG. 170. — Sound traveling against wind. Solid lines indicate actual wave fronts.

If we remember that sound always travels in a direction perpendicular to the wave front, it will be clear that the observer is here left in a sort of sound shadow, all that reaches him being what little is diffracted in his direction.

231. Another factor in the loudness of sound is the presence of some large elastic body with which the vibrating body is rigidly connected. Thus a violin string when removed from any sounding board gives a thin, weak sound; but when stretched over the body of the violin, the sound is full and resonant and less prolonged. The explanation of this phenomenon, which is as follows, was also given by Stokes (*Papers*, Vol. IV, p. 299): —

When a solid body moves through the air slowly, the air flows from in front toward the rear, almost exactly in the same manner as if the solid were moving through an incompressible fluid; but when the motion becomes very rapid, the air in front is to some extent compressed, and equalization of pressure between the front and rear does not occur in exactly the same manner as before. If, however, the vibrating body be thin and small, as a piano wire, the equalization of pressure between front and rear is very much facilitated. But in the case of a broad flat body, such as the sounding box of a guitar, there is less chance for the lateral escape of the air and hence, corresponding to every slightest vibration of this board, a wave of condensation and rarefaction is emitted. For the air can no longer behave as an incompressible fluid unless the frequency of vibration be very much less than that employed in music. Thus, although the amplitude of the sounding board is much less than that of the string, the actual amplitude at the ear is enormously increased by its use. In the reënforcement of sound in this manner the two fundamental factors are, then, the size and frequency of the vibrating body.

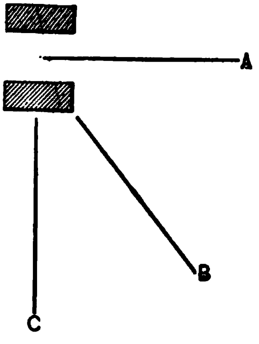


FIG. 171.—Stokes's explanation of effect of sounding board.

Stokes illustrates this principle most beautifully by holding near a tuning fork a piece of cardboard in the position indicated by the plane *B* in Fig. 171, where the two small rectangles represent a horizontal cross section of the fork. If the cardboard be placed in the symmetrical positions *A* or *C*, the air in front of the vibrating prong has no difficulty in getting around to the rear, but when the card is held in a diagonal position, with its edge near the edge of the fork, as at *B*, these side currents are hindered, and waves of compression and rarefaction, instead of annulling each other, go to increase the intensity of the sound. The increase in intensity on shifting the cardboard from position *A* to position *B* is easily observed by a large group of hearers.

B. Pitch

232. On clamping a piece of steel or brass spring, say a piece of clock spring, in a vise, one observes not only that the spring vibrates more rapidly as it is made shorter, but also that it *emits a higher note*. See Fig. 172.

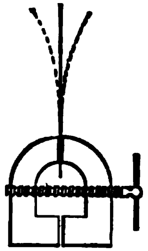


FIG. 172. — Illustrating the fact that pitch varies with frequency.

In the case of a wire, whether stretched from one fence post to another or between two fixed pins in a piano, there is no doubt but that it vibrates more rapidly the more it is stretched; we observe also that the note it emits is higher and higher the more it is stretched.

From such experience we might suspect the pitch of a note to depend upon the frequency (§§ 48, 190) of the vibrating body. But we need more direct and conclusive evidence to thoroughly establish such a conclusion. A card held against a rotating toothed wheel will furnish us just such evidence. Here we compel the card to make one vibration every time it passes a tooth. The frequency of the vibrating card is the number of teeth passed per second. When the wheel slows up, the pitch of the note produced by the card falls; when the wheel increases in speed, the pitch of the note rises.

If still further evidence be desired, try the following experiment: —

Into the speaking-tube of a manometric flame sound two notes of different pitch in rapid succession. Or, what is better still, use two manometric flames placed near each other and viewed in the same rotating mirror. It is then easily seen that when the flames are excited by notes of the same pitch, the notches in the image of each flame are at the same distance apart.

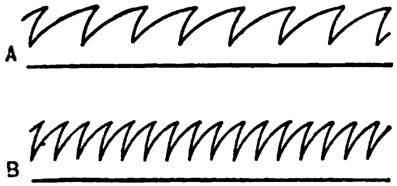


FIG. 173. — *A* and *B* are two notes alike in all respects except pitch.

But if either flame be excited by a note of higher pitch, the notches on this flame are closer together. And if one note be an octave above another, we see that the notches are exactly twice as close together in the high note as in the low. See Fig. 173.

Such evidence leads us to think that the pitch of any tone depends simply upon the frequency of the vibrating body which produces it; or, what is the same thing, upon the period of the vibrating body.

For, if we denote the frequency by *n* and the period by *T*,

$$n = \frac{1}{T}.$$

Defining equation for frequency.

The numerical value of any pitch is the frequency *n*. A tuning fork which makes three hundred vibrations per second is said to have "a pitch of three hundred."

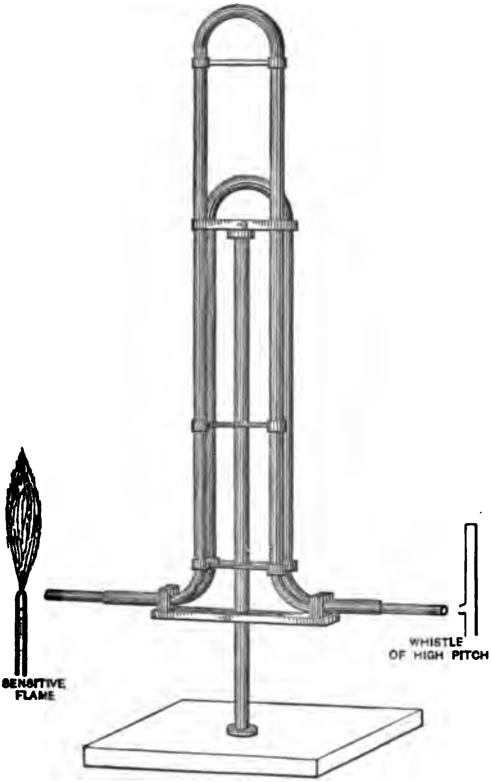


Fig. 174.

233. There are many laboratory methods for the actual determination of the pitch of sound, most of them based upon the characteristic equation of wave motion; namely,

$$V = nl. \qquad \text{Eq. 76}$$

The following must suffice as a type of them all. Take a small whistle, such as that devised by Galton, for the source of sound whose pitch is to be measured. This source is placed at one end of a Y-shaped tube which opens into two parallel tubes. These parallel tubes, at their farther end, again unite into a Y-shaped tube, as shown in Fig. 174.

If, now, a gas flame which is just on the point of flaring be placed at the end of this second Y, it will serve as a very sensitive means for detecting any disturbance in the air at that point. The U-shaped portion of one of these parallel tubes is so arranged as to telescope upon itself, somewhat after the fashion of a trombone.

In this manner the length of path from the whistle to the flame is constant through one tube and variable through the other. At the first Y, the wave which leaves the whistle is divided into two parts. These two parts reunite at the second Y, but the phase in which they meet depends upon the difference of length between the two routes which they travel. If this difference is a whole number of wave lengths, the two waves will reënforce each other; but if their path differs by an odd number of half wave lengths, they will interfere and produce silence. As the telescope is moved along, these silent points will be indicated by the sensitive flame. If now we measure the difference in length of the variable tube between two successive silent points, we shall have the wave length of the tone which is emitted by the whistle. Knowing the velocity of sound in air V, we may now easily compute n from Eq. 76.

One advantage of the sensitive flame is that it may be used even when the pitch of the whistle is so high as to be inaudible to the human ear.

C. Quality

234. But two notes may have the same pitch, and may have the same loudness, and yet be quite different. Two tenor voices singing the same note with equal loudness may be easily distinguished; for, as we are accustomed to say, they have different **qualities.** Strike the same note upon a piano and upon an organ; — let them be equally loud; no one finds any difficulty in distinguishing the piano from the organ. Two of our friends may converse in the same key and with equally loud voices, but we do not hesitate in telling their voices apart, even though they stand at our back, or perhaps in the dark. Their voices have a different quality. These differences are most strikingly illustrated in the various instruments of the orchestra, where we recognize with great ease just which instruments are being used. A guitar string when plucked at its middle emits a note of different character from that of the same string when plucked near one end.

How shall we explain these differences, so familiar to every one? Difference of amplitude produces difference of loudness. Difference of frequency produces difference of pitch. But what produces difference of quality?

The most direct answer to this question is obtained by means of the manometric flame. Into the speaking tube of such a flame pronounce as nearly as possible, with the same pitch and loudness, each of the vowels of the alphabet. It is observed at once that the ribbon of flame seen in the mirror is different in each case; to each particular vowel corresponds a wave of different shape. The notches in the ribbon have not the same form for any two vowels. **It is then the form of the wave which determines the quality of any note.**

It is well to remember that, while to every different quality there corresponds a different form of wave, Helmholtz has proved that the converse of this is not true. Quality depends simply on what components combine to produce the wave, not at all upon the *phases* in which they combine.

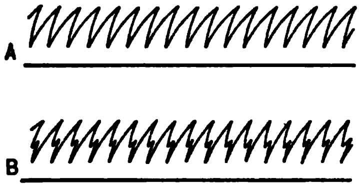


FIG. 175.—Two notes of the same pitch and loudness, but of different quality.

In Fig. 175 is shown the appearance of two flames each excited by a note of the same frequency and same loudness, but of different quality. In this case we see not only that the form of the wave in *A* is different from that in *B*, but we see just *how* it differs.

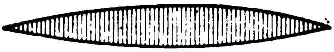
In *B* there is present a small wave having twice the frequency of that of *A*. So that *A* represents the fundamental, while *B* represents a note in which both the fundamental *and* the octave are present.

In the case of the vowels which we have just examined, this difference of wave form is brought about by changes in the cavity of the mouth. Pronounce the vowels *a* and *o* in succession, and observe the manner in which you change the air space inclosed within your mouth during the interval between the pronunciation of the two vowels.

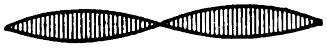
In general, experiment shows that the wave form which determines the quality of any tone depends upon the presence of vibrations more rapid than the fundamental, the fundamental being defined as the lowest note which a body can give out. These other and higher notes are called *overtones* or *harmonics.*

235. The presence of these harmonics in a piano wire may be easily shown by holding down a key of, say, middle C, and then striking several times a note one or two octaves below. It will then be found that when the vibrations of the lower note have been damped out, the upper note is still sounding, owing to resonance between it and one of the harmonics of the lower note.

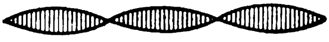
The resonators devised by Helmholtz (which are simply hollow spheres tuned to respond to certain definite tones), when connected with a manometric flame, form a very sensitive means for detecting those harmonics which have the same frequency as the resonators.



A. FUNDAMENTAL 1:1



B. FIRST OVERTONE 2:1



C. SECOND OVERTONE 3:1

FIG. 176.

The existence of these overtones may easily be shown in any stretched string. Damp the string at the middle point. If now it be bowed at a point one quarter of the length of the string from either end, the string will vibrate as shown in Fig. 176, *B*, and the octave of the fundamental will be heard. Evidently, therefore, the first overtone in a stretched string is the octave of the fundamental. For (§ 215), $V = 2\,nL = 2\,n_1L_1$, where n is the frequency of the fundamental and n_1 is the frequency of the first overtone; and since $\frac{L}{L_1} = 2$, it follows that $\frac{n_1}{n} = 2$. It may be shown in like manner that the pitch of the second overtone is to that of the fundamental as 3 : 1. The third overtone is two octaves above the fundamental, and so on.

A better method still of showing these overtones is by aid of two open organ pipes (Fig. 177), one of which is an octave above the other. When both are sounded at the same time, the first overtone of the lower pipe can readily be made to beat with the fundamental of the higher pipe. **It is these higher, and generally feebler, tones which give the distinguishing quality to any particular note.**

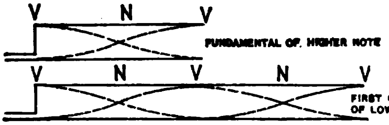


FIG. 177. — Showing wave length of overtone in one pipe equal to wave length of fundamental in other.

236. Here is perhaps the place to mention a useful distinction between the terms "note" and "tone." When, for instance, a violin string is bowed, the whole musical sound emitted is called a **note**. But, as we now see, each of these notes is made up of a series of irresolvable notes which are called **tones**. The lowest of these tones is the one which determines the pitch of the note, and is called the **fundamental**; the rest are called **overtones** or **upper partials**. If the frequencies of these overtones are exact multiples of the frequency of the fundamental, they are called **harmonics**.

Digression on Beats

237. When two notes are slightly out of unison, they have slightly different wave lengths in air. The consequence is that they reach our ears with a difference of phase which is constantly changing. At one instant, they interfere in such a way as *almost* to produce silence; at the next instant, they combine in such a way as to produce a sound louder than that of either alone.

Most people are familiar with the peculiar rise and fall that attends the sound of a large bell after the clapper has struck it for the last time. This is due to the interference of two different segments of the bell, such as *A* and *B* in Fig. 178, which travel around the rim owing to lack of perfect symmetry or homogeneity in the metal. The same phenomenon is easily shown with a glass bell or an iron ring.

Two tones which interfere in this way to produce this alternate rise and fall of intensity are said to **beat**. These alternations of loudness are called **beats**.

This phenomenon is most readily shown by means of two tuning forks which are in unison. If the forks are in exact unison to begin with, a little wax placed on the prong of either one will slow it up; for the wax adds to the inertia of the fork, and does not add to its elastic force. Consequently, whatever difference of phase may exist at any instant between the two forks, this difference will continuously change, and hence will follow a series of beats. The more wax placed on the prong, the more rapid the beats. How do you explain this? Two guitar strings are easily adjusted to unison. Tighten either one ever so little and they beat.

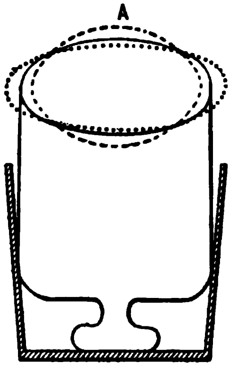


FIG. 178.—A vibrating bell jar: really equivalent to four different vibrating bodies.

It is easily seen from the theory of waves in general that the number of beats per second is numerically equal to the difference in frequency between the notes beating. For the interval between two successive beats is simply the time taken for one train of waves at any point, say at the listening ear, to gain 360° (2π radians) in phase over the other train. Each train of waves travels at the same speed; but as these trains pass any one point their difference of phase is continually changing. If one note makes 500 vibrations per second and another 502 per second, it is evident that at any point the latter will gain in phase, over the first, 4π radians each second. These two notes will be in the same phase twice each second, and in opposite phase twice each second. They will therefore beat twice each second. **In general, if the frequency of one note is n and of another n', the number of beats per second will be $n - n'$.** Absence of beats is the simplest test of unison for notes of nearly the same pitch.

VIII. RELATION OF TONES TO EACH OTHER. THE MUSICAL SCALE

238. Up to the present moment we have been studying the nature of a single train of waves in the air, the propagation of a single musical tone. We now proceed to consider some of the relations of musical tones to each other. Here we find ourselves already in the domain of **Music.**

But there are some experimental facts which must be viewed as a sort of connecting link between Music and Physics. These constitute the

Scientific Basis of Music

There are few ears which have the ability to determine, on hearing a single musical tone, just what the pitch of the note is. Frequency is a difficult thing for the human ear to determine. But there are *very many* observers who, on hearing *two* notes, can tell what the *ratio* of their frequency is. If two keys on a piano are struck, there is no difficulty in saying whether their pitches are related as 3 : 2 or 1 : 2, or in some other ratio; but not more than one person in a hundred can make an intelligent guess as to the frequency (absolute pitch) of either note.

Definition of Musical Interval

239. The interval between two notes is defined as the ratio of their frequencies. Two notes having the same pitch, whatever their quality or intensity, are said to be in unison. The interval in this case is not zero, but unity. If the ratio be 2 : 1, the interval is said to be an

Octave

A note whose pitch (frequency) is 400 is, then, an octave *above* a note whose pitch is 200. A note whose pitch is 100 is an octave *below* one whose pitch is 200. The impressions produced upon the human ear by a note and its octave are strikingly similar. So alike, indeed, are the two sensations that many fairly well-trained musicians sometimes mistake the one for the other. So far as the author is aware, this similarity has never been satisfactorily explained.

The Major Chord

240. Still more remarkable is the following fact concerning the human ear — a fact which has been well known for more than two thousand years, a fact which is fundamental in all music, but which, up to the present century, had baffled all attempts at explanation.

The fact is that any three tones whose frequencies are as 4 : 5 : 6 always make a pleasing impression upon the ear. This combination of notes is known as the **major triad**; or, taken together with the octave of the fundamental, is called the **major chord**. The universal judgment is that three such notes, whatever their frequencies, produce harmony when sounded together.

The explanation of this, which we owe to Helmholtz, is based upon the disagreeable impression resulting from beats.

Helmholtz likened the effect of beats on the human ear to the unpleasant sensation of a flickering light upon the eye. When, however, the beats become *very* rapid, they cease to be disagreeable, — just as an electric light alternating in intensity ceases to be painful when the alternations are very rapid. Helmholtz showed that beats are most disagreeable when they occur at the rate of 30 per second; but they can be detected even when they run as high as 132 per second. He next showed that the physical basis for the harmony of the major chord lies in the fact that neither between notes whose frequencies bear the simple ratios 4 : 5 : 6, nor between their overtones, nor between their overtones and fundamentals, are there any disagreeable beats.

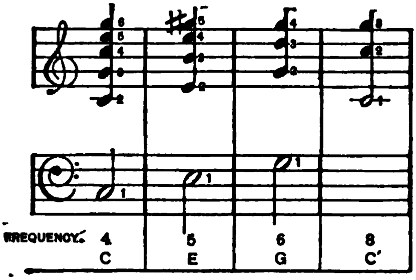


Fig. 179. — Major chord represented by half notes; harmonics represented by quarter notes.

In Fig. 179 are represented the totality of tones in the major chord up to the sixth harmonic.

Beginning with the fundamental C, it will be observed that none of the overtones are so near the fundamental or each other as to beat and produce dissonance.

Consider next the note E, whose frequency is five; its fourth harmonic is in unison with the fifth harmonic of the fundamental. But the third and fifth harmonic of E are each more or less dissonant with the fourth and sixth of the fundamental.

When we come to the note G, whose frequency is six, we find only one harmonic, the third, which is not already present in the fundamental. This third beats with the fourth and fifth of the fundamental and distracts somewhat from the harmony. But the octave C′, it will be observed, introduces no harmonics whatever that are not already in the fundamental. This almost complete freedom from beats thus explains the remarkable harmony of the major chord.

The Major Scale

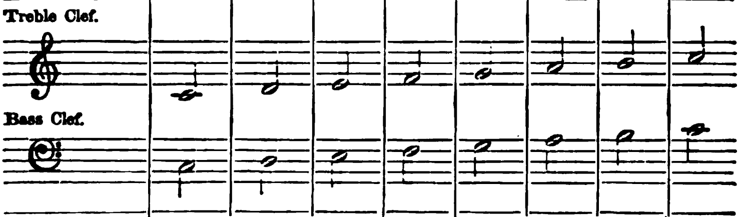
241. Musicians divide the interval between any note and its octave into a series of seven smaller intervals called **tones** and **semitones**. The notes which make up the octave are denoted by the letters of the alphabet and are built up out of three major triads, as follows: —

$$\begin{aligned} C:E:\ \ G &= 4:5:6. \\ F:A:2\,C &= 4:5:6. \\ G:B:2\,D &= 4:5:6. \end{aligned}$$

Let us call the lowest note of the octave C, and denote its frequency by "m"; these proportions then amount to a series of six equations which we may solve to find the frequency of the other six notes in the octave. Thus,

$$\begin{array}{rlrl} & & C &= m. \\ C:E &= 4:5. & \therefore\ E &= \tfrac{5}{4}\,m. \\ C:G &= 4:6. & \therefore\ G &= \tfrac{3}{2}\,m. \\ F:2\,C &= 4:6. & \therefore\ F &= \tfrac{4}{3}\,m. \\ F:A &= 4:5. & \therefore\ A &= \tfrac{5}{3}\,m. \\ G:B &= 4:5. & \therefore\ B &= \tfrac{15}{8}\,m. \\ G:2\,D &= 4:6. & \therefore\ D &= \tfrac{9}{8}\,m. \end{array}$$

TABLE OF MUSICAL NOMENCLATURES

NAME OF NOTE.	C	D	E	F	G	A	B	2 C
Frequency in terms of fundamental.	m	$\frac{9}{8}m$	$\frac{5}{4}m$	$\frac{4}{3}m$	$\frac{3}{2}m$	$\frac{5}{3}m$	$\frac{15}{8}m$	$2m$
Intervals.		$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$
Name of note in vocal music.	do	re	mi	fa	sol	la	si	do
Treble Clef. Bass Clef.								
Orchestral Pitch.	264	297	330	352	396	440	495	528
Scientific Scale.	256	288	320	341	384	427	480	512

The preceding table indicates also the ordinary musical notation, a beautiful form of shorthand. The character employed tells the reader the time for which any note is to be sounded, while the vertical position of the note among the lines and spaces gives its pitch. The modern system of musical notation is, then, essentially that invented by Descartes in 1637 and universally employed in analytical geometry. **The bass and treble are connected by the fact that the C represented by the first line above the bass is identical with the C on the first line below the treble.** This note is called **Middle C** on the piano. Ordinarily its frequency is about 264.

242. The scale thus formed and represented in the preceding table is called the natural or diatonic scale. Between the successive notes it will be observed that there are three kinds of intervals; namely,

C–D, *F–G*, *A–B*, $\frac{9}{8}$ — called a **major tone**,
D–E, *G–A*, $\frac{10}{9}$ — called a **minor tone**,
and *E–F*, *B–C*, $\frac{16}{15}$ — called a **major semitone**.

If all music were written in the key of *C*, this diatonic scale would suffice. But on instruments with fixed notes, such as the piano and guitar, it would be impossible to pass to other keys without enormously complicating the instrument. To avoid this difficulty Bach (1685–1750) invented the so-called **tempered scale**, in which the octave is divided into twelve equal intervals, or half notes, each tone having a frequency $2^{\frac{1}{12}}$ that of the preceding. This interval is very nearly the semimajor tone. The student will now see why the black key on the piano is omitted between *B* and *C*, and also between *E* and *F*.

The names of the notes employed in vocal music are derived from the initial syllables of the following stanza, from an old Latin hymn: —

“ *ut* queant laxis *re*sonare fibris
*Mi*ra gestorum *fa*muli tuorum
*Sol*ve polluti *la*bri reatum
*Sa*nct *Jo*annes.”

Poggendorff, *Geschichte der Physik*, p. 802.

IX. APPLICATION OF PRECEDING PRINCIPLES TO VARIOUS INSTRUMENTS

Wind Instruments

243. Of these the organ pipe is perhaps the simplest and most typical. There are two kinds, *open* and *closed*. In each of them two trains of waves interfere to produce stationary waves. By an **open pipe** is meant one which is not closed at either end; by a **closed pipe** is meant one which is closed at one end only. These two types are shown in Fig. 180.

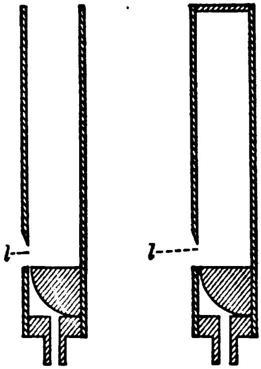


FIG. 180. — Section of open and closed organ pipes.

One train of waves is produced by blowing against the lip *l*, of the pipe; another train is produced by reflection from the upper end of the pipe.

In the case of the open pipe, reflection occurs when the incident train strikes the quiet air at the end of the pipe; in the case of the closed pipe, reflection occurs when the wave strikes against the wooden end of the pipe. In either case, the whole behavior of the air in the organ pipe is exactly that of a small portion of the air in a Kundt's tube (§ 223).

In the case of the open organ pipe, it is evident that each end is an antinode, *i.e.* a place where no compression occurs, but where great motion is always possible. Consequently, an open organ pipe is simply that portion of a Kundt's tube which lies between two consecutive antinodes, as shown in Fig. 181, where *N* denotes a node and *A* an antinode. It is evident, therefore, that the wave length of the fundamental is just twice the length of the open pipe which emits it.

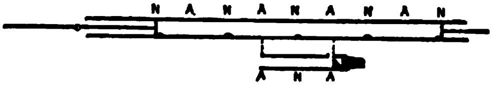


Fig. 181.—Correspondence between the open organ pipe and Kundt's tube.

As a matter of fact this rule is not quite exact; for it is found that the fundamental wave length is a little more than twice the length of the pipe. This is due to the fact that the inertia of the air surrounding the open end of the pipe has the effect of slightly increasing the effective length of the pipe, very much in the same way that the velocity of a rifle ball is found to increase even after the ball has left the muzzle of the gun. Just as the dead air about the muzzle of the gun forms a tube, so to speak, in which the gaseous products of the burning powder continue to expand, so the quiet air about the end of the organ pipe has the effect of practically extending the length of the actual circular pipe by about 0.6 of its radius, a quantity which is called the "end correction."

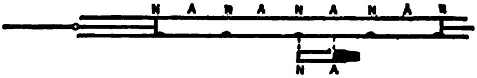


Fig. 182.—Correspondence between closed organ pipe and Kundt's tube.

In the case of the closed pipe, no longitudinal motion of the air is possible at the stopped end; consequently, this end must be a node, while the lower (*i.e.* the open) end is an antinode. The correspondence is indicated by Fig. 182.

It is at once evident that the length of a closed pipe is one quarter of a wave length of its fundamental.

The student will find it interesting to prove that an open pipe yields the complete series of harmonics, 1, 2, 3, 4, 5, 6, etc.; while the closed pipe gives only the odd notes 1, 3, 5, 7, 9, etc. Make a sketch, after the style of Fig. 177, showing the distribution of nodes and loops in both open and closed pipes.

A very direct means of experimentally locating the nodes and loops in an open pipe is to replace the bottom of a pill box with a thin stretched membrane, — say very thin paper, — place a little sand in the box, and lower it into the sounding tube, holding the membrane of the box horizontal. The sand distinctly announces the positions of the loops — becoming silent at the nodes.

To this class of instruments belong also the **flute**, the **clarinet**, the **cornet**, and the **trombone**. In the first two of these instruments the vibrating column of air is made to break up, by means of stops, so as to yield either the fundamental or overtones at the will of the player. The opening of a stop at any point on the tube insures an antinode at that point. While in the trombone and in the cornet the player controls the pitch by varying the *length* of the air column.

Stringed Instruments

244. Of these perhaps the simplest and most typical is the **guitar**. This instrument has six strings, three of silk covered with silver wire, and three of catgut. The range of the instrument is about three octaves. The lowest string yields the note E; the highest open string, 4 E; or when held at the middle, 8 E. Three methods of varying the pitch are illustrated in the guitar: —

1. To vary the pitch n of any one string a screw is employed to change the tension T. As the tension increases, the force restoring the string to its position of equilibrium becomes greater. The frequency therefore increases.

2. To pass from one note to another on the same string, "frets" are employed. The finger holds the string down over the fret and thus changes the length of the string.

$$V = nl = 2\,nL = \text{a constant.} \qquad \therefore n \propto \frac{1}{L}.$$

3. In passing from one string to another, the pitch of the lower strings is diminished by increasing their density. This is accomplished by wrapping them with silver wire. The strings would be too clumsy and too rigid if the same mass were given to the string by use of catgut. The wrapping of silver wire adds nothing to the restoring force, but does increase the mass to be moved, and hence diminishes the frequency.

These three facts are most briefly and definitely summarized by writing Eq. 82 (§ 201) as follows: —

$$V = 2\,nL = \sqrt{\frac{T}{\mu}} \qquad \text{Eq. 82}$$

from which it follows that, other things remaining constant, —

(i) n varies as $\sqrt{T}$.

(ii) n varies as $\frac{1}{L}$.

(iii) n varies as $\frac{1}{\sqrt{\mu}}$.

These three statements are generally known as "Mersenne's Laws." They appear to have been derived experimentally by Mersenne and Galileo. All three are clearly explained by Galileo as early as 1638 in his *Dialogues on Two New Sciences. First Day.*

In the **violin** the length of the string is no longer determined by the position of the fret, but is under the entire control of the player. The violin, is, therefore, much more responsive to the will and to the skill of the player than is the guitar.

The **viola** is an instrument pitched a perfect fifth lower than the violin; or, to put it in another way, the viola omits the upper string of the violin and adds one string lower than any on the violin.

In defiance of its name, the **violoncello** is a larger instrument than the viola and is indeed pitched just one octave below it.

The **piano** is an instrument of between 80 and 90 strings, or sets of strings, each of which is employed to produce one note only. The tension and length of each string are intended to remain constant. Anything which changes the tension of a string once adjusted is said to put the piano "out of tune."

The **harp** is essentially a piano in which the strings are plucked by the fingers, instead of being struck by hammers.

The Aeolian Harp, though not a stringed instrument as its name and structure might suggest, is from an acoustical point of view a highly instructive instrument. Every one has listened to these Aeolian tones on a winter's night. Lord Rayleigh describes, *Phil. Mag.*, 29, 444 (1915), the following simple device by which they may be produced. A brass wire *ABC* about one millimeter in diameter is bent into the shape indicated in Fig. 183. A cork is fitted on to a glass tube and the ends of the wire are thrust through the cork so that the wire is held close in front of the nozzle of the glass tube. This nozzle is drawn down until it has a diameter of only one or two millimeters. A rubber tube *R* is attached to the large end of the glass tube in order to convey the sound to the observer's ear.

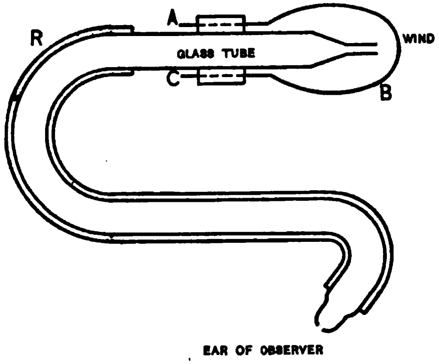


Fig. 183.—Lord Rayleigh's Æolian harp.

If now this wire be placed in the crack of a door or window where the wind — even a very gentle one — blows upon it in the direction indicated by the arrow, the familiar Aeolian tones are distinctly heard. Their explanation lies in the fact that just behind the portion of the wire indicated by *B* there is formed a series of unstable vortices in the air. The instability of these vortices may perhaps be likened to that of a flag flapping in a steady breeze.

Vibrating Membranes. The Human Voice.

245. Of this class, the **drum** is the well-known example. But by all odds the most interesting and most marvelous instrument of this, or indeed of any class, is that which produces the **human voice**. Here, in general, three factors enter: —

(*a*) The vibrations of the so-called vocal cords.
(*b*) The vibrations of the tongue.
(*c*) The vibrations of the lips.

The vocal cords consist of membranes situated one on each side of the throat (larynx). By changing the muscular tension of these one easily changes the pitch of his voice. By altering the form of the mouth one easily changes the overtones, and hence the quality of the tone. In this manner the various vowel sounds are produced.

246. The vibrations of rods which cannot here be discussed are also full of interest. We merely mention as examples the xylophone and triangle of the orchestra, tuning forks, Geneva musical boxes, Marloye's harp.

An acoustical device of unusual interest, though having no connection with music, is the **ear guard** worn by soldiers to protect the tympanum from injury by the firing of heavy rifles or by the bursting of shells containing high explosives. In order to have any practical value this instrument must allow the soldier to hear ordinary sounds and must at the same time shut out the tremendous pulses sent through the air by the big guns. This is accomplished by inserting in the outer ear a small cell, something like a centimeter in diameter. The essential features of this cell are the following: a sensitive delicate diaphragm, *E*, held in position by two very thin flat-ring washers represented by light shading in Fig. 184. On each side of this diaphragm is fitted a disk of fine gauze. These disks are also held in position by

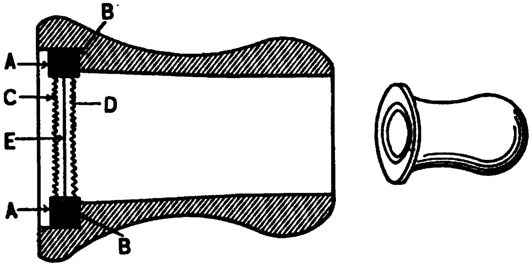


FIG. 184. — Cross section, enlarged, of soldier's ear guard. Full size is shown in small figure on right. *A* and *B* represent washers; *C* and *D*, gauze disks; *E*, the sensitive diaphragm.

washers. The delicate diaphragm is untouched except at its edges. In this respect, it resembles a telephone diaphragm. Moderate sounds are readily transmitted by it. The violent pulses due to gun-fire or explosion of shells bring this diaphragm up against the gauze, thus stopping the motion and preventing the transmission of dangerous pressures.

Problems

257. Approaching a large building at night, I stamp my foot on the pavement and 0.6 second later I hear the echo. How far away is the building? *Ans.* 99.6 m.

258. What is the pitch of a sound whose wave length *in air* is 33.2 cm.

259. Explain how the violinist varies the pitch of any one string by placing his finger upon it.

260. A well is 1800 cm. deep. How long a time will be required for a sound to travel from the bottom to the top of the well? How long will a stone be in falling from the top to the bottom of the well? How long, then, after the stone is let fall before you hear the splash? *Ans.* Total interval = 1.969 sec.

261. A Kundt's tube is filled with hydrogen, and it is then found that the distance between the successive dust heaps is 3.8 times as great as when the same note is sounded in the tube filled with air. From this compute the speed of sound in hydrogen. *Ans.* 1261.6 m. per sec.

262. The brass rod used to excite a certain Kundt's tube is just 1 m. long; the dust heaps which it produces in air are 99 mm. apart. What is the speed of sound in brass? *Ans.* 3353 m. per sec.

263. An open organ pipe 166 cm. long will yield a note of what pitch? *Ans.* 100.

264. A closed organ pipe 63 cm. long will yield what note?

265. Explain how it is that the speed of sound in a gas increases as the temperature rises. Show that an equation of the following form describes this phenomenon:

$$V = 332.0\ (1 + 0.00181\ t^\circ) \text{ meters per second.}$$

266. Two telegraph sounders are placed on the same electric circuit so that they click each five times a second. One of these sounders is fixed; the other I can carry off to some distance. How far away shall I have to take the movable one in order to hear its nth click at the same instant that I hear the $(n + 1)$th click of the fixed sounder? This is the " unison " method for measuring the speed of sound. *Ans.* 66.4 m.

267. In how many ways could you change the pitch of a string which now emits the note C, so that it would emit the note G in the same octave?

268. Would increase of temperature change the pitch of an organ pipe? Would it change the ratio of the frequencies of a series of pipes? *Shearer*, 1265.

269. Make a diagram of the frets on a guitar and show how, starting from any one string, any other string may be tuned by the use of beats.

270. Assuming the density of hydrogen as 0.0000896, find the speed of sound in hydrogen under normal atmospheric pressure of 76 cm.

271. Assuming the density of marsh gas to be 0.000727, find the temperature at which the speed of sound in air will be equal to the speed of sound in marsh gas at 0°C.

272. A brass and an iron rod have each the same size, shape, and density. Through which would you expect sound to travel more rapidly? Why? How much more rapidly?

273. How do you explain the loud sound produced by the crack of a whip?

274. Show by diagram on the blackboard how a closed organ pipe gives only the odd harmonics.

275. Explain, by diagram on the blackboard, how a high-pitched tuning fork when moved rapidly to or from a wall is heard to beat with the train of waves reflected from the wall.

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"Design! The very word disturbs our quiet discussions about *how* things happen with restless questionings about the why of them all."

CLERK MAXWELL, 1854.

CHAPTER VII

THEORY OF HEAT

247. In an earlier chapter we found that work is always required to overcome friction. The energy thus spent is immediately or ultimately frittered away into heat. The student is therefore justified in suspecting at once that, since energy is indestructible, heat is a form of energy; and the sequel shows that the evidence is the strongest possible for believing that this intangible, invisible something which we call "heat" is not a substance at all, but a form of energy.

Unfortunately heat is something concerning which our two most important senses — sight and hearing — give us no direct information. We are dependent therefore upon our temperature sense, which is not very keen, and upon certain laboratory devices whose indications can be felt, seen, or heard.

The subject of heat is one which is so intimately connected with all the various parts of Physics that it is well-nigh impossible to offer any systematic analysis; but perhaps the following order of treatment will serve our purpose: —

I. Distinction between Heat and Temperature.
II. Measurement of Temperature. Thermometry.
III. Quantity of Heat. Calorimetry.
IV. Transfer of Heat.
V. Some Effects of Heat.
VI. Nature of Heat.

I. DISTINCTION BETWEEN HEAT AND TEMPERATURE

248. Most people will agree that a cup of water taken from a boiling kettle has, at the instant it is dipped out, the same temperature as that of the boiling water. Most people will likewise agree that for heating purposes the large kettle is much more effective than the small cup of water. A water bag or a water bottle that holds a gallon will give out more heat and give it out longer, other things being equal, than one which holds only a pint. If the temperatures of the two vessels of water are the same, the larger is said to contain more heat. If a bath tub be half filled with cold water, one can heat it more by pouring in a gallon of boiling water than by putting in a quart of boiling water; for, although the temperatures are the same, the gallon contains more heat than the quart. These simple facts are cited merely to show that popular notions concerning the distinction between heat and temperature are perfectly clear. They are also correct. But before the word "temperature" can be admitted to the rather select vocabulary of Physics, it must be defined in unmistakable English. This we now proceed to do.

Imagine three vessels of water, *A*, *B*, *C* (Fig. 185), each containing a different quantity of water. If *A* and *B* are placed side by side, in contact, and *B* thereby gains heat, — be it ever so little, — *A* has imparted heat to *B*; and *A* is said to have a higher temperature than *B*.

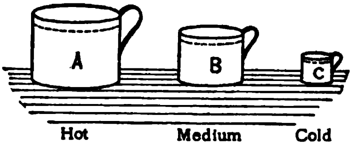

FIG. 185.

Put *C* in contact with *B*; if *B* thereby loses heat, — be it ever so little, — heat has passed into *C*; and *C* is said to be at a lower temperature than *B*.

When, in general, one body is placed in contact with another, the difference of temperature between the bodies is that which determines which way the heat flows. If the heat flows from *A* to *B*, *A* has the higher temperature; but if the heat flows from *B* to *A*, then *B* has the higher temperature.

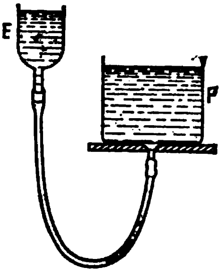

FIG. 186.

Consider two vessels of water, *E* and *F* (Fig. 186), connected by a rubber tube. Does the water always flow from the large vessel to the small one? What *does* determine the direction of the flow? If the water flows from E to F, does the surface of the water in F rise by the same amount that the surface in E falls?

Returning now to Fig. 185, we observe that the analogy is very perfect. Whether heat flows from A to C or *vice versa* depends, not at all upon the size of the vessels, but upon their difference of heat level or, more accurately, upon their difference of temperature.

249. The following definitions will now be clear to every one who has mastered the foregoing facts: —

"DEFINITION OF TEMPERATURE. — **The temperature of a body is its thermal state considered with reference to its power of communicating heat to other bodies.**

"DEFINITION OF HIGHER AND LOWER TEMPERATURE. — *If when two bodies are placed in thermal communication, one of the bodies loses heat, and the other gains heat, that body which gives out heat is said to have a higher temperature than that which receives heat from it.*

"COROLLARY. — *If when two bodies are placed in thermal communication, neither of them loses or gains heat, the two bodies are said to have equal temperatures or the same temperature. The two bodies are then said to be in thermal equilibrium.*

"LAW OF EQUAL TEMPERATURE. — *Bodies whose temperatures are equal to that of the same body have themselves equal temperatures.*" — MAXWELL, *Theory of Heat*, Ch. II.

The point in these definitions which calls for emphasis is the fact that temperature is a state of a body. "It is a condition and not a theory which confronts us."

250. In estimating temperatures, the student must be warned against drawing incorrect inferences from his sensations. For, in judging the temperature of the atmosphere, it is well known that our estimate is greatly influenced by any wind that may be blowing and by the presence of moisture in the air. If the hand be dipped into hot water and immediately afterward into tap water, the tap water will feel distinctly cold. But if a piece of ice be held in the hand for a moment, the same tap water will immediately after feel distinctly warm. The indications of thermometers such as are usually employed in the laboratory are much more independent of their previous history than are the sensations of the human hand.

II. MEASUREMENT OF TEMPERATURE. THERMOMETRY

251. The mercurial thermometer is an instrument which belongs quite as much to the household as to the physical laboratory. But the principles upon which it is constructed are not so well known as the instrument itself.

Since heat itself is invisible and intangible, and can be perceived only through its effects upon bodies, we must employ some one of these effects for the measurement of heat. **For ordinary purposes the universal choice has been change in size, which always accompanies a change in temperature.** We might take a brass rod 10 centimeters long, and then define one degree as that difference in temperature which is required to lengthen the brass rod by $\frac{1}{1000}$ millimeter. Such an instrument would be portable and would give the same reading for the same temperature; but it would be impossible to measure such a minute elongation except by means of elaborate apparatus.

If a glass bulb fitted with a hollow stem were partially filled with water, we might define one degree as that difference of temperature which would increase the volume of water by $\frac{1}{1000}$ of itself. But, unfortunately, water does not always *increase* in volume as its temperature rises. Moreover, each particular volume of the water does not correspond to one particular temperature. As we shall see presently, a body of water at 6° has the same volume as at 2°. A water thermometer would, therefore, be ambiguous in its readings. Hence water is never, in these days, used for making thermometers.

252. But there is one substance, viz. mercury, which has shown itself eminently adapted to the purpose. Among the advantages of mercury may be mentioned the following: —

(*a*) It is easily prepared in a pure state.

(*b*) It does not wet glass or stick to it.

(*c*) There is a difference between its rate of expansion and that of glass sufficient to make changes of temperatures easily read.

(*d*) To each particular volume corresponds a definite temperature.

(*e*) It does not freeze except at temperatures comparatively low, and does not boil except at temperatures comparatively high.

(*f*) Compared with water and most solutions it requires a very small amount of heat to raise its temperature through a given range.

The indications of temperature which are given by the mercurial thermometer hinge upon the fact that mercury expands with rise of temperature more rapidly than does glass. If, therefore, a glass tube having a bulb blown at one end be partially filled with mercury and immersed in a bath at higher temperature than its own, the mercury will rise in the tube. If the instrument is immersed in a bath of lower temperature, heat will flow from the mercury to the bath, and the mercury will descend in the tube. When the mercury is neither rising nor falling, it is fair to suppose that the thermometer is at the same temperature as the bath, or, at least, the part of the bath immediately about the thermometer.

Manufacture of a Mercurial Thermometer

253. The steps in the process are as follows: —

1. A piece of thick-walled capillary tubing of very uniform bore is selected.
2. A bulb is blown on the end of this tube.
3. The bulb is filled with mercury. It is then highly heated and "sealed off."
4. The tube of the thermometer is graduated.

This last step is one which demands further consideration.

Fixed Points of a Thermometer

254. It so happens that there are two temperatures which can be easily produced and reproduced at almost any place and at any time: one of them is the melting point of ice, the other is the boiling point of water.

It has been agreed upon by the whole scientific world, that

these two temperatures — the melting point of ice and the boiling point of water — should be called zero and one hundred degrees, respectively. Together they are known as the fixed points of the thermometer. The interval between these two fixed points is divided into 100 steps — degrees — in such a way that the apparent change in volume of the mercury between any two successive steps is exactly $\frac{1}{100}$ part of the total apparent change in volume between 0° and 100°. This is known as the " centigrade " scale," and is generally indicated by writing a " C." after the numerical value of the temperature. Thus " 48° C." means " 48 degrees on the centigrade scale."

The first step in the graduation of a thermometer is, then, to determine the fixed points, and mark them on the tube. Next the 50° point is so chosen that the volume (not the length) of the tube between 50° and 100° is equal to the volume between 50° and 0°. In like manner, the 25° point must halve the volume between 0° and 50°.

The manner in which these points are determined will be considered in the laboratory.

Temperature Scales of Fahrenheit and Réaumur

255. Besides the centigrade scale which, in a modified form, was suggested by Celsius, there are two others which are widely used in everyday life. In the one proposed by Fahrenheit, the volume between the fixed points is divided into 180 parts; while the zero is placed at a point which is 32 of these degrees below the melting point of ice.

In the other, proposed by Réaumur, the volume between the freezing point and the boiling point of water is divided into 80 degrees, the former of these two points being employed as zero. It is important to notice that these three thermometers, Celsius, Fahrenheit, and Réaumur, differ in no essential respect. They are identical in principle, mode

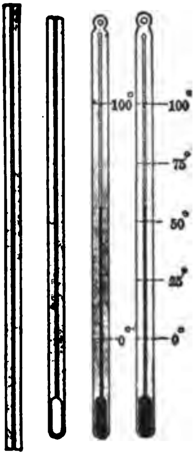

FIG. 187. — Showing four stages in the manufacture of a mercurial thermometer.

of construction, and use, the only difference being in the size of the degree employed.

256. Essential Features of a Good Thermometer.

1. It must be easily portable.
2. It must be permanent.
3. It must always give the same reading when subjected to the same temperature.
4. It must be possible for the user to test the correctness of its graduation, and to determine any errors in its graduation.
5. It must be relatively small, so that when placed in contact with a second body, the temperature of the second body will not be seriously affected.

Historical Development of the Thermometer

257. Although the thermometer is one of the oldest instruments of physical science and has a history covering three hundred years, there is even now scarcely a year goes by that does not bring either some new form of thermometer or some important improvement.

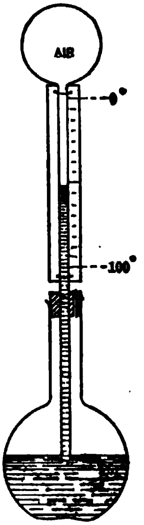

FIG. 188. — Galileo's thermometer.

In Winkelmann's *Handbuch der Physik,* Pernet traces the development of the instrument through a series of steps which are somewhat as follows: —

(i) Some time previous to 1603, Galileo had made and used in lectures at Padua a thermoscope or indicator of temperature, which in its essential features is shown in Fig. 188.

A glass bulb with a rather long and uniform stem is inverted with its lower end placed under the surface of some colored liquid, such as wine. But before the stem is placed in the liquid the bulb must be heated to a temperature a little higher than any temperature it is afterward desired to measure. At the standard temperature, say that of the room, the liquid rises to a certain mark on the scale and remains there, except for variations of the barometer. When taken out of doors the air in the bulb expands or contracts and the water in the tube thus falls with rise of temperature and rises with a fall of temperature. This simple device is surprisingly sensitive, and proved useful, even in Galileo's time, for the detection of fever.

(ii) As noted by Pascal in 1643 the readings of this thermometer are dependent upon the simultaneous readings of the barometer. To avoid this difficulty, Father Schoot, in 1657, sealed the liquid off from the atmosphere by bending the tube and sealing a bulb on each end as shown in Fig. 189. What this device indicates is evidently the *difference* in temperature between the two bulbs A and B. It is therefore called a *differential thermometer*.

(iii) In each of the preceding instruments a gas is employed as a thermometric substance; but in 1631 a French physician, Jean Rey, first made use of the expansion of a liquid for the measurement of temperature. A glass bulb, provided with a stem and filled with water, was used by him to distinguish between normal blood temperature and that of fever patients.

(iv) The first liquid thermometer to secure independence of atmospheric pressure was made about 1641 by sealing off the end of the glass stem. About this time the Academy of Sciences at Florence introduced the use of mercury and a scale graduated so that one end indicated the lowest temperature of winter, the other, the highest temperature of summer.

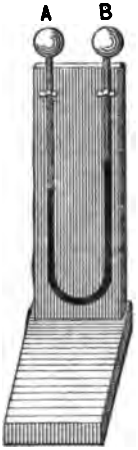

FIG. 189. — Schoot's differential thermometer.

(v) These rough and inaccurate standard temperatures were partly replaced by the one excellent standard in 1664; for it was in this year that Robert Hooke, Secretary of the Royal Society of London, suggested the melting point of ice as *the* standard temperature. In the year following Huygens proposed the boiling point of water as the standard, neither of these men having yet recognized the fact that at least *two* fixed points are absolutely necessary in order to make the thermometric readings from one instrument comparable with those from another.

(vi) This important step, the use of two definite fixed points, was first proposed in 1688 by Dalance, who adopted the melting points of ice and butter as the two temperatures in question. Newton constructed a thermometer on this principle, using the melting point of ice as zero and the temperature of his own body as 12°. He also tested the uniformity of the bore in the stem of the thermometer, thus introducing a process which is now called calibration. As a thermometric substance he employed linseed oil.

(vii) The next advance is due to Fahrenheit (1686–1736), who was born in Danzig, but spent most of his life in Holland and England. He introduced the modern mercury thermometer with cylindrical bulb and stem, about the year 1714. For fixed points he used various temperatures, among them that of a mixture of salt and ice, 0°; melting ice, 32°; "blood heat," 96°; boiling water, 212°. Which pair were taken as standard in any particular case appears to have been determined largely by the range of temperatures for which that particular instrument was intended.

(viii) The Réaumur scale, described above, was introduced about 1730. Little interest attaches to this thermometer other than its wide use in domestic and industrial circles.

(ix) In 1742, Anders Celsius, professor of astronomy at Upsala, proposed a scale with the zero at the boiling point of water and with 100° at the melting point of ice. The inversion of this scale, which gives us the modern centigrade instrument, is due to Christin in France (1743), possibly also to Strömer, in Upsala (1750).

(x) The range of the ordinary mercury thermometer extends from − 39° C. (the freezing point of mercury) to the neighborhood of + 357° C., which is the boiling point of mercury. But by filling the upper part of the thermometer stem with nitrogen or carbon dioxide under high pressure it is possible to use mercury thermometers up to 550° C. Not only so, but by making the bulb and stem of quartz instead of glass, the mercury thermometer becomes available for temperatures even up to 755° C.

(xi) For extremely low temperatures, such as are now produced by the use of liquid air, glass thermometers are filled with liquid pentane, instead of mercury. The melting point of pentane is lower than the boiling point of air (− 185° C.); hence this thermometer covers the entire range from melting ice to liquid air.

The Gas Thermometer

258. Convenient as the mercury thermometer is, it has one serious defect, inherent in the glass tube which holds the mercury. When a glass vessel is heated through any range of temperature and again cooled to its initial temperature, the volume of the vessel does not return to its initial value until after the lapse of several days, or even several weeks. This uncertainty of volume in the glass envelope means uncertainty of temperature indications on the thermometer. If, instead of mercury, one uses some gas — preferably hydrogen — which expands much more rapidly than mercury, this irregularity in the volume of the bulb becomes negligible. It was the German physicist Lambert (1728–1777) who first proposed the use of air as a thermometric substance; and it was the French physicist Regnault (1810–1878) who first rendered this suggestion practicable by his exact measurements of the properties of gases and vapors.

The principle of the gas-thermometer is that a given mass of gas when placed in a closed vessel *of constant volume* will assume a pressure which is entirely determined by the temperature. The manner in which the gas is employed is indicated in Fig. 190. The bulb, *B*, is filled with dry hydrogen, since this substance is more nearly than any other a "perfect" gas. See § 283. This bulb, *B*, is sealed to the lower arm of a syphon barometer, *H*, which serves to measure the pressure of the inclosed hydrogen. To begin with, one brings the bulb, *B*, to the temperature of melting ice. Then the mercury reservoir, *R*, is lifted or lowered until the surface of the mercury in the short arm of the barometer tube stands at a certain fiducial mark, *f*. The pressure on the hydrogen is then measured by the difference of level, h_0, between the mercury sur-

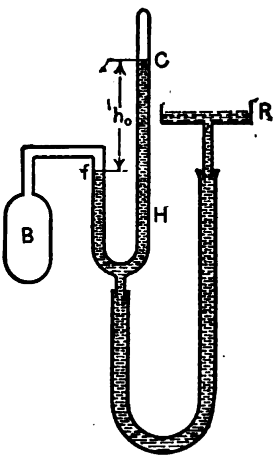

FIG. 190. — The gas thermometer.

faces at f and c. If now the bulb be placed in a steam bath and the mercury brought back to the fiducial mark at f, the difference of level (which we will now call h_{100}) is found by experiment to be 1.367 h_0. **Temperatures on the gas thermometer are defined as proportional to pressure.** Hence, if T_0 is the temperature of melting ice and T_{100} the temperature of boiling water, one may write,

$$\frac{T_{100}}{T_0} = \frac{h_{100}}{h_0} = 1.367 \qquad \text{Eq. 88}$$

If we agree to use the centigrade scale, as is universally done, then we have by definition,

$$T_{100} - T_0 = 100 \qquad \text{Eq. 89}$$

From these two simultaneous equations we see that $T_0 = 273°$ and $T_{100} = 373°$ approximately. Other temperatures are measured by interpolation between these two or between other known temperatures such as those of melting ice and boiling sulphur, which latter is 444°.6 C.

All temperatures measured in this manner are said to be referred to the "gas scale." This is the scale adopted by the International Committee on Weights and Measures in 1887 and is the standard scale to which are referred all accurate measures of temperature.

Measurement of High Temperatures; Pyrometry

259. The determination of temperatures which lie above, say, the boiling point of sulphur, 445° C., demands methods which constitute almost a separate science — pyrometry — and one which cannot here be considered. It may, however, be mentioned that these methods are practically all based upon one or more of the three following principles, which the student will meet later.

(i) The variation of electrical resistance with temperature, giving rise to what is known as "electrical resistance thermometry" and "platinum temperatures," since the resistance used is generally that of a platinum wire. Valuable both for moderately high and for very low temperatures; especially useful between 400° and 1200° C. It was the work of Professor H. L.

Callendar of London that made the resistance thermometer an instrument of precision.

(ii) The variation of electromotive force in a circuit made of two wires of different material when the temperature of one of the junctions of these two wires is changed, is the basis of a method which leads to "thermoelectric thermometry."

(iii) The variation with change of temperature of either (*a*) the total amount of energy radiated by a hot body, or (*b*) the energy (brightness) of a particular wave length (color) of light radiated by a hot body.

The total energy varies according to a law first determined by Stefan and Boltzmann; the partial energy according to one investigated by Wien and Planck. §§ 273, 274.

TABLE OF STANDARD TEMPERATURES

SUBSTANCE	PHENOMENON	TEMPERATURE		
		CENTIGRADE C	FAHRENHEIT F	ABSOLUTE K
Helium	Boiling point	−270°, −268°	−454°, −450°.4	3° to 5°
Hydrogen	Boiling point	−253	−423	20
CO_2	Sublimation in Gasoline	−78	−108	195
Mercury	Freezing point	−39	−38	234
Water	Freezing point	0	+32	273
$Na_2SO_4 + 10\,H_2O$	Transformation into anhydrous salt	+32.38	90.28	305.38
Water	Boiling point	100	212	373
Naphthaline	Boiling point	218	424	491
Tin	Melting point	232	450	505
Benzophenone	Boiling point	306	583	579
Lead	Melting point	327	621	600
Zinc	Melting point	419	786	692
Sulphur	Boiling point	444.6	832.3	717.6
Antimony	Melting point	630	1166	903
Silver	Melting point	960	1760	1233
Gold	Melting point	1063	1945	1336
Copper	Melting point	1083	1981	1356
Nickel	Melting point	1452	2646	1725
Palladium	Melting point	1549 ± 2	2820 ± 3.6	1822 ± 2
Platinum	Melting point	1755 ± 5	3191 ± 9	2028 ± 5
Tungsten	Melting point	3000 ± 100	5400 ± 180	3273 ± 100
Electric arc	Positive crater	3600 ± 150	6512 ± 270	3873 ± 150
Sun	Surface	6000 ± 500	10800 ± 900	6273 ± 500

These are the basis of what is known as "blackbody temperatures," since the methods are strictly accurate only for bodies which are perfectly "black." These bodies are realizable in the laboratory in a variety of ways. The method is available for the highest temperatures known; such as the melting point of tungsten, 3000° C.

The preceding table gives a fair idea of the standard temperatures in use to-day. In the last column the temperatures are given in terms of the "Absolute Scale" introduced by Lord Kelvin, and therefore indicated by his initial letter "K." This "absolute" scale of temperatures will be explained later in § 307.

III. QUANTITY OF HEAT

260. We have already learned that temperature is merely a condition determining the direction of the flow of heat, very much as pressure is a condition determining the direction of the current when two vessels of compressed air are connected. And just as we need a means of measuring the amount of air which escapes from either of the two vessels into the other, so we need a method of estimating the quantity of heat which passes from one body to another when they are brought into contact with a difference of temperature.

When a hot body is brought into the neighborhood of cooler bodies, we have excellent reason for thinking *something* passes from the hotter one because the cooler ones always rise in temperature. If a cold body is brought into the midst of hotter ones, we observe again that when the temperature of the cold body rises it is always at the expense of the hotter surroundings.

Let us call this something which passes "heat," and proceed to discover what factors determine its amount. The simplest method of doing this is perhaps through the following series of experiments: —

(i) Take equal masses of water and mix them together. It will be found that the temperature of the mixture is an exact mean between the two original temperatures, whatever the masses may have been. This statement is not exactly true, although practically correct. For Rowland at Baltimore discovered in 1878 that as one proceeds from 0° C. to 30° C. it takes less and less heat to make a pound of water one degree warmer; or, as we shall presently see, the specific heat of water diminishes from 0° to 30° C.

(ii) Now take *unequal* masses of water, which we may indicate by m_1 and m_2, having temperatures t_1 and t_2 respectively. On mixing these two, it is found that the final temperature θ is obtained by multiplying the mass of each by its original temperature, adding these products, and dividing by the sum of the masses. This result seems first to have been formulated by the Russian physicist Richmann (1711–1753); it is known as Richmann's rule, and is most clearly expressed by the following equation: —

$$\theta = \frac{m_1t_1 + m_2t_2}{m_1 + m_2}. \qquad \text{Eq. 90}$$

If we assume that as much heat is given up by one mass of water as is gained by the other, then the experimental fact expressed by Richmann's rule shows that the amount of heat given up or absorbed by any body depends upon the mass and the change of temperature. This inference will be clearer if we rewrite Eq. 90 in the following form: —

$$m_1(t_1 - \theta) = m_2(\theta - t_2).$$

This equation may indeed be considered as a definition of equal quantities of heat; or, if you like, this equation may be considered as the application of the law of the conservation of energy to the isolated system made up of two masses of water, m_1 and m_2.

Unit of Heat

261. Having once discovered a method of duplicating a given quantity of heat, the measurement of heat becomes possible so soon as a unit of heat has been agreed upon. For this purpose we have only to place equal to unity both the mass and the change of temperature in the preceding equation, a process which leads to the following widely employed definition. **Unit of heat is that quantity which will raise the temperature of one gram of water through one degree Centigrade. This unit is called a gram calorie.**

Engineers more frequently employ the British Thermal Unit (written " B.T.U."), which is the amount of heat required to raise the temperature of one pound of water one Fahrenheit degree.

As noted above, the amount of heat required to raise one gram of water 1° C. varies with the temperature. How great this variation is will be seen from the accompanying table. In precision work, the "20° calorie" — the one indicated in the table — is widely used.

TEMPERATURE	SPECIFIC HEAT OF WATER
0° C	1.0094
5	1.0054
10	1.0027
15	1.0011
20	**1.0000**
25	0.9992
30	0.9987
35	0.9983
40	0.9982
45	0.9983
50	0.9987

From KAYE AND LABY'S *Tables.*

SPECIFIC HEAT

262. So far we have been dealing with a single substance, namely, water, a most convenient material because it is easily obtained in large quantities and in a state of high purity. Now mix two liquids which differ in chemical composition but have no chemical action upon each other. It was discovered by the Scottish chemist, Black (1728–1799), that under these circumstances Richmann's rule must be modified. He found that equal masses of different substances when changing temperature by equal amounts do not give out equal amounts of heat, and was led to assign to each substance a certain constant which would indicate the specific effect of heat in raising the temperature of that particular substance. If we denote this constant by c, then Richmann's rule in its generalized form becomes

$$\theta = \frac{m_1c_1t_1 + m_2c_2t_2}{m_1c_1 + m_2c_2}. \qquad \text{Eq. 91}$$

This equation, which may be written

$$m_1c_1(t_1 - \theta) = m_2c_2(\theta - t_2), \qquad \text{Eq. 92}$$

shows that, in general, the amount of heat which a body gives up, without change of state, depends, in general, upon three factors, namely, mass, difference of temperature, and a specific constant which is now called " specific heat " and which is defined as follows: —

DEFINITION OF SPECIFIC HEAT

263. The ratio between the amount of heat required to raise the temperature of any body one degree and that required to raise the temperature of an equal mass of water one degree, is known as the specific heat of the body.

The *numerical value* of the specific heat of any substance is then the quantity of heat (number of calories) required to change the temperature of one gram of the substance by one degree.

Let us denote by Q the quantity of heat added to a body whose mass is m and whose specific heat is c; let $t_1°$ be the initial and $t_2°$ the final temperature of the body; accordingly the change in temperature produced by Q will be $t_2° - t_1°$. Then

$$c = \frac{Q}{m(t_2° - t_1°)} \qquad \text{Defining equation for specific heat.} \qquad \text{Eq. 93}$$

becomes the defining equation for the average specific heat between the temperatures $t_1°$ and $t_2°$.

The definition of specific heat here given is strictly equivalent to the following: In Eq. 92, two substances are referred to. Let the one indicated by the subscript "1" be water. Let us arbitrarily agree that $c_1 = 1$. Then solving for c_2, one obtains Eq. 93.

Another useful conception is that of " heat capacity of a body," which is defined as the amount of heat in calories required to raise the temperature of that particular body one centigrade degree. Evidently, therefore, the heat capacity of a body is measured by the product of its mass and specific heat.

Various applications of Eq. 93 will be taken up in the laboratory under the head of *calorimetry*.

TABLE OF SPECIFIC HEATS

SOLIDS	TEMPERATURE IN DEGREES CENTIGRADE	SPECIFIC HEAT	LIQUIDS	TEMP.	SPECIFIC HEAT
Aluminium	20° C	0.219	Alcohol, ethyl	0° C	0.547
Brass	0	0.088	Alcohol, methyl	12	0.601
Copper	0	.093	Mercury	0	.033
Diamond	10.7	.1128	Petroleum	21–58	.511
Glass, crown	10–50	.161	Sea water	17.5	.938
Glass, flint	10–50	.117	Brine		
Graphite	10.8	.160	(density = 1.2)	15	.72
"	977°	.467	Olive oil	7	.47
Ice	−21–0	.502	Turpentine	18	.42
Iron	15	.119			
Lead	15	.030			

From KAYE AND LABY'S *Tables.*

Problems

276. A hot iron ball weighing 10 lbs. is immersed in 16 lbs. of water at 14° C. The temperature of the water is thereby raised to 49° C. Find the temperature of the iron ball. *Ans.* 519°.6 C.$^+$

277. One places in a certain vessel 500 cc. of water at 15° C. How much heat must be imparted to the water before it begins to boil? *Ans.* 42,500 calories.

278. One hundred grams of a certain substance at 80° C. is immersed in 550 grams of water at 2°. The temperature of the water rises to 6° C. Find the specific heat of the substance. *Ans.* 0.29.

279. One mixes two lbs. of mercury at 20° C. with 2 lbs. of boiling water. What is the resulting temperature?

280. It is required to prepare a bath of 40 gallons of water at a temperature of 70° F. having at disposal a supply of boiling water and of tap water at 40° F. What quantity of each must one take? *Ans.* 33 + gallons cold and 7 − gallons hot.

281. A 5 lb. iron ball is taken out of an electric furnace and immediately immersed in 12 lbs. of water at 13° C. The resulting temperature is 63° C. Assume the specific heat of iron to be 0.12 and find the temperature of the furnace.

IV. TRANSFER OF HEAT

264. Having now defined temperature and quantity of heat in such a way that we can measure each of them, we next consider the various modes by which heat is transferred or diffused. Of these three general methods are known, viz. (1) Conduction, (2) Convection, and (3) Radiation. In nearly every case of the transference of heat, however, all three of the processes are operating simultaneously.

1. *Conduction of Heat*

265. The handle of a spoon may become very hot when the bowl of the spoon is placed in a cup of tea. The process by which the heat is transferred from the lower to the upper part of the spoon is called "conduction." The same spoon, when used with ices, may become very cold. Here, again, the process by which the heat is transferred, this time from the handle to the bowl of the spoon, is called conduction.

In this process heat is transferred by contact of neighboring particles. If there is any relative motion of the parts of the body, this is not large enough to be visible. How rapidly the heat is handed on from one point to another in a body depends greatly upon the composition of that body. Every one knows that he can hold the stick of a burning match for 30 or 40 seconds without difficulty; but a copper wire of the same size and shape held in a match flame for that length of time becomes unbearably hot.

An iron and a copper rod, each of the same length, say half a meter, and of the same cross section, each having one end placed in the same flame, will become unequally heated at equal distances from the junction (see Fig. 191). For if at equal distances from the flame on each rod be placed either bits of wax or matches, those on the copper will melt or burn earlier than those on the iron. A series of bicycle balls attached with wax, three centimeters apart, along each bar will melt off in such a way as to show the progress of temperatures in the bars.

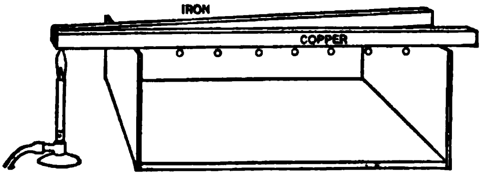

FIG. 191.—Illustrating the fact that copper is a better conductor than iron.

It is here to be borne in mind that the rapidity with which the iron and the copper change temperature at any point depends, not only upon the rate of conduction, but also upon the specific heats of iron and copper. In the present experiment, however, these two specific heats are so nearly equal that the relative conductivities of iron and copper are fairly represented.

A good experiment to illustrate differences in conductivity is the following, the results of which are familiar to us all: Take a piece of woolen goods, a piece of glass, and a piece of iron. Put them in some cold place where their temperature will fall 50° or 60° F. below that of the human body. If these three bodies are now examined, the iron will feel very cold to the hand because it conducts off the heat of the hand most rapidly; the glass will feel moderately cold; while the woolen piece is quite comfortable, being a very poor conductor. Now place these same bodies in an oven where they will reach a temperature 50° or 60° F. above blood heat; the iron will be uncomfortably hot, because it conducts heat to the hand so rapidly, while the woolen goods feels merely warm, being unable to give up its heat quickly. Here is a case, then, where sensation is determined largely by power to conduct heat, a property which is called "conductivity."

266. It is a curious circumstance that all substances which are good conductors of electricity are also good conductors of heat. The discovery of this fact by Wiedemann and Franz in 1853 led men to think that there must be an intimate connection between the process by which heat is conducted and that by which electricity is conducted. But this connection has only recently been worked out, mainly through the efforts of Drude, Thomson, and Lorentz. Thomson showed in 1897 that small charges of negative electricity, entirely dissociated from matter, can be obtained in a vacuum tube from any metal whatever. These small charges are called "electrons." Professor O. W. Richardson and others have shown that these same small particles of negative electricity are given off by any highly heated metal. The bold assumption which Drude made is that, in a cubic centimeter of any conducting substance, there is an enormous number of these electrons which are free from ordinary molecules of matter. These are called "free" electrons. The fundamental idea in Drude's explanation of heat conduction is that the free electrons take up and pass on, by collision, the kinetic energy of the molecules of ordinary matter. The larger the number of free electrons in any substance the greater the conductivity of this substance. Further consideration of this view must be postponed until the kinetic theory of gases has been studied.

267. Among the various applications of thermal conductivity none is perhaps more important than the classical invention of the safety lamp by Sir Humphry Davy. The explosive mixture of gases which occurs in many mines and which is known as "fire damp" must, of course, be protected from naked flames. For this purpose Davy suggested that the flame be inclosed in a small cage of wire gauze. The effectiveness of this device rests upon the fact that, when the fire damp penetrates the hood and burns, the flame cannot strike back through the meshes of the gauze because the heat of combustion is carried off so rapidly, *owing to the conductivity of the iron*, that the temperature just outside is always lower than that of ignition. That this is true may be shown either by lowering a piece of flat wire gauze over an ordinary Bunsen flame (Fig. 192), when it will be seen that the flame does not pass through the meshes, or by first allowing the gas to pass through the gauze and then lighting it above. In the latter case it will be observed that the flame remains entirely upon the upper side. Since human nature is what it is, the lock on the safety lamp which prevents the miner from lighting his pipe from it is a feature quite as essential perhaps as the wire gauze.

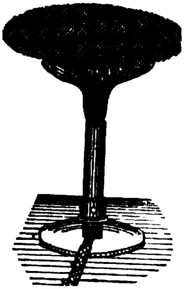

FIG. 192.—Illustrating the principle of the Davy safety lamp.

Measure of Conductivity

268. Were we to attempt to melt ice by means of a copper bar, one end of which is held in a flame at constant temperature, the other end on the ice, we should find the *rate* at which the ice is melted depends upon the dimensions as well as upon the composition of the bar. If the bar were made twice as thick, the heat would be transferred nearly twice as rapidly. But if the cross section remained constant while the bar were made twice as long, we should find the ice melting only about half as rapidly as before.

By such experiments, it has been found that the rate at which heat " flows " through a solid from one surface to another when these surfaces are at constant, but different, temperatures depends upon four things only, viz. (1) the substance from which the solid is made, (2) the distance apart of the surfaces, (3) the area of the surfaces, and (4) the difference in temperature between the surfaces. Let H denote the amount of heat which passes during the time t from one surface whose area is s to another of equal area. Then if k represents the thermal conductivity of the substance, θ_1 the temperature of the first surface, and θ_2 the temperature of the second, we may describe the above facts by writing

$$H = k\frac{(\theta_1 - \theta_2)\,st}{l}, \qquad \text{Eq. 94}$$

where l is the distance apart of the surfaces.

Since all the quantities except k have already been defined, this is evidently the defining equation for thermal conductivity. The modern theory of conduction is almost entirely the creation of a single genius, the French physicist Fourier (1768–1830), whose results are contained in his *Analytical Theory of Heat.*

In actual cases which occur in the laboratory we do not generally measure the heat-flow across a limited surface drawn in a solid, but a bar or some body of definite form is selected and the flow from one end to the other is measured. In any such case where l is considerable, as it generally is, it is clear that one has to reckon also with the amount of heat which escapes sidewise from the lateral surface of the bar. It is these " boundary conditions " which introduce many of the most difficult problems in experimental and mathematical physics. Professor E. H. Hall has devised a clever method for experiments of this kind, in which he manages continually to send back into the sides of the bar just as much heat as escapes, thus eliminating, instead of correcting for, the surface loss of heat.

In the case of gases and liquids, it is found that the conductivities are, in general, very small compared with those of solids, as will be seen from the following table: —

Table of Thermal Conductivities

[Values expressed in C.G.S. units, i.e. in $\left(\frac{\text{Calories}}{\text{cm. sec. degree}}\right)$]

Solids		Liquids		Gases	
Aluminium	0.353	Alcohol	0.00042	Air	0.0000568
Brass	.226	Ether	.00030	Carbon dioxide	.0000307
Carbon	.0004	Mercury	.0201	Hydrogen	.000327
Copper	.918	Petroleum	.000355	Nitrogen	.0000524
Cork	.0007	Water	.0012	Oxygen	.0000563
Glass { from	.0005				
to	.0023				
Iron	.164				
Lead	.080				
Silver	1.096				

Problems

282. Give in words a definition of thermal conductivity derived from Eq. 94 by making all the quantities on the right, except k, unity.

283. From Eq. 94 find the dimensions of thermal conductivity. Observe that temperature will cancel out on each side of the equation.

284. A cylindrical copper vessel is filled with boiling water and placed on a flat cake of ice at 0° C. The bottom of the copper cylinder is 2 millimeters thick and has an area of 30 cm.^{2}. How many calories of heat will be transmitted through this bottom plate per second?

285. On passing through the iron shell of a locomotive boiler from the furnace space to the water space, the temperature falls at the rate of 800° C. per centimeter. The amount of heat transmitted across one square centimeter of this surface per second is 130 calories. Find the thermal conductivity of the iron.

286. A pond of water is covered with a sheet of ice 10 cm. thick. The temperature of the water just under the ice is 0° C., while the temperature of the overlying air is $-8°$ C. Assume the thermal conductivity of the ice to be 0.005 and find how much heat will pass through each square meter of surface in an hour.

2. *Convection of Heat*

269. The toy balloons which, after being filled with hot air, may be seen rising on each Fourth of July, illustrate in an exquisite manner how the principle of heat conduction combined with the principle of Archimedes leads to a large number of our most important and vital phenomena.

The air inside the balloon is heated by conduction from the flame at the mouth of the balloon, and then rises, filling the balloon with a gas whose density is much less than that of the surrounding air. The buoyant force thus produced lifts the balloon into the upper atmosphere, where it finds a position of equilibrium determined by the temperature and pressure of the surrounding air.

If a cup of tea be too hot to drink, we may cool it by adding cream or cold water. Here the heat of the tea is transferred to the cooler liquid by the process of conduction; but the hot and cold portions of the tea do not need to be stirred; they will mix, although a little more slowly, of themselves. **This automatic mixing, which is a mere mechanical process, is called convection.**

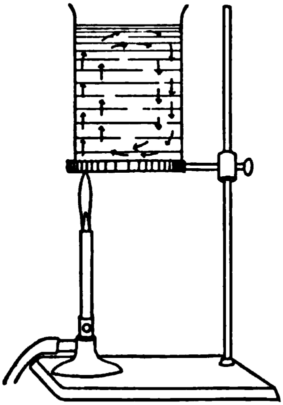

FIG. 193. — Convection currents in water.

The hot air of a chimney rises, mixes with the outside air, and gives some of its heat to the outside air. The hot air rises because its density is less than that of cold air. This process of carrying the hot air up the chimney is, again, convection. A beaker of water to which a flame is applied on one side, as in Fig. 193, becomes equally heated all through. First of all, the water just over the flame becomes hot by *conduction* through the glass. Then, by *convection,* the hot water just over the flame is displaced by the colder water, which is heavier and therefore sinks to the bottom as indicated by the arrows. The potential energy of the system is thus less than if the cold water remained on top.

This cold water, in turn, becomes heated by *conduction* through the glass. **These currents, produced by differences of density and easily detected by small particles in the water, are called "convection currents."** They are exhibited on a gigantic scale in the equatorial regions of the earth, where the lower layers of the atmosphere, in contact with the water of the oceans or with the land, become highly heated by *conduction*, and then rise by *convection*. The air rushing in from north and south constitutes the well-known trade winds.

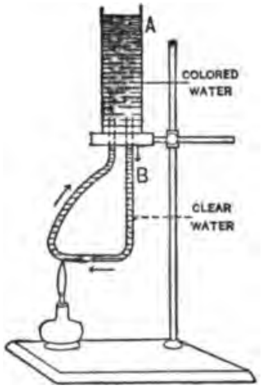

Fig. 194. — Circulation in tubes, brought about by convection currents.

Perhaps the best illustration of convection currents is obtained by the simple experiment indicated in Fig. 194. *B* is a glass tube with both ends fitting into a cork in the bottom of a lamp chimney. The chimney *A* is filled with colored water; but the tube *B* is filled with clear water. The circulation becomes very evident in a few seconds after a flame is applied to one side. Observe that here again the *heat* is communicated to the water by *conduction* through the glass, while the *water* is transferred from one part of the vessel to another by the mechanical (not thermal) process of *convection*.

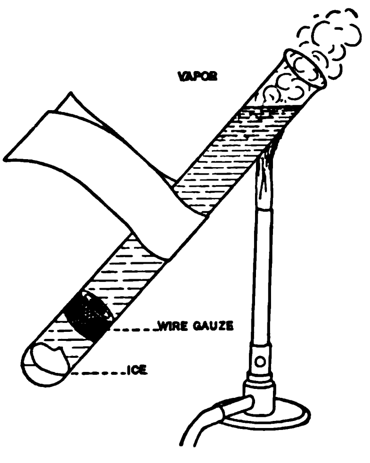

Fig. 195. — Illustrating the fact that water is a poor conductor of heat.

From the preceding it will be evident that in fluids we have, in general, both processes, conduction and convection, always at work; while in solids, convection is necessarily impossible. When, however, heat is applied to the upper layers of a fluid, convection may be reduced to a minimum.

To illustrate, put a small piece of ice, say two grams, in the bottom of a test tube, and hold it at the bottom by means of a little coil of wire or wire gauze (see Fig. 195). The tube may be filled with water and held over a flame until the water in the top of the tube boils. The heated portions, being lighter than the cold, remain on top, while water is such a poor conductor of heat that the ice in the bottom is not easily melted.

In all the varied processes of heat conduction, whether aided by convection or not, it will be observed that the heat is transferred from places of higher temperature to places of lower temperature. *In this respect* heat behaves like steam and all other fluids in flowing from points of higher pressure to points of lower pressure. We shall later find electricity behaving in a similar way, flowing from places of higher to places of lower electrical pressure.

3. *Radiation of Heat*

270. When the hand is held some inches at the side of, or underneath, an incandescent electric lamp, the sensation of heat is distinctly recognized. The same is true of the hand placed beneath a heated metal ball (see Fig. 196). We hold our hands before an open grate fire to warm them. How does the heat pass from the fire to the hands? Certainly not by conduction. For air is one of the very poorest conductors known (see Table, § 268). Nor is it a case of conduction aided by convection currents. For, in each of the three cases cited above, the currents due to convection are such as to cool the hand.

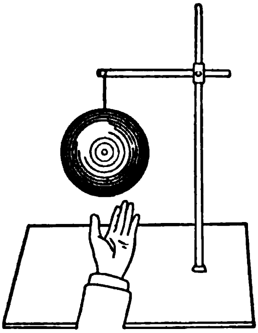

FIG. 196. — Radiation of heat downward from a hot iron ball.

Indeed, one of the earliest observations made after the discovery of the air pump was that both heat and light pass through a vacuum with the utmost ease. There is every reason for believing that the space which separates us from the sun is more nearly a perfect vacuum than any other known; yet across this vast, and so-called empty, region, the earth daily receives enormous quantities of heat. But by what process? We call it *radiation*. But what is radiation?

The question here raised can be answered (but only very imperfectly) by anticipating some of the results of optics and thermodynamics. At this point we must content ourselves with saying that there is most excellent evidence for thinking that light is a form of energy, that light consists in a wave motion, that this wave motion is propagated in a medium, called the *ether*, which permeates all bodies and fills all interstellar space, and that this wave motion travels in free space (*i.e.* space devoid of matter) at the rate of 300 million meters per second, but generally more slowly in space which is not devoid of matter.

Now the experimental evidence for thinking that radiant heat is of exactly the same nature as light is overwhelming. And all the more important laws of radiant heat and of light are included under one single and general statement of the Laws of Radiant Energy.

If, however, we once take for granted the evidence for thinking radiant heat a wave motion, we may immediately predict interference, reflection, refraction, and other properties of waves.

271. These results have been amply verified by more than two centuries of experiments, covering the work of Newton, Melloni, Stewart, Kirchhoff, Tyndall, Langley, and others. The following experiment by Newton sets forth the fact and the explanation with essential truth and clearness: —

> "If in two large cylindrical vessels of glass inverted, two little thermometers be suspended so as not to touch the vessels, and the air be drawn out of one of these vessels, and these vessels, thus prepared, be carried out of a cold place into a warm one; the thermometer *in vacuo* will grow warm as much, and almost as soon, as the thermometer which is not *in vacuo;* and when the vessel is carried back into the cold place, the thermometer *in vacuo* will grow cold almost as soon as the other thermometer. Is not the heat of the warm room conveyed through the *vacuum* by the vibrations of a much subtiler medium than air, which, after the air was drawn out, remained in the *vacuum?* And is not this medium the same by which light is refracted or reflected, and by whose vibrations light communicates heat to bodies, and is put into fits of easy Reflection and easy Transmission? And do not the vibrations of this medium, in hot bodies, contribute to the intense-

ness and duration of their heat? And do not hot bodies communicate their heat to contiguous cold ones, by the vibrations of this medium propagated from them into the cold ones? And is not this medium exceedingly more rare and subtile than the air, and exceedingly more elastick and active? And doth it not readily pervade all bodies? And is it not (by its elastik force) expanded through the heavens?" — *Optics*, Book III, Query 18.

A strong argument for thinking that radiant heat is propagated by the same mechanism as ordinary light is found in the fact that after a solar eclipse the heat rays and light rays reappear, as nearly as can be observed, at the same instant. From this and evidence to be adduced later it appears that heat waves, light waves, and electric waves all travel with the stupendous speed of 300 million meters per second and are essentially identical except as regards wave length.

Qualitative Illustration

272. To show the presence of these heat rays and to show that a lens can form an image of any hot object, the following procedure is perhaps as simple as any. Take an iron ball, *S*, an inch or two in diameter; heat it to a temperature of 200° or 300° C. The ball will then emit no light of its own. If now this ball be suspended in front of a rock-salt lens, *L*, as shown in Fig. 197, it will be found, by moving a thermopile along the line *AC*, that practically all of the heat falling upon the lens is concentrated in a small area about the point *B*. For when the thermopile is placed at *B*, the deflection of the galvanometer is much larger than on either side of *B*. This point *B* is indeed the thermal

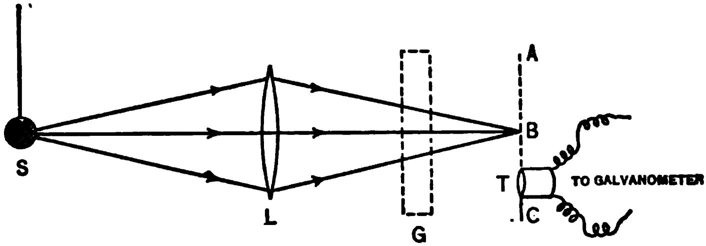

FIG. 197. — Illustrating the refraction and absorption of heat rays.

image or focus of the hot body, S. The rock-salt lens collects the heat rays just as it does the light rays.

If a piece of ordinary plate glass be inserted at G, it will be observed that the heat is almost entirely shut off; in other words the glass plate is almost entirely opaque to the heat rays given off by this hot iron ball.

Herein lies the explanation of the glazed sash which are used over hot-beds in the early spring. The glass will allow the light rays and the short heat rays from the sun to pass through. These heat the black soil of the hot-bed by which they are absorbed. But the heat rays emitted by the black soil are of the very long variety which cannot pass out through glass; hence they are trapped in the hot-bed.

Instead of the rock-salt lens used in Fig. 197, one may conveniently substitute a concave mirror or a Florence flask filled with carbon bisulphide.

Measurement of Radiation

273. The first step toward discovering the laws of any phenomenon is the invention of some method for measuring the quantities involved. There are three principal methods for determining the amount of radiant energy which falls upon a body. **These are each dependent upon the fact that when radiant energy is incident upon a well-blackened body — such as one covered with lampblack — practically all the incident energy of wave motion is transformed into heat energy.** The measurement of radiation then reduces itself to the measurement of a rise in temperature — generally very small. Bodies which absorb all the radiant energy falling on them are said to be "perfectly black." The three methods are the following: —

(i) *Thermocouple.* The essential feature of this instrument is a junction of two different metals, connected in series with a delicate galvanometer. See § 412. This junction is a body of small heat capacity and is coated with lampblack and platinum black so as to absorb (*i.e.* transform into heat energy) radiations of all wave lengths. The Italian physicist, Melloni, first made this device effective; later it was refined by the German physicist

Rubens. A new form was given to it by Professor C. V. Boys of London, who suspended a small wire circuit of two different metals between the poles of a magnet as seen in Fig. 198. The junction marked J_1 is shielded from the radiation, while that marked J_2 is blackened and exposed to the radiation. The difference in temperature thus produced generates an electric current in the circuit and the current causes the thermocouple to rotate. By means of this instrument, which is called a "radiomicrometer," placed at one end of a large room, Boys has been able to show to an audience the radiation from a single candle placed at the other end of the room.

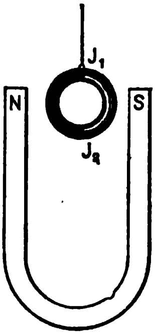

FIG. 198.—Boys' radiomicrometer.

More recently, however, A. H. Pfund of Baltimore has placed the thermocouple in a high vacuum and rendered it so sensitive that he easily measures the radiant energy from the brighter stars. Some idea of the extraordinary sensibility of such an instrument may be obtained from the fact that, if a candle-flame, placed at a distance of 18 miles, were focused upon the junction of this couple by means of a 3 ft. mirror, a deflection of one millimeter would be observed.

Still more recently Dr. W. W. Coblentz of the Bureau of Standards, working at the Lick Observatory, has succeeded in making a vacuum thermocouple which will measure the radiation from stars of only the 6th magnitude, which is equivalent to detecting the radiation from a candle at a distance of 53 miles. This assumes, of course, that there is no absorption in the intervening space. In other words, if the earth's atmosphere could be eliminated, one might sit in an office building in New York City and measure the radiation from a single tallow candle burning in another office building in Trenton, N.J.

(ii) *Radiometer.* This instrument, invented by Sir William Crookes, has been so improved by Professor E. F. Nichols that it is now available for quantitative work. It consists essentially of a horizontal bar delicately suspended in a properly exhausted vessel. At each end of the bar is attached a thin glass disk blackened on one side. The radiation is admitted to the instrument through a window of fluorite which is remarkably transparent to a wide range of wave lengths. On striking the black surface, the radiant energy is degraded to heat and the black disk repelled — probably owing to the bombardment of the smallest particles of air, which, in a vacuum of this degree, are reflected freely from the disk to the wall of the containing vessel and acquire greater kinetic energy as the temperature rises. So delicate is this instrument that by means of it, Nichols and Hull, and Lebedew, have been able to measure accurately the pressure exerted by a beam of sunlight — a vastly smaller effect than that of the radiant energy which comes to us in such a beam, and which must be eliminated before the light pressure can be measured.

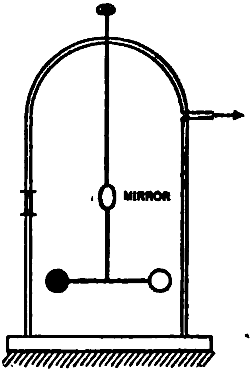

FIG. 199. — The radiometer.

(iii) *Bolometer.* Here the blackened body is a thin strip of metal which forms one arm of a Wheatstone bridge, an electrical instrument which will be met in the laboratory. Before the radiation is allowed to fall upon this strip, the galvanometer reading is zero. But the radiation heats the wire and thus increases its electrical resistance. This throws the bridge out of "balance," and the galvanometer needle is deflected through an angle which is proportional to the amount of the incident radiation. The late Secretary of the Smithsonian Institution, Professor Langley, who perfected this method, showed that differences of temperature of even less than a millionth of a degree centigrade could be detected in this way.

Laws of Radiation

274. By means of such methods as those just indicated two general results connecting radiation and temperature have been obtained.

(i) *Stefan-Boltzmann Law.* It was first suggested by Stefan, afterwards predicted from theory by Boltzmann, and has been amply verified by experiment, that **the total radiation** (*i.e.* **radiation of all wave lengths) of any perfectly black hot body is directly proportional to the fourth power of its absolute temperature.** This statement is often spoken of as the "Fourth-power Law."

Premising that the absolute scale of temperature begins with a zero 273° C. below that of melting ice, and denoting the total power radiated by E, this law may be expressed as follows: —

$$E = C(t^\circ + 273)^4 \quad \textbf{Stefan-Boltzmann Law.} \qquad \text{Eq. } 95$$

where C is a constant throughout a wide range of temperatures, and t° is the ordinary centigrade temperature of the body.

(ii) *Displacement Law.* It has also been found that if the various radiations be separated, say by a prism, so that the different wave lengths are sent off in different directions, and if the radiation of each wave length be measured, that wave length at which the radiation is most intense becomes shorter and shorter as the temperature rises, a fact which is true for all bodies. **And accurate experiment shows that this variation in the case of black bodies occurs in such a way that the product of the wave length λ_m of maximum radiation and the absolute temperature is constant**; or, in terms of algebra, —

$$\lambda_m(t^\circ + 273) = \text{Constant.} \qquad \text{Eq. } 96$$

Upon laws such as these are based the optical and radiation methods for high temperature measurement described in § 259, and widely employed, not only by investigators, but also by engineers.

(iii) *Stewart-Kirchhoff Law.* Let us denote by H the amount of radiant energy which falls upon a body at temperature t° in one second: and by h denote the amount of radiation which this body absorbs in one second when at the same temperature. The ratio $\frac{h}{H}$ is what Kirchhoff calls the **absorption** of the body, and is denoted by A. A body which absorbs all the radiation falling upon it is said to be **absolutely black**; its absorption is unity.

Let us now denote by E the amount of radiant energy which this same body, at this same temperature, would emit in one second in virtue of its hotness only. This quantity E is called the **emission** of the body. Now Balfour Stewart in England and Kirchhoff in Germany independently proved that

I. **The ratio of the emission to the absorption of any body depends upon the temperature only;** **and**

II. **This ratio is numerically equal to the emission of a perfectly black body at the same temperature and wave length.**

The quantitative form in which these laws are expressed is due to Kirchhoff. From these two statements it is evident that a piece of polished metal will not emit so strong a light as a piece of metal covered with lampblack and heated to the same temperature. For since the absorption of polished metal is less, its emission will also be less.

This result is in thorough accord with what we might expect **if radiant heat consists in a wave motion.** For in our study of vibrating bodies (see *Resonance,* § 204), we have found that any source of vibration, *e.g.* a tuning fork or a pendulum, can absorb only those vibrations which are of the same period as its own. A source of vibrations is a source of radiant energy. It has been proved that the red-hot carbon **emits waves of all lengths.** It might, therefore, be expected that the carbon when cool would **absorb waves of all lengths,** and thus appear black, *provided* the disturbances involved in the radiation of heat are ordinary mechanical disturbances such as we have been studying under the subject of dynamics and sound. The fact that any body, placed in an inclosure maintained at a constant temperature will attain and maintain this temperature is the simplest and most cogent reason for thinking that any good absorber is also a good radiator.

Once in possession of this general law, we have the explanation of a host of important household and meteorological phenomena. For instance, land being, in general, a much better absorber than water for radiant energy, it follows that the surface of the land will cool more quickly than the surface of a lake on a clear night; conversely, it will rise in temperature more rapidly than the water during the day when the sun is shining. Indeed, the ground at the bottom of a lake or river may radiate energy through the water so rapidly as to drop the temperature of the lake bottom below the freezing point, thus producing a coating of ice which clings fast to the bottom and is known as "anchor ice."

Air is a comparatively poor absorber of heat: and yet it is estimated that rather less than half the solar energy falling upon our outer atmosphere penetrates to the surface of the solid earth. That portion of the sun's radiant energy which *does* pass through the atmosphere strikes the ground, is there transformed into heat and raises the temperature of the air just over it by conduction and convection.

V. Some Effects of Heat

275. Among the multitudinous effects of heat the following two have been selected as being perhaps the most important.

1. Change of Dimensions or of Stress.
2. Change of Molecular State.

Change of Dimensions

1. *Expansion of Solids*

276. The familiar fact that solids as a rule increase in size as their temperature rises is illustrated in the manufacture of large guns, which are built up of coaxial cylinders, the inside diameter of each one being a trifling bit smaller than the outside diameter of the preceding. When the outside cylinder is sufficiently heated it will slip over the inside one, and, on cooling, will shrink so as to form what is practically a single piece of metal, but one which is stronger than a single casting. Wagon tires are shrunk on to wooden wheels in the same manner. On account of the changes in length caused by temperature variations, engineers generally rest one end of a girder bridge, not directly upon the pier, but upon an iron roller as shown in Fig. 200. A moderate drop in temperature would shorten a 100 ft. steel girder by half an inch.

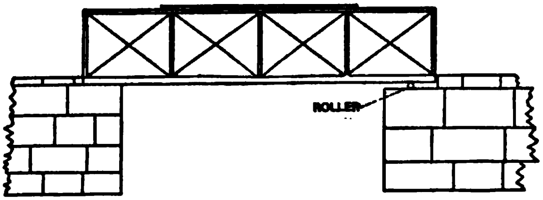

FIG. 200. — Roller providing for expansion of iron bridge.

Coefficients of Expansion

Many accurate experiments by methods which must be studied in the laboratory have shown that the change in length is, for a moderate range of temperature, proportional to the change in temperature.

If a bar whose length is l centimeters changes in length by e centimeters, while its temperature changes by $t°$, the change in unit length per degree is $\frac{e}{lt°}$. This increment in length experienced by each unit of length for one degree rise in temperature is called the **coefficient of linear expansion** for the material of which the bar is made.

Let us denote this coefficient by a, then

$$a = \frac{e}{lt°}. \qquad \text{Defining eq. for coef. linear expansion.} \qquad \text{Eq. 97}$$

The three quantities on the right-hand side of this equation can each be measured in the laboratory, and the coefficient a thus determined.

We are accordingly led to the following expression for the length of a bar l_t at any temperature t, when l_0 is its length at the zero of temperature: —

$$l_t = l_0(1 + at°) \qquad \text{Eq. 98}$$

Observe that the value of the coefficient of linear expansion will vary from one thermometric scale to another. Measurements of the highest precision show also that this coefficient itself is a function of the temperature, thus rendering the length of a bar a function of both the first and second power of the temperature as expressed in the following equation: —

$$l_t = l_0(1 + at + bt^2). \qquad \text{Eq. 99}$$

In general, the coefficient b is very small.

277. In the case of many alloys the expansion is profoundly modified by the percentage composition. Taking advantage of this fact, the French physicist Guillaume has succeeded in discovering a nickel-steel alloy — about 36% nickel — for which the linear coefficient a has a value of less than one part in a million. The value of this linear coefficient is as follows: —

$$a = (0.965 - 0.00017\, t^{\circ})10^{-6}.$$

Owing to the fact that the size of a body constructed of this metal is practically invariable with temperature, the substance has been called "invar." The usefulness of such a metal for scales of length employed in geodetic measurements and for clock pendulums is at once apparent. —

TABLE OF LINEAR COEFFICIENTS

(Centigrade Scale)

Glass	0.0000085	$= \frac{1}{120000}$
Platinum	0.0000085	$= \frac{1}{120000}$
Steel	0.000012	$= \frac{1}{85000}$
Brass	0.000018	$= \frac{1}{55000}$
Copper	0.000019	$= \frac{1}{52000}$
Aluminium	0.000023	$= \frac{1}{43000}$
Zinc	0.000029	$= \frac{1}{34000}$
Invar	0.00000096	$= \frac{1}{1000000}$

Study this table to discover why platinum rather than copper wire is fused into the glass of an incandescent lamp bulb, and into vacuum tubes.

Anomalous Expansion

278. Under certain circumstances it is possible for a substance to have a coefficient of expansion which is *apparently* negative. That is, the substance may shorten instead of lengthen on being heated, as the following experiment shows: —

Take a piece of ordinary rubber tubing from three to five feet in length. Into each end fasten a three-way connecting tube of brass, as shown in Fig. 201. These can be wired in very easily. The upper end of the tube is attached to a support. To the lower end is fixed a heavy stretching weight. Cork up one of the openings at the top, and into the other introduce a current of steam. The rubber will contract to such an extent as to be easily seen at once by the whole class. We might be inclined to say that under these circumstances rubber has a negative coefficient of expansion. If, however, the heavy weight be replaced by a light one, we shall find that the rubber lengthens when the steam is introduced. Not only so, but if, instead of measuring the length, one measures the volume of the rubber he finds that this increases on heating. The anomaly in *linear* expansion arises from the fact that *stretched* rubber is a substance which, at any point, has different properties in different directions. Such substances are called "anisotropic." All crystals except those belonging to the regular (or cubic) system are anisotropic. We shall find another and more important anomaly in the expansion of water.

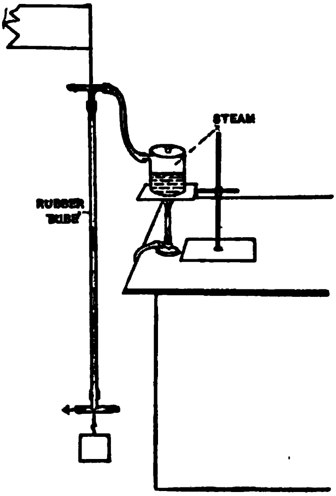

Fig. 201.—The contraction of rubber on heating.

Problems

287. Prove by squaring each side of Eq. 98 that for any solid the coefficient of superficial expansion — that is, the rate at which area increases with temperature — is twice the linear coefficient.

288. Prove in like manner that the coefficient of voluminal expansion is three times the linear coefficient.

289. Look up the subject of compensated pendulums in some encyclopedia or larger treatise on physics, and be able to explain the principle by means of a diagram on the blackboard.

290. A wagon tire 300 cm. in circumference is heated until its length has increased by 1 cm., *i.e.* by one third of one per cent. By what per cent has the area inclosed by the tire increased?

Ans. Two thirds of one per cent.

291. For every degree (centigrade) that iron is heated it increases in length by one thousandth of one per cent. When the temperature rises from 0° to 30°, how much longer will an iron bridge of 200 feet span become? *Ans.* 0.72 inch.

292. Explain, by diagram on blackboard, a system of hot-water heating such as might be used in a private residence. Include in your diagram, boiler, flow pipe, return pipe, radiators, and expansion tank.

293. Explain, by diagram on blackboard, just how a strip of zinc soldered to a strip of sheet copper, as indicated in Fig. 202, can be employed to announce (by the ringing of a bell) when a room is too hot and when too cold.

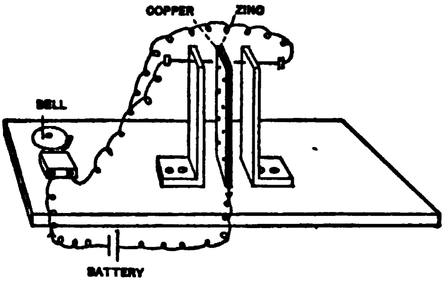

Fig. 202. — Device for announcing change of temperature.

294. An iron cannon ball of radius 5 cm. is heated until its radius has been increased by $\frac{1}{50}$ of one per cent. Find by what per cent the volume of the ball has been increased.

295. Long horizontal steam pipes are always supported in such a way that they are free to move at one end. Why?

296. A brass wire 100 ft. long is fixed at its upper end and carries a scale pan at its lower end. When a mass of one kilogram is added to the scale pan the wire is elongated by $\frac{1}{16}$ inch. To what extent must the temperature of the wire be lowered in order to restore the wire to its original length?

297. A clock with a steel pendulum is adjusted to beat seconds at a temperature of 20° C. Find how many seconds it will gain each day when its temperature falls to 0° C.

298. An iron rail 30 feet long is laid during the summer when the temperature is 30° C. Find its length in winter when the weather is 10° C. below zero.

299. An iron telegraph wire is stretched between two poles which are 400 ft. apart, and has a sag of 5 ft. and a stress of 10,000 lb. per square inch. What stress will be caused by a drop in temperature sufficient to make the sag only 4 ft.?

300 (*a*). Suppose the limit of accuracy in measuring the standard meter to be $\frac{1}{20000}$ mm. What variation in temperature of the bar — imagine it to be platinum — would be permissible during the measurement?

300 (*b*). An electric current is sent through a copper wire which is 120 cm. long at room temperature, 20° C. The effect of the electric current is to heat the wire and lengthen it by 0.8 cm. Find the temperature of the wire.

300 (*c*). At what temperature is the reading of a Centigrade thermometer the same as that of a Fahrenheit?

CHANGE OF DIMENSIONS

2. *Expansion of Liquids*

279. Since liquid bodies are defined (§ 129) as those which have no shape of their own, but assume the shape of the containing vessel, it is evident that a liquid body has no characteristic edges or lines, and hence no coefficient of linear expansion. But liquid bodies have characteristic volumes, and these do change when heated.

Definition of Coefficient of Expansion of a Liquid

If we denote by V_1 and V_2 the volume of a liquid at the temperatures t_1 and t_2 respectively, and if we call the coefficient of expansion b, its defining equation becomes

$$b = \frac{V_2 - V_1}{V_1 (t_2 - t_1)}. \quad \text{Defining eq. for cubical expansion.} \qquad \text{Eq. 100}$$

When, therefore, we speak of the coefficient of expansion of a liquid, we shall always refer to its **coefficient of cubical expansion, *i.e.* to that fraction of itself by which a volume of liquid increases when heated through one degree.**

Of all liquids, water and mercury are perhaps the two of most importance in the physical sciences. We shall consider these briefly.

Mercury

280. The very elegant method by which Regnault * determined the density of mercury at various temperatures, and hence its coefficient of expansion, has long served as a model for succeeding experimenters. The process is as follows: —

Two U-tubes, filled with mercury, as shown in Fig. 203, are connected at the bottom by a tube of small caliber; and this tube contains a little air which serves to separate the two columns of mercury. We have then really two U-tubes of mercury joined by a capillary tube of air.

---

* *Regnault* (b. 1810, d. 1878), a French physicist distinguished for his experimental skill.

In the portion of the tube marked C, the pressure must be the same throughout the volume of air. For if the pressure due to each of the mercury columns were *not* the same, the mercury surfaces would move one way or the other until these pressures became equal.

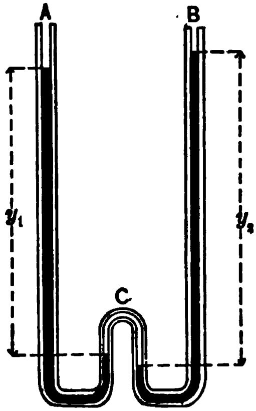

FIG. 203. — Regnault's experiment.

Suppose now that we heat the mercury in the branch B until it reaches a temperature t_2, and that in the branch A we cool to the temperature t_1. The density of the mercury in B will diminish on heating; consequently the difference of level indicated by y_2 will increase. The density of the mercury in A will increase on cooling, and hence the difference of level in A will diminish; call this difference y_1. The pressure in C due to mercury in A is, by the laws of Hydrostatics, y_1D_1g; while the same pressure in C due to B is y_2D_2g.

But these pressures must be the same. Hence,

$$y_1D_1g = y_2D_2g.$$

$$\therefore \frac{y_2}{y_1} = \frac{D_1}{D_2}.$$

$$\therefore \frac{y_2 - y_1}{y_1} = \frac{D_1 - D_2}{D_2}.$$

By measuring y_2 and y_1, Regnault was thus enabled to determine the fraction of itself by which the *density* of mercury is changed when its temperature is varied from $t_1°$ to $t_2°$.

But how can one obtain from this fraction the value of the coefficient of expansion?

Consider any constant mass of mercury M, whose volume is changing in consequence of a variation in temperature. The defining equation for density gives

$$M = V_1D_1 = V_2D_2.$$

$$\therefore \frac{V_2}{V_1} = \frac{D_1}{D_2}.$$

$$\therefore \frac{V_2 - V_1}{V_1} = \frac{D_1 - D_2}{D_2}.$$

$$\therefore \frac{V_2 - V_1}{V_1} = \frac{y_2 - y_1}{y_1}.$$

Now substituting in Eq. 100,

$$b = \frac{y_2 - y_1}{y_1(t_2 - t_1)}, \qquad \text{Eq. 101}$$

where y_1 and y_2 are differences of level corresponding to temperatures t_1 and t_2 respectively, and b is the coefficient of cubical expansion. These quantities y_1 and y_2 are, however, difficult to measure with accuracy.

Of course, we might take a single tube of mercury, Fig. 204, and measure its height at two different temperatures; but here the difficulty is that the glass vessel containing the mercury expands with the heat, so that the volume of the mercury is no longer proportional to the height of the column. In this way we should obtain only the **apparent expansion** of mercury. **It is the apparent expansion which is used in glass and mercury thermometers.**

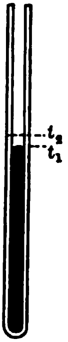

FIG. 204.—Apparent expansion.

The beauty of Regnault's method is that the height of the column depends only on the temperature, and not at all upon the containing vessel. Regnault thus measured the **absolute expansion**. The value which he thus obtained for the average coefficient of expansion of mercury between 0° and 100° was 0.0001815.

The most accurate of recent values is probably that obtained at the Imperial College of Science in London by Callendar and Moss, namely, 0.0001820536. These two investigators have immensely improved upon Regnault's method by using six pairs of hot and cold columns, instead of one, also by the use of platinum thermometers extending throughout the column, as well as by improved electric heating. A platinum resistance thermometer is capable of measuring temperature changes as small as 0°.0001 C. —*Phil. Trans.* 1911.

Water

281. From many points of view, water is a remarkable substance. This fact is strikingly exhibited in its behavior under heat. In general, warm liquids are less dense than the same liquids cold; and this is true of water except between the temperatures 0° C. and 3°.98 C. The manner in which water expands under heat is clearly set forth in the curve, Fig. 205.

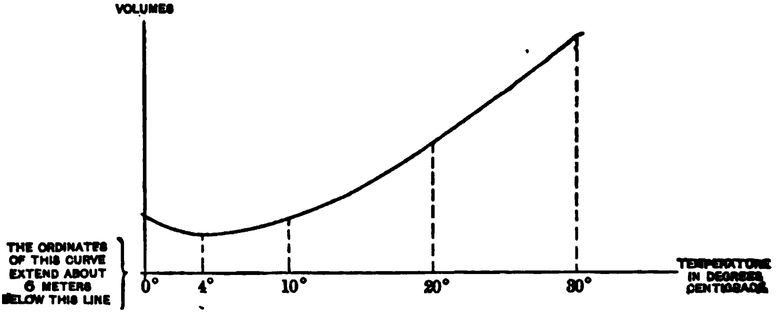

Fig. 205. — Expansion of water.

The coefficient of expansion of water has recently been determined at the Reichsanstalt * by the method which Regnault employed for mercury. The results are summarized in the following table: —

Density of Water

Temperature Centigrade	Density	Volume
0°	0.999867	1.000132
3.98	1.000000	1.000000
10	0.999727	1.000272
15	0.999126	1.000874
20	0.998229	1.001773
25	0.997071	1.002937
30	0.995673	1.004345
35	0.994057	1.005977
40	0.992241	1.007819

The preceding figure (205) is merely a graphical description of the results contained in this table. Or, if desired, a still briefer description of the facts may be obtained in the following empirical equation,

---

* The *Reichsanstalt* is the great national physical laboratory of Germany, located in Charlottenburg, a suburb of Berlin.

$$V_t = V_4\left(1 + \frac{(t^\circ - 4^\circ)^2}{144000}\right), \qquad \text{Eq. 102}$$

where V_t is the volume of any given mass of water at t° C. and V_4 the volume of the same mass at 4° C.

The maximum density of water at 4° C. may be demonstrated by loading a small hollow glass sphere with sand until it will just sink in water at 0°. If placed in a vessel of ice water, it will sink; if now the ice be removed the water will gradually rise in temperature, the float will also rise, and after the temperature passes 4° C. the float will again sink. The enormous economic importance of this anomaly of water is seen in the fact that it prevents our lakes and rivers from freezing except over the surface.

Problems

301. Graham's compensated pendulum is made of a steel rod which carries at its lower end a glass cylinder containing mercury. Show that the linear expansion of the steel (which extends to the base of the cylinder) will be very approximately counterbalanced by the cubical expansion of the mercury when the height of the mercury is $\frac{1}{16}$ the height of the steel pendulum.

302. A large thermometer bulb, made of glass, and placed in melting ice, holds 200 g. of mercury. How much mercury will overflow when placed in a bath of steam at atmospheric pressure?

303. A mercury barometer in a laboratory at 15° C. reads 756.2 mm. What are the three most important corrections which must be made in order to obtain the true barometric height?

304. A glass cylinder which weighs 5 g. in air, weighs only 2.994 g. in water at 20° C. Find its weight in water at 40° C.

305 (*a*). Compare the density of water at 30° as obtained
(i) from the table in § 281, with that obtained
(ii) from Eq. 102.

305 (*b*). What are some of the limitations of liquid thermometers, and what objections are to be urged against resistance thermometers?

305 (*c*). What causes a glass bottle to break when placed on a hot stove?

CHANGE OF DIMENSIONS OR OF STRESS

3. *Expansion of Gases*

282. We have already seen (§ 257) that the first thermometers were made by inclosing a certain mass of gas in a glass bulb whose stem was sealed by immersing the end in a liquid. Such a device is at once the simplest and earliest for studying the effects of heat upon gases.

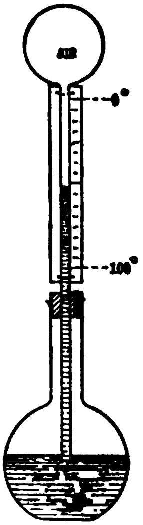

FIG. 206. — Galileo's thermometer.

Having substituted mercury for water in the thermometer of Galileo (see Fig. 206), Amontons, in 1699, discovered that the *change of pressure* produced by heating through a range of 100° was the same whatever the amount of air inclosed in the bulb.

Law of Charles

By definition, the volume of a gas is always the volume of the containing vessel. Boyle's law states the experimental fact that to each volume of gas there corresponds a definite pressure, provided the mass and the temperature remain constant. If, however, the temperature varies, the effect may be

(1) a change of volume while the pressure remains constant; or

(2) a change of pressure while the volume remains constant; or

(3) a change of *both* pressure and volume.

To determine the effect of heat on a gas, we must then either

(1) Keep the gas at a constant pressure and measure the ratio of increase of volume to original volume; or

(2) Keep the gas at constant volume and measure the ratio of increase of pressure to original pressure,

in each case for a rise of one degree in temperature.

These two quantities were first roughly measured by Charles, afterwards more accurately by the French chemist, Gay-Lussac (1778–1850).

The capital discovery was thus made that: —

(*a*) These two quantities are the same for any one gas.

(*b*) They have very approximately the same numerical value for *all* gases, namely, $\frac{1}{267}$. Later and more accurate observations have shown that this value is more nearly 0.003665 = $\frac{1}{273}$.

The actual measurement of these quantities is a somewhat difficult matter, requiring many precautions, and may well be reserved for the student's second or third year in physics. But the algebraic description of Gay-Lussac's *results* is a matter of the utmost simplicity. It is as follows: —

Let V_0 and V_t represent the volumes of a given mass of gas at the temperatures 0° and $t°$ respectively; then, so long as the pressure remains constant, —

$$V_t = V_0(1 + 0.003665\,t^\circ). \quad \text{Law of Charles.} \qquad \text{Eq. 103}$$

If, in like manner, P_0 and P_t represent the pressures of a given mass of gas at the temperatures 0° and $t°$ respectively, then, so long as the volume remains the same, —

$$P_t = P_0(1 + 0.003665\,t^\circ). \qquad \text{Eq. 104}$$

Gay-Lussac's result may be summarized by saying that the pressure of a given mass of gas at any temperature $t°$ varies directly as a factor $(1 + at)$ where a has the value $\frac{1}{273}$.

Another method of describing Charles' law is the following: — Make simultaneous measurements of the *volume* and *temperature* of any given mass of gas inclosed in a vessel under constant pressure. Plot the temperatures (measured on the gas scale explained in § 258) as abscissae and the volumes as ordinates; then all the points thus determined will fall on a straight line OP, as shown in Fig. 207.

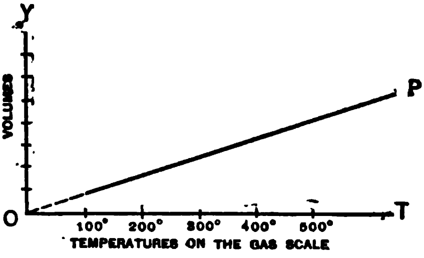

FIG. 207.—Graphical representation of Charles' law.

Boyle's Law

283. Boyle having already shown that, at constant temperature, the pressure varies inversely as the volume, we may combine his law with that of Charles into the following single expression, thus: —

$$P = \frac{C\left(1 + \frac{t^\circ}{273}\right)}{V},$$

where C is a proportionality factor.

Clearing of fractions,

$$PV = \frac{C}{273}(273 + t^\circ),$$

or

$$PV = RT, \qquad \text{Eq. 105}$$

where $R = \frac{C}{273}$ and $T = t^\circ + 273$.

In other words, if we measure temperatures from a zero which is 273° C. below the melting point of ice and denote these temperatures by T, we find that the pressure and the volume each vary directly as the temperature. **A gas which obeys Boyle's law exactly is said to be an ideal or perfect gas.** Temperatures measured on this scale are called "ideal gas temperatures." Perfect gases do not occur in nature; but Lord Kelvin has proved that if they *did* occur, temperatures measured by use of any one of them would coincide with absolute temperatures, which we shall meet a little later.

The Gas Constant

The laws of Boyle and Charles are each completely expressed by Eq. 105. The proportionality factor R which there appears is a constant for any given mass of any particular gas. The value of R in any particular case must be determined by experiment. If the body of gas selected for experiment be one gram of air, then R turns out to be 2.8791×10^6 ergs/1° C. But if one selects for his experiment a pound of air, and measures volumes in cubic feet, temperatures in Fahrenheit degrees, and pressures in pounds per square-foot, then $R = 53.18 \frac{\text{foot-pounds}}{1^\circ F.}$.

Chemists have shown, however, that if one chooses a mass of *any* gas whose weight in grams is numerically equal to the molecular weight of the gas, the volume of this mass (at 0° C. and 760 ^{mm} Hg.) will be 22412. cubic centimeters; or more technically, the volume of a gram-molecular weight of any gas is 22412. $cm.^3$ Accordingly if we agree always to use a gram-molecular weight, Boyle's law becomes, *for any gas whatever,*

$$PV = RT, \text{ where } R = 83.15 \times 10^6 \frac{ergs}{1^\circ\ C.}.$$

If instead of using a single gram-molecular weight, one uses m gram-molecular weights, then the law becomes

$$PV = mRT \qquad \text{Eq. 106}$$

where R has the value just given, namely 83.15 million ergs per degree. This is the form of the gas-law which is now used in chemistry, engineering, and physics.

KINETIC THEORY OF GASES

284. Any satisfactory theory of gases must explain at least the following observed phenomena: —

1. The Law of Boyle.
2. The Law of Charles and Gay-Lussac.
3. Dalton's Law — A mixture of gases having no chemical reaction on each other exerts at any given temperature a total pressure which is equal to the sum of the partial pressures which would be produced by each gas separately if it alone occupied the containing vessel at the given temperature.
4. The Hypothesis of Avogadro — that "the masses of molecules are in the same ratio as the densities of the gases to which they belong," or what amounts to the same thing, *the number of molecules in equal volumes of all gases is the same.* It is here supposed, of course, that the temperature and likewise the pressure is the same for each gas.

Assumptions

285. The kinetic theory of gases, which is largely the creation of Bernouilli, Clausius, and Maxwell, begins with the following assumptions: —

(i) That a gas consists of a large number of small smooth particles which rebound with unchanged speed when they strike the wall of a containing vessel at the same temperature as that of the gas.

(ii) That the motion of these gas particles is not appreciably affected by gravity.

(iii) That the particles are so small as seldom to collide with each other.

(iv) That the particles exert no appreciable attraction upon each other.

From the first assumption it would follow that the kinetic energy of such a system ought to remain constant so long as the walls of the containing vessel remain at the same temperature.

Definitions

286. Let P = pressure of gas, measured in bars (*i.e.* in dynes/cm.^{2}).
V = volume of containing vessel, in cm.^{3}
N = total number of particles — molecules.
$n = \frac{N}{V}$ = number of particles per cm.^{3}
m = mass of each particle.
u = square root of mean square of the speed of the particles.
$= \sqrt{v_x^2 + v_y^2 + v_z^2}$ where v_x, v_y, v_z are the components of the mean velocity of all particles along the axes of X, Y, and Z, respectively.

We proceed to show that the pressure may be computed from simple dynamical principles to be

$$P = \frac{nmu^2}{3}.$$

Since the particles are supposed to move at random in all directions, the squares of the v_x, v_y, and v_z are each equal to the other; and hence

$$u^2 = 3\,v_x^2 = 3\,v_y^2 = 3\,v_z^2,$$

or the mean value of $v_x^2 = \frac{1}{3}\,u^2$. Eq. 107

The fundamental hypothesis of the gas theory is that the pressure on any wall of the containing vessel is due to the momentum which is delivered to that wall per second by the gas particles. Imagine the vessel to have two parallel walls perpendicular to the axis of X and these walls to be a centimeters apart. The Y and Z components of the velocity being each parallel to these walls will contribute nothing to the pressure on these walls.

Limiting our consideration now to a single particle, this particle will travel to and fro from one of these walls to the other $\frac{v_x}{2a}$ times per second. At each impact the velocity of the particle changes by $2v_x$ and its momentum by $2mv_x$. The momentum imparted to the wall per second by this particle is therefore $2mv_x, \frac{v_x}{2a}$ or $\frac{m}{a}v_x^2$. But momentum per second means average force; so that the expression just obtained gives us the mean force which a particle rebounding from one of these walls to the other would exert upon each of them.

Now consider N particles; then if the total force on each of these walls be denoted by F, we shall have

$$F = \frac{m}{a}Nv_x^2.$$

But since $v_x^2 = \frac{1}{3}u^2$, it follows that —

$$F = \frac{mNu^2}{3a}. \qquad \text{Eq. 108}$$

If s is the area of the wall over which this force is exerted, the pressure will be

$$P = \frac{F}{s} = \frac{Fa}{V}.$$

Substituting the value of F from Eq. 108, we have

$$P = \frac{1}{3}\frac{Nmu^2}{V} \qquad \text{Eq. 109}$$

$$= \frac{1}{3}nmu^2,$$

which is the fundamental equation of the kinetic theory of gases, and a result which is to be interpreted as follows: —

Laws of Boyle and Charles

287. If now we make the common and perhaps reasonable assumption that the kinetic energy resident in any mass of gas is the kinetic energy of translation of the small particles and that the ideal gas temperature (§ 283) is proportional to the mean kinetic energy of all the particles, we have $T \propto \frac{1}{2} mu^2$, and

$$T = \frac{Cmu^2}{2},$$

where C is an unknown proportionality constant.

Let R be a constant such that

$$R = 2/3\ C \times \text{total number of particles} = 2N/3\ C.$$

Then it follows from Eq. 109 that

$$PV = RT, \qquad \text{Eq. 105}$$

which is precisely the law of Charles and Boyle as derived by experiment.

Dalton's Law

Total pressure = Sum of partial pressures

288. This follows at once, since we assume that the particles are so small as not to interfere with one another; hence the pressure would vary as the total number N of different particles of all kinds in the vessel, or

$$P \propto N.$$

Avogadro's Hypothesis

289. For any two gases, we may write from our fundamental equation

$$P_1 = \tfrac{1}{3} n_1 m_1 u_1^2$$

and

$$P_2 = \tfrac{1}{3} n_2 m_2 u_2^2.$$

Since the temperatures and pressures are the same for each gas we have

$$T_1 = T_2 \therefore \tfrac{1}{2} m_1 u_1^2 = \tfrac{1}{2} m_2 u_2^2$$

and

$$P_1 = P_2 \therefore \tfrac{1}{3} m_1 u_1^2 n_1 = \tfrac{1}{3} m_2 u_2^2 n_2.$$

But these equations can be true simultaneously only when

$$n_1 = n_2,$$

which is Avogadro's hypothesis.

V. EFFECTS OF HEAT (*Continued*)

CHANGE OF MOLECULAR STATE

290. There are probably not many people whose memories reach back to years so early that they recall their surprise at first seeing a piece of ice melt in their tiny hands and apparently disappear. The change from water to steam which occurs in a boiling teakettle is equally familiar. These two instances of **melting** and **boiling** are so typical of many similar cases met with in nature that they deserve close attention.

Melting

291. (i) In the first place, it is well known that water and ice have the same chemical composition and practically the same chemical properties. A similar statement concerning solid and liquid paraffin would be true.

(ii) In the second place, a glass of water so long as it contains ice and is stirred does not become either hotter or colder on standing. The ice may melt away; but, so long as there is any ice left, the water will remain approximately at what we call "the temperature of melting ice."

(iii) Thirdly, the temperature of melting ice is slightly changed — always lowered — by adding to the ice any impurity which is soluble in water. If therefore one wishes to use melting ice as a standard of temperature which shall be exact to within, say, $\frac{1}{100}$ of a degree or less, he should use ice which has been made from distilled water.

(iv) Fourthly, the temperature of melting ice can be changed by placing the ice under pressure. The most convenient method of showing this is perhaps the following: —

Choose a block of ice having a section of from 100 to 200 square centimeters. Support it, as indicated in the figure, upon two stools or boxes. Over the ice tie a loop of steel piano wire about one half millimeter in diameter. Hang a mass of about 25 kilograms on the wire. In this way you can subject the ice to a pressure of several hundred kilograms per square centimeter.

Compute the area of the wire in contact with the ice. The weight of the mass attached divided by this area will roughly give the pressure. The block of ice will, in general, be not far from its melting temperature; but the melting will all occur at the surface until pressure is put on the wire; it will then be observed that the ice just underneath the wire melts also and lets the wire cut its way through the block. But the water behind the wire freezes at once, so that the ice remains a solid block although it has just been cut through. The effect of pressure is, then, to produce melting, *i.e.* to lower the freezing point of water.

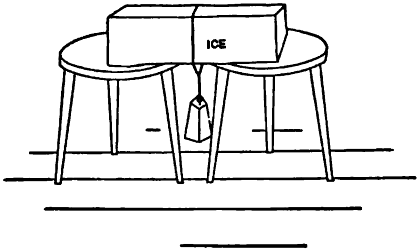

Fig. 208. — Bottomley's experiment showing effect of pressure on melting point of ice.

This process of melting under pressure and again freezing is known as "regelation." In this principle lies the explanation of the fact that cold snow will not "pack" into a good snowball; also of the fact that glaciers apparently "flow" down a mountain side as if ice were a very viscous fluid. In front of points of great pressure the ice melts; the pressure being by this means transferred to some other point, the water again freezes, but in a new configuration. In this manner the glacier gradually adapts itself to the shape of the valley as it proceeds downward. Glacier speeds vary from a few inches to three feet a day, depending upon the slope of the mountain, and upon the distance of the point of observation from the middle of the stream.

Our experience with ice and all other solids may be summarized in the following statement: —

Experimental Law of Melting

"The pressure remaining the same, there is a definite melting point for every solid; and (provided the mass be stirred) however much heat be slowly applied, the temperature of the whole remains at the melting point till the last particle is melted." — Tait, *Heat*, Ch. VIII.

Boiling

292. (i) In boiling, just as in melting, we have to deal with two states of matter which have the same chemical composition; that is, steam and water have the same chemical properties.

(ii) As in melting, so in boiling, it matters not how much heat you give to the water in a teakettle, its temperature does not change after boiling has once begun. One can easily satisfy himself of this with an ordinary mercury thermometer.

(iii) The analogy between melting and boiling goes still further, for the boiling-point is also changed by the addition of any impurity which is soluble in water; but while the melting point is lowered by such an impurity, the boiling point is practically always *raised* by it.

(iv) The fourth step in the analogy is that the temperature of boiling water is very much changed by a change of pressure.

This is most easily proved by boiling in a kettle of water a bottle half filled with water. If this bottle be corked while still boiling, it may be removed with so little air inclosed in it that the pressure on the inclosed water soon becomes greatly less than that of the atmosphere. Under these circumstances, the water in the bottle will continue to boil long after it has reached a temperature not uncomfortable to the hand.

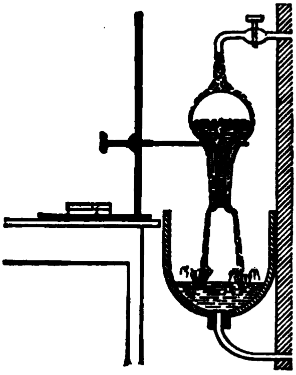

FIG. 209. — Franklin's experiment showing boiling point lowered by diminution of pressure.

In the laboratory this phenomenon is most commonly shown by boiling over a Bunsen flame a Florence flask half filled with water. Cork the flask while boiling. Remove it at once to a support previously prepared and cool with cold water. Violent boiling will occur during the cooling process. To show that to each different pressure corresponds a definite boiling point, a very simple method is that indicated in Fig. 210.

The water under examination is placed in a Florence flask that can be heated over a Bunsen burner. Into the neck of this flask is sealed a thermometer and a glass tube leading to a vacuum chamber. A Woulff bottle with three necks makes a convenient vacuum chamber and serves also as a trap for either water or mercury. To a second neck of the Woulff bottle connect a Bunsen filter pump or some more convenient form of air-pump. In the third neck seal an inverted U-tube, with its outer end dipping into a vessel of mercury, as shown in the figure. From the height, *h*, to which the mercury rises in this tube one can easily determine the pressure over the surface of the water. For each different *h*, the thermometer registers a different boiling point. It will thus be seen that the boiling point of water is much more sensitive to changes of pressure than is the melting point of ice. Not only so, but the effects of pressure in these two cases are exactly opposite: the melting point of ice is lowered by pressure; the boiling point of water is raised by it.

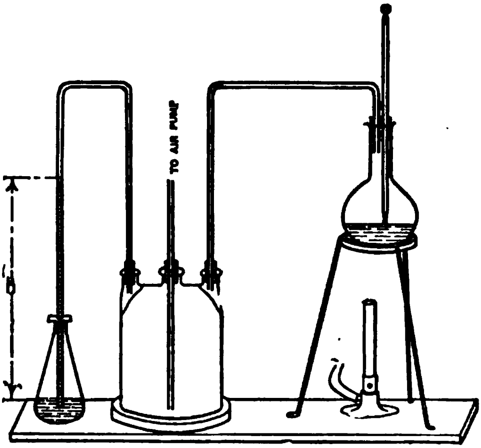

FIG. 210.—Boiling point raised by increase of pressure

Our experience concerning boiling may be summarized as follows: —

"**The pressure remaining the same, there is a definite boiling point for the free surface of every liquid; and (provided the mass be stirred) however much heat be applied, the temperature of the whole remains at the boiling point till the last particle is evaporated.**" — TAIT, *Heat*, Ch. IX.

293. It can be shown that liquids are undergoing evaporation at all temperatures. But the region over the free surface of a liquid may contain so much vapor from the liquid that condensation will occur as rapidly as new vapor is formed. The atmosphere over the liquid is then said to be **saturated.**

The processes of **boiling** occur when the temperature has reached a point where vapor is formed in the interior of the liquid, and rises to the top. But unless the pressure of this hot vapor were equal in pressure to that at the free surface of the liquid, the bubble of vapor would collapse in the liquid.

The temperature of boiling (what we ordinarily call the boiling point) is then simply that temperature at which the saturated vapor of the liquid exerts a pressure equal to the pressure at the free surface of the liquid.

PRESSURE OF SATURATED WATER VAPOR

TEMPERATURE CENTIGRADE	PRESSURE IN ATMOSPHERES	TEMPERATURE CENTIGRADE	PRESSURE IN ATMOSPHERES
0°	0.006	90°	0.691
10°	0.012	100°	1.000
20°	0.023	110°	1.415
30°	0.042	120°	1.962
40°	0.072	140°	3.576
50°	0.121	160°	6.120
60°	0.196	180°	9.929
70°	0.306	200°	15.380
80°	0.466	220°	22.882

The preceding table, due to Regnault, shows us very clearly just how the boiling point of water varies with the pressure.

BOILING POINT OF WATER UNDER VARIOUS BAROMETRIC PRESSURES

BAROMETRIC HEIGHT	0	1	2	3	4	5	6	7	8	9
	° C.									
680 mm.	96.91	96.95	97.00	97.03	97.07	97.11	97.15	97.20	97.24	97.28
690	97.32	.36	.40	.44	.48	.52	.56	.59	.63	.67
700	97.71	.75	.79	.83	.87	.91	.95	.99	98.03	98.07
710	98.11	98.14	98.18	98.22	98.26	98.30	98.34	98.38	.42	.45
720	98.49	.53	.57	.61	.65	.69	.72	.76	.80	.84
730	98.88	.91	.95	.99	99.03	99.07	99.10	99.14	99.18	99.22
740	99.25	99.29	99.33	99.37	.41	.44	.48	.52	.56	.59
750	99.63	.67	.70	.74	.78	.81	.85	.89	.93	.96
760	100.00	100.03	100.07	100.11	100.15	100.18	100.22	100.26	100.29	100.33
770	100.37	.40	.44	.47	.51	.55	.58	.62	.66	.69
780	100.73	.76	.80	.84	.87	.91	.94	.98	101.01	101.05

— From KAYE AND LABY'S *Tables.*

The preceding table, which is also due to Regnault, summarizes the results of experiment in the immediate neighborhood of 100° C. It will be found useful in determining the error of the 100° point on a mercury-in-glass thermometer.

Distinction between a Gas and a Vapor

294. With such apparatus as that indicated in Fig. 210, adapted of course to accurate laboratory measurement, it is readily shown not only that the pressure of a saturated vapor of any given substance depends upon its temperature, but also that it depends upon nothing else. Every student should clearly grasp the idea that the saturated vapor pressure does *not* depend upon the volume of the vapor. The simplest and clearest illustration of this is perhaps the following: —

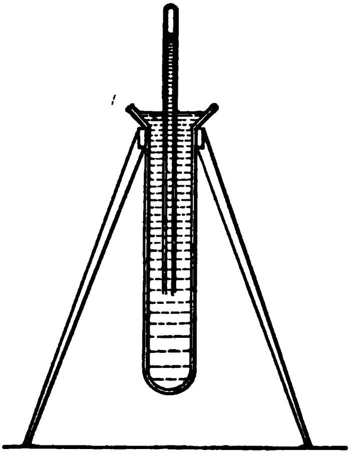

FIG. 211.

Fill a barometer tube with mercury and invert it over a deep basin of mercury as shown in Fig. 211. At the top of the tube there will be a small Torricellian vacuum. With a penfiller, having its tip bent as indicated in the accompanying figure, a few drops of sulphuric ether may be introduced into the bottom of the barometer tube. The ether at once rises to the surface of the mercury and the Torricellian vacuum becomes filled with the vapor of ether. The result is that the mercury column falls from a height of 76 to a height of perhaps 36 centimeters; for the pressure of this vapor is determined by the temperature of the room, which is probably about 20° C. The entire volume of the tube above the mercury is filled with saturated ether vapor.

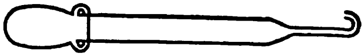

FIG. 212.

If now the barometer tube be raised or lowered in the mercury bath, it will be observed that the height of the *mercury* column inside the tube does not vary, so long as the temperature remains constant. See Fig. 213, where one and the same barometer tube is shown in three different positions. In other words, the saturated vapor pressure does not vary with the volume. How does the mass of *vapor* vary as the tube is raised and lowered?

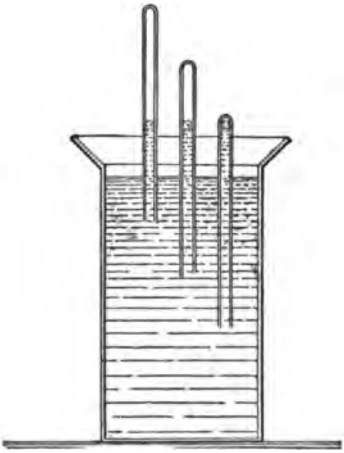

Fig. 213. — Illustrating the fact that vapor pressure is independent of volume.

Now take a similar tube, fill it with mercury, and invert it so as to form a barometer as before; but into the Torricellian vacuum introduce now a few bubbles of air, instead of ether. The behavior of the mercury column as the tube is raised and lowered is now totally different. When the volume is halved the pressure is now doubled and so on; in short, the gas obeys Boyle's law and the vapor does not.

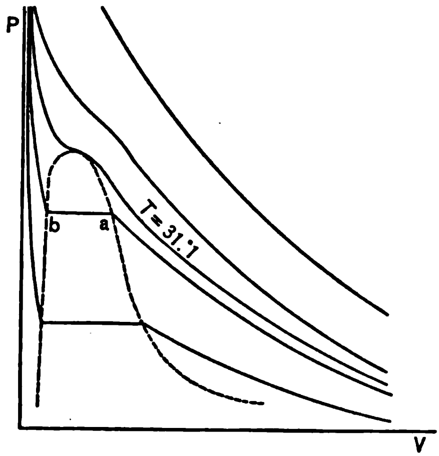

Fig. 214. — Isothermals for carbonic acid gas.

This entire question as to the difference between a gas and a vapor was cleared up about 1869 by Andrews at Glasgow, who showed that two conditions must be fulfilled in order to condense a gas into a liquid, viz.: —

(i) The temperature must fall below a certain critical point, and

(ii) The pressure must, for each particular temperature, rise to a certain definite value.

He illustrates this by plotting the pressure as a function of the volume for a given mass of carbonic acid gas, at various temperatures. When the compression occurs at a constant temperature, the curves which connect pressure and volume have the forms shown in Fig. 214 and are called "isothermal curves." The horizontal portion such as that indicated by *ab* in the figure represents a state in which the substance is partly gaseous and partly liquid. Thus at *a* the pressure assumes a constant value and retains it until the condensation reaches the point *b*. It is this constancy of pressure which, as we have seen above, indicates saturation of the vapor.

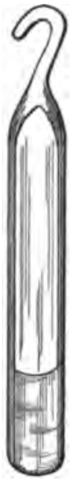
FIG. 215. — The critical tube.

Now Andrews discovered that after reaching a certain temperature (in the case of carbonic acid gas, about 31° C.) the isothermals ceased to have any region of saturation, *i.e.* they no longer showed any horizontal portions.

In other words, when condensation from a gas to liquid takes place above a certain temperature, no free surface occurs in the liquid; the change is a perfectly continuous one.

295. A very satisfactory mode of illustrating the passage of a liquid from the region below to the region above the critical temperature is by use of what is called a "critical tube," that is, a thick-walled glass tube a little less than half filled with ether and closed at both ends. When this tube is suspended in a test tube filled with melted paraffin and is heated still further with a Bunsen burner, it will be observed that presently the liquid expands very rapidly, the meniscus flattens, and finally disappears. The critical temperature has now been exceeded. The ether now fills the whole tube and has become a gas. After the flame is removed, a hazy cloud forms in the middle of the tube, a meniscus reappears, rapid contraction occurs, the critical temperature has again been passed, and the ether has now returned from a state of gas to a state of liquid and vapor.

296. The temperature of the isothermal at which the substance ceases to show a free surface during compression is known as the critical temperature. The point on the isothermal at which the horizontal portion just disappears is called the **critical point**. The pressure and the volume corresponding to this point are called the **critical pressure** and **critical volume** respectively. The *numerical value* of the critical volume will, of course, depend upon the amount of gas under experiment. It is therefore agreed, by general consent, to take for the critical volume that volume which will be assumed, at the critical point, by a mass of gas which occupies one cubic centimeter at 0° C. and 760 mm. of mercury.

We are now in a position to define "vapor" and "gas." **"Vapor" is the name given to a substance in the gaseous condition when below the critical temperature; while the term "gas" is confined to substances in the gaseous condition when *above* the critical temperature.**

When a vapor is at its maximum pressure for a given temperature, or at its minimum temperature for a given pressure, the vapor is said to be *saturated*.

TABLE OF CRITICAL DATA

SUBSTANCE	SYMBOL	CRITICAL TEMPERATURE	CRITICAL PRESSURE IN ATMOSPHERES	CRITICAL VOLUME
Air		− 140 ° C.	39	0.00468
Carbon dioxide .	CO_2	31.1	73	.0066
Carbon monoxide	CO	− 141.1	35.9	.00505
Ether	$(C_2H_5)_2O$	197	35.8	.0158
Hydrogen . . .	H_2	− 234.5	20	.00264
Nitrogen . . .	N_2	− 146	33	.00517
Oxygen	O_2	− 118	50	.00426
Water	H_2O	365	194.6	.00386

— From KAYE AND LABY'S *Tables.*

LIQUEFACTION OF GASES. LIQUID AIR

297. The principles involved in the liquefaction of gases have been well understood ever since the work of van der Waals (§ 182) and Andrews (§ 294) became known; but it was only in 1896 that a simple process for applying these principles was discovered by Linde. As early as 1885, the Russian physicist, Wroblewski, had succeeded in cooling air below its critical temperature and in applying sufficient pressure to liquefy it; but his method is complicated, slow, and laborious. About the same time, Sir James Dewar of London accomplished the same result by essentially the same method. Practically the only method in use to-day is that of Linde, which we proceed to describe.

The pressure which a gas exerts when confined in a closed vessel might lead one, at first thought, to assume that there exists between the gas molecules a repulsive force; but it is evident that this assumption is unnecessary; for the pressure may arise as we have just seen (§ 286) from the motion of the molecules. And indeed Kelvin and Joule showed by a justly celebrated experiment, in 1854, that the particles of which a gas is composed attract one another, just as all other pairs of bodies do.

The results of the Joule-Kelvin experiment were embodied by van der Waals in his expression of Boyle's law. The effect of the attraction of the gas-particles is to render the pressure of the gas a little greater than it would be if it were dependent upon motion alone; for as a matter of fact the molecules of a compressed gas are *pulled* together as well as *pushed* together. Accordingly van der Waals replaces the single factor "P" in Boyle's law by a factor of two terms, "$P + \frac{a}{V^2}$." His term "$\frac{a}{V^2}$," representing the attraction of the molecules, is called "the cohesive pressure," while his P represents the purely external pressure.

It was also recognized by van der Waals that the entire volume of the containing vessel is not available for compression; the available space is only that which remains when one subtracts from the total volume of the containing vessel the volume occupied by the molecules themselves. Hence van der Waals replaces the single term "V" by "$V - b$," and obtains:—

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT, \qquad \text{Eq. 75}$$

which is known as van der Waals' Equation.

As a consequence of this "cohesive pressure," a gas on expanding will have to do a certain amount of internal work in overcoming the attraction of each of its molecules for the others. This work is done at the expense of the heat (kinetic energy) of the gas itself. In the case of air, at ordinary temperatures, this cooling effect amounts to about ¼ ° C. for a difference of pressure of one atmosphere between the compressed air and the region into which the air escapes. The expanding gas will also be cooled in consequence of the external work which it does in lifting the earth's atmosphere: but this cooling will merely compensate the heating produced by compression, unless a part of the heat of compression has already been removed by a stream of cooling water.

298. The manner in which Linde used this fact to produce liquid air will be clear from an examination of Fig. 216. On the right are shown two pumps for compressing the air. The low pressure cylinder *H* first takes the air, already freed from carbon dioxide, partly compresses it, and then hands it over to the high pressure cylinder *F*, which brings the compression up to about 200 atmospheres.

The air, now warm from compression, next passes through the trap *G*, which catches most of the water remaining in it; thence it passes through the cooling bath, *E*, to the liquefier, which consists simply of three concentric copper tubes wound into a compact coil. At the lower end of this coil is situated a valve *B*, which connects the innermost of these tubes with the outermost. As soon as this valve *B* is opened, the air escapes from the very high pressure of the innermost tube into the low-pressure region of the outermost tube; this expansion is accompanied, as we have seen above, by a slight drop in temperature. Consequently the air which returns through the outer tube will continually reduce the temperature of the on-coming air in the inner tube. The result is that the air escaping at the valve *B* keeps getting colder and colder, owing to this fore-cooling and expansion. Presently the temperature is reduced to a point below – 140° C., which is the critical temperature of air; and, a little later, the temperature drops to – 191°, at which point air becomes a liquid, forms in drops, and falls like rain into the flask *C*, where it is collected.

299. Vessels for handling liquid air are practically all provided with double walls, so that the air between these walls can be pumped out; in this manner heat is prevented from reaching the liquid air by conduction. These vessels were invented by Sir James Dewar in 1892. Starting with a couple of quarts of liquid air, in one of the Dewar flasks, it will require more than three days of summer weather to evaporate it all. In this way liquid air is easily kept at atmospheric pressure during a time long enough for many practical purposes. The boiling point of oxygen is more than ten degrees above that of nitrogen. Which of these two gases would you therefore expect to evaporate from liquid air more rapidly?

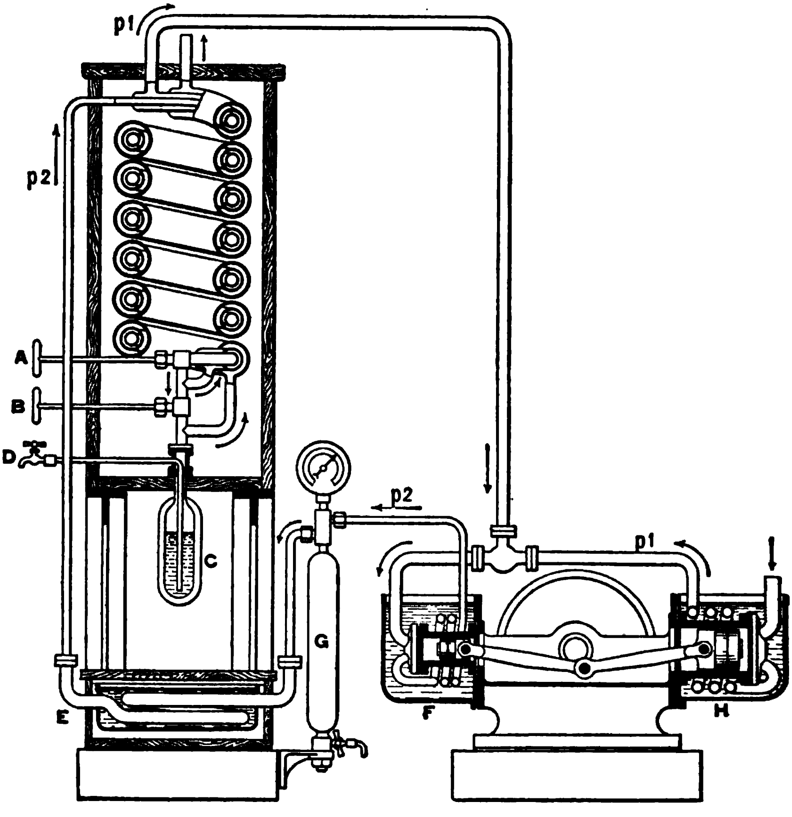

FIG. 216. — Linde apparatus for liquefying air.

300. Many meteorological occurrences with which we are most vitally concerned, as well as some of the most exquisite of natural phenomena, such as geysers, are merely special cases of change of state brought about by change of temperature. Thus *dew, fog, clouds,* and *rain* are four forms of water which make their appearance when the air is saturated with water vapor at any temperature and afterward more or less cooled below this temperature. The manner in which the necessary cooling is brought about is not always the same.

In the case of dew, the bodies upon which the moisture is deposited are cooled mainly by radiation; these bodies in turn cool the adjacent layers of air below the point of saturation, or "dew point," as it is sometimes called. The result is that the moisture formerly in the air or evaporating from the ground is now deposited over the surface of the stone, plant, or whatever the body may be. As we have already learned (§ 293), there is, for each particular temperature, a certain pressure which water vapor can sustain; when the amount of vapor in the air is at any time greater than what is sufficient to supply this pressure, the excess of moisture is naturally precipitated.

In the case of clouds and rain, the conditions which bring about the cooling of the air are, mainly: —

(i) The intermixture of two masses of air at different temperatures;

(ii) A land breeze bringing the moist air of the ocean over the cooler land.

(iii) Convection currents which lift masses of moist air into higher altitudes and thus allow them to cool by expansion.

(iv) The presence in the air of small particles of dust upon which the moisture can condense; the discovery of Aitken and Kelvin. See § 173. We now know that electrically charged ions of any gas will also serve as centers of condensation.

The Geyser

301. In the case of geysers, which were first satisfactorily explained by the great German chemist, Bunsen, in 1847, we have a crevasse, generally in an old lava bed, the heat of which is sufficient to raise the temperature of the spring water, which feeds into this crevasse, above the boiling point. The water in the lower part of the crevasse (which has usually become closed up on the sides so as to resemble a tube) is under considerable pressure, owing to the superincumbent water. Its temperature therefore rises above 100° C. If now any thin, horizontal layer of water in this tube becomes displaced — tilted perhaps — so that a part of it whose temperature is, say, 102° C., gets into a region where the boiling point is only 101°.9 C., the result is that some of the water is transformed into steam. This steam pushes some of the water out of the crevasse and thus slightly diminishes the pressure on the water below. This diminution of pressure is followed by a much larger conversion of water into steam, which in expanding expels a large quantity of water, until the entire mass is cooled to the normal boiling point or below. Much of the water is wasted and does not fall back into the tube; this waste is supplied by springs. The above process of heating and explosion is then repeated, at intervals which are quite regular, so long as the sources of heat and water remain. In the geysers of the Yellowstone Park these periods range from a few minutes to a considerable number of days.

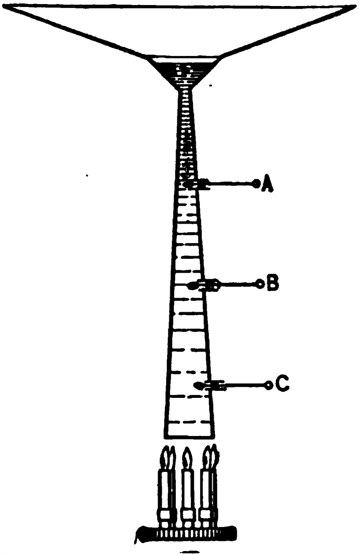

FIG. 217. — Bunsen's model of geyser.

In Fig. 217 is shown a model suggested by Bunsen, consisting of a sheet-iron tube, the upper end of which is soldered into a basin to catch the water when falling back. Three thermometers, *A*, *B*, *C*, corked into the side of the upright tube, prove that the geyser goes into action only when the temperature conditions are those described above. A triple or quintuple Bunsen flame at the bottom takes the place of hot lava in nature.

Heat of Fusion

302. It has been found by experiment that **a definite amount of heat is required just to melt one gram of a substance,** *i.e.* to change it from a solid to a liquid **without changing its temperature.** This quantity of heat is, in general, different for each different substance and is called the **Heat of Fusion** of the substance. For the last hundred years this quantity has been known as the "Latent Heat" of the substance. This name is, however, rather misleading; for, as we shall presently see, it is exactly during the present century that men have learned that heat is a form of energy which, when employed to melt a body, disappears as heat *of any kind,* and assumes some other form of energy concerning which we know comparatively little. There is possibly one justification for the name in the fact that when a substance freezes, instead of melting, this internal energy is returned in the shape of heat. Before water at 0° C. can turn into ice it must give off a very considerable amount of heat.

If H calories be required just to melt m grams of any substance, the heat of fusion F of this substance is measured as follows: —

$$F = H/m. \qquad \text{Defining equation for heat of fusion.} \qquad \text{Eq. 111}$$

In the case of ice, the heat of fusion has been found by the Bureau of Standards to be very nearly 79.75 calories per gram.

Heat of Vaporization

303. In a strictly analogous manner, experiment shows that **it takes a definite amount of heat just to change one gram of any liquid into vapor without changing the temperature.** This quantity, for any particular substance, is called its **Heat of Vaporization.**

In the case of water boiling under standard atmospheric pressure (760 millimeters of mercury), it is found that 539 calories are required to evaporate one gram without changing its temperature. The heat of vaporization of water is, therefore, said to be 539 calories per gram.

It is now clear why surf-bathing in windy weather is such a chilling process, and why damp clothes are so disagreeably cold. The heat required to evaporate a single gram of water, even after it has been heated up to the boiling point, is enormous — 539 calories — and this heat must, in the instances cited, be furnished at the expense of the human body.

VI. The Nature of Heat

304. In our study of the effects of heat, we have nowhere considered the question as to what this something is which changes the size of a body, alters its temperature, melts and vaporizes it.

Up to the beginning of the nineteenth century it was thought to be a form of matter somewhat like air, only of much greater tenuity; for in this way it was easy to explain the fact that, in conduction, the amount of heat leaving one system of bodies is always equal to that entering the other. This substance, which was given the name of "caloric," was supposed to be so "thin" that it would penetrate solid bodies with the utmost ease. It was supposed to be without weight because hot bodies do not weigh any more than the same bodies when cold.

That there is no such substance as "caloric" and that heat is a form of energy was established through the efforts of a large number of investigators. But the picture will perhaps be essentially correct if we say that this accomplishment is mainly due to the joint labors of Rumford, Davy, and Joule.

Count Rumford (1753–1814), who was by birth an American, but by residence an Englishman and a German successively, was greatly impressed by the fact that in the boring of cannon, the metal chips came out very hot. In some experiments which he tried in the arsenal at Munich he found that the heat generated by a blunt tool was proportional to the time during which the tool was driven and not proportional to the amount of abraded metal. The caloricists explained the heat which made its appearance in the chips by saying that the specific heat of brass in a finely divided condition was less than that of solid brass. But Rumford showed by experiment that this was not true; and he suggested that this "something" which a system of bodies can continue to furnish in unlimited quantities can hardly be a material substance.

Sir Humphry Davy (1778–1829), the English chemist, showed that by rubbing together two pieces of ice, in a room whose temperature was below the melting point of ice, he could not only melt the ice, but obtain a product of abrasion, so to speak, namely, water, whose specific heat, so far from being less, is more than twice as large as that of ice. Rumford's experiments were in this manner thoroughly confirmed.

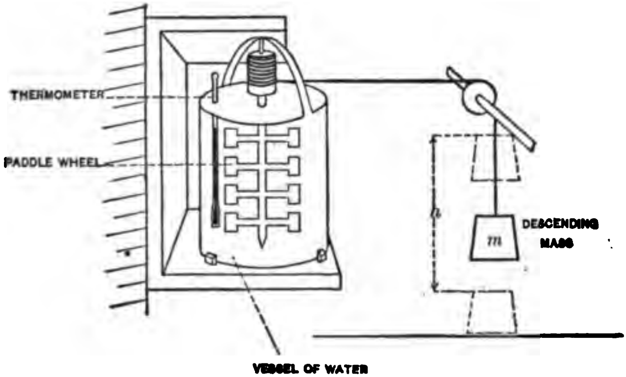

Fig. 218. — Joule's experiment to determine the mechanical equivalent of heat.

First Law of Thermodynamics

305. It remained for Mr. James P. Joule (1818–1889), an English physicist, to discover the exact relation between heat and mechanical energy. He arranged a paddle wheel in a vessel of water in such a way that the paddle wheel could be kept in rapid rotation by means of a descending clock weight. By placing a thermometer in the water, he found that the more the water was stirred, the warmer it became. **But, what is more important, he measured the work done by the descending weight and the corresponding heat produced by the paddle in churning the water, and found that in every case the ratio between these two quantities was the same.**

Let us denote by W the work done in raising the mass m (Fig. 218) through the height h; then

$$W = mgh.$$

Barring a slight correction for friction at the bearings, this is also the amount of work which the descending weight does on the paddle wheel.

Let us denote by M the mass of the water stirred, and by $t_2° - t_1°$ the rise in temperature produced by the stirring. Then the heat H acquired by the water is

$$H = M(t_2° - t_1°).$$

The capital discovery of Joule was that for every unit of mechanical energy which *disappears* as the weight descends, a definite and constant quantity of heat *appears* in the stirred water; *i.e.* H is proportional to W.

In terms of algebra,

$$\frac{W}{H} = \frac{mgh \text{ ergs}}{M(t_2° - t_1°) \text{ calories}} = J = \text{a constant}.$$

By the use of this equation, Joule determined the numerical value of this constant, J, and found it to be very nearly 42,000,000.

The physical meaning of this constant, which is called the **Mechanical Equivalent of Heat,** is as follows, viz. **42 million ergs of work must be done in order to produce one calorie of heat; in like manner, when one calorie of heat disappears, some other form of energy equivalent in amount to 42 million ergs of work always appears.**

Joule expressed his results in terms of foot-pounds, by saying that the amount of work which will raise the temperature of a pound of water one degree Fahrenheit is 772 foot-pounds. It is in this form that English engineers always use the mechanical equivalent of heat. The same result may be expressed in the metric system by writing

$$J = 424 \text{ gram-meters per calorie.}$$

The inference, then, is, since work disappears as heat appears, and *vice versa*, that heat is a form of energy. **The principle expressed in the equation**

$$W = JH, \qquad \text{Eq. 112}$$

is known as the First Law of Thermodynamics.

Having shown heat to be a form of energy, we see that **the first law of thermodynamics is merely a special case of the Law of the Conservation of Energy.**

The transformation of heat into forms of energy other than mechanical will be discussed in following chapters.

Second Law of Thermodynamics

306. About the middle of the nineteenth century Clausius and Kelvin established the general principle that **it is impossible for a self-acting machine, unaided by any external agency, to convey heat from a body at one temperature to another body at a higher temperature.** Kelvin showed, by an argument which must be reserved for advanced study, that it is impossible by use of the most perfect imaginable heat engine to convert into mechanical work more than a certain definite fraction of the heat in any body. Let us suppose an engine working under the most favorable circumstances and receiving an amount of heat H_1 at a temperature of t_1. Every heat engine disposes of a part of the heat which it receives to a body at some lower temperature t_2. Let us denote by H_2 this heat given out by the engine to the refrigerator whose temperature is t_2. Then Kelvin showed that, if temperatures are measured on the ideal gas scale, the maximum possible value for the ratio between useful work and heat taken in — a quantity generally called "efficiency" — is $\frac{t_1 - t_2}{t_1}$. Or, putting this same result in terms of algebra, we have

$$\text{Efficiency} = \frac{H_1 - H_2}{H_1} = \frac{t_1 - t_2}{t_1}. \qquad \text{Eq. 113}$$

This efficiency equation may be considered as a quantitative expression for the second law of thermodynamics.

Absolute Scale of Temperatures

307. Still another way of putting this result is to say that the efficiency of the most perfect heat engine working under the most favorable conditions is a function only of the temperatures of the source and refrigerator respectively.

One of the important applications of the second law of thermodynamics is the determination of temperature by a method which is independent of the particular thermometric substance employed. Such temperatures are said to be "absolute." An outline of the method by which this is done is as follows: Eq. 113 may be transformed by simple algebra so as to read

$$\frac{H_1}{H_2} = \frac{t_1}{t_2}. \qquad \text{Eq. 114}$$

With clear vision, Kelvin saw that the ratio $\frac{H_1}{H_2}$ is independent of the material, and being a function of the temperature alone, may be used as a measure of the ratio of two temperatures on a new and absolute scale.

The experimental determination of the numerical value of this ratio $\frac{H_1}{H_2}$ for a substance working between the boiling and freezing points of water was made by Kelvin and Joule, and found to be $\frac{373}{273}$. This is, of course, also the value of $\frac{t_1}{t_2}$, on the ideal gas scale, for the same two fixed points. In terms of algebra

$$\frac{t_1}{t_2} = \frac{373}{273}. \qquad \text{Eq. 115}$$

In order to definitely determine either of these two temperatures on the absolute scale, it remains only to discover another equation between them. This we have in the universal convention that the difference between the melting point of ice and the boiling point of water shall be 100°, or, calling t_1 the temperature of steam and t_2 the melting point of ice,

$$t_1 - t_2 = 100°. \qquad \text{Eq. 116}$$

Eliminating t_1 and t_2 respectively between these last two equations, we have

Temperature of melting ice $= t_2 =$ 273° on the absolute centigrade scale, and
Temperature of boiling water $= t_1 =$ 373° on the absolute centigrade scale.

It thus appears that the zero on the absolute scale is identical with that on the ideal gas scale.

From Eq. 113 it follows that **the absolute zero is simply the temperature of a refrigerator at which a perfect engine would convert into mechanical work *all* the heat supplied to it.**

Observe that the *size* of the degrees here employed is determined by Eq. 116, and that they are centigrade. If $t_1 - t_2$ had been fixed at 180°, the size of the degrees would have been Fahrenheit and the absolute zero would then have been 461° below that of melting ice.

Heat Engines

308. We have seen how Joule transformed mechanical energy into heat by means of his raised weight and paddle. The heat engine is a machine devised to accomplish the reverse operation, viz. to transform heat into mechanical energy. The most important of these engines are included in the three following groups: —

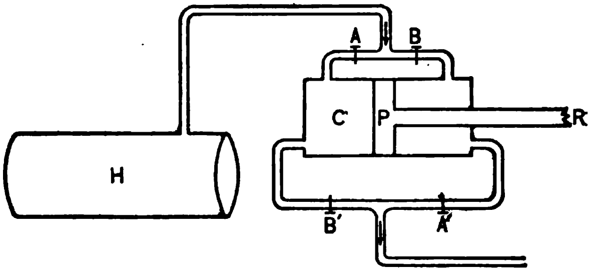

FIG. 219.

(i) *The reciprocating steam engine.* This is the ordinary steam engine, of which the common railroad locomotive is an example. Its essential features are the following: a *boiler*, *H*, in which heat, furnished by combustion of fuel, is employed to evaporate water; a *cylinder, C,* in which slides a close-fitting *piston head, P,* and a *piston rod, R,* as indicated in Fig. 219; a steam chest containing an automatic valve which admits the steam alternately to one end and to the other of the cylinder. By means of a connecting rod this to-and-fro motion of the piston is transformed into a circular motion of the shaft and pulley in a manner which will be clear from an inspection of Fig. 220. This form of engine is very largely the creation of the Scottish engineer, James Watt.

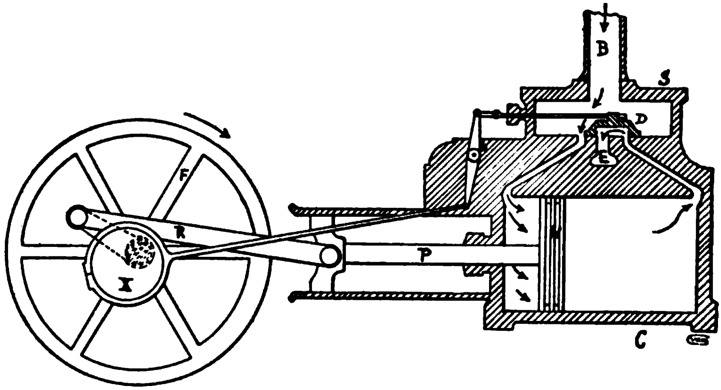

FIG. 220. — The idea of the steam engine. *B*, pipe from boiler; *S*, steam chest; *D*, slide valve; *C*, cylinder; *E*, escape pipe; *H*, piston head; *P*, piston rod; *R*, connecting rod; *F*, flywheel; *X*, eccentric.

In marine engines (and in some others also) the exhaust steam is collected, condensed into water, and again returned to the boilers. The same water is thus used over and over again. The steam is not used up, but is merely a "working substance," a carrier to transfer the energy of the combining coal and oxygen in the furnace to the machinery which is doing mechanical work.

The student who would know more of the history or the theory of the steam engine should consult the very readable article of Professor Ewing on this subject in the *Encyclopædia Britannica;* also the biographical sketch of James Watt there given.

(ii) *The steam turbine.* When water escapes from pressure, its energy becomes kinetic — $\frac{1}{2}mv^2$. If a stream of water, say a slender jet, is directed against a flat surface, such as a door, it exerts a pressure upon this surface. If the door gives way and moves in the direction of the force, work is done upon it — *Fs* — and this work is performed at the expense of the kinetic energy of the moving water. For many years the miners of the west used a form of water-wheel — the hurdy gurdy — made up of flat vanes attached to a shaft. This wheel was made to run by playing a jet of water on the vanes. The Pelton waterwheel of the present day employs the same principle, but by use of *curved* vanes is able to utilize a much larger proportion of the kinetic energy of the water.

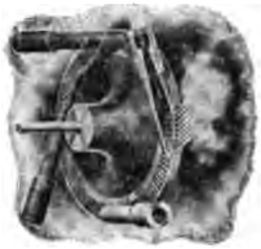
Fig. 221. — The De Laval steam turbine.

When steam escapes from pressure the velocity factor of its kinetic energy is very great. Consequently a jet of steam directed against a resisting surface exerts upon it a force similar to that of the wind, with which we are all familiar; but the force due to a jet of steam is likely to be of far greater intensity.

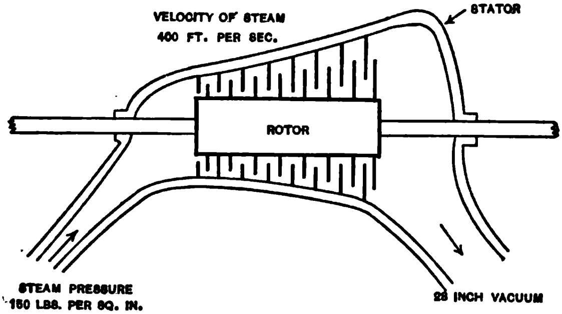

Fig. 222. — Longitudinal section of Parsons turbine.

A rough sort of steam engine may be made by running steam through an ordinary Pelton water-wheel; while the De Laval steam turbine is simply the Pelton wheel properly designed for the use of a light and expansive medium — steam.

The disadvantage of the De Laval wheel is that it must run at an excessively high speed in order to absorb a large fraction of energy from the steam which has freely expanded and has thereby acquired an enormous speed.

We have seen that in the ordinary reciprocating engine the steam does work by expanding the volume of the space in which it is confined.

In the Parsons turbine this principle is also made use of to some extent in order to absorb the energy from the steam and yet to keep the speed of the steam down to about four hundred feet per second.

To accomplish this the cylinder (stator), instead of being fitted with a piston, is provided with a coaxial shaft (rotor) which is covered with small blades *inclined to the axis of the shaft,* on the same principle that the blades of a windmill are inclined to the direction of its shaft. Corresponding to each movable blade on the rotor, there is a *fixed* blade on the *inside* of the stator.

Accordingly we may think of the passage of the particles of steam between these blades as similar to their passage through a long rectangular pipe, one side of which is constantly receding from the other and is therefore absorbing energy. The impulse principle — that of the De Laval turbine — is also employed in the Parsons to abstract still another portion of the energy of the steam.

This type of engine is especially adapted for the smooth running of steamships and dynamos. The steam turbine owes its initial development largely to the English engineer, Parsons.

(iii) *The internal combustion engine.* The two preceding engines derive their energy from fuel which is burned outside the cylinder. But there has recently been perfected a type of machine which burns its fuel (either gas or a spray of oil) right inside the cylinder. When the proper fuel is used, the products of combustion are entirely gaseous, so that the cylinder easily clears itself. These are the little engines so familiar to us on automobiles. In most of them there are four steps through which the " working substance " is taken. These are indicated in Fig. 223. These four steps constitute what is known as the " Otto cycle," after the German engineer, Otto, who did much to perfect the modern gas engine. In these engines the piston does work only when moving in one direction; they are therefore said to be "single acting," to distinguish them from the ordinary reciprocating steam engine, which does work both on the forward and on the backward stroke, and is hence said to be "double acting." The use of intake and exhaust valves *I* and *E*, shown in Fig. 223, will be sufficiently clear from the diagram. The intake valve admits the explosive mixture — say, air and gasoline vapor; compression then occurs; explosion is then produced by an electric spark; the products of combustion, CO_2, etc., are then swept out through the exhaust valve *E*. These are the four steps in the Otto cycle. Nearly all motor cars (and aëroplanes) use four-cycle engines of this type. Naturally the cylinders get very hot and have to be cooled with water. The water-jackets are not shown in the figure. The water, in turn, gets hot and has to be cooled. For this purpose motor cars are provided with radiators.

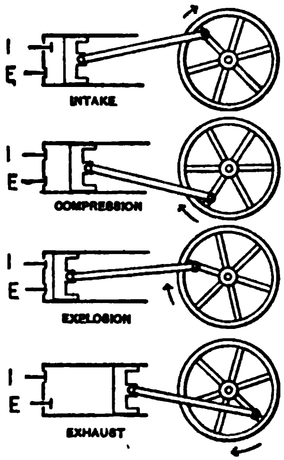

Fig. 223.

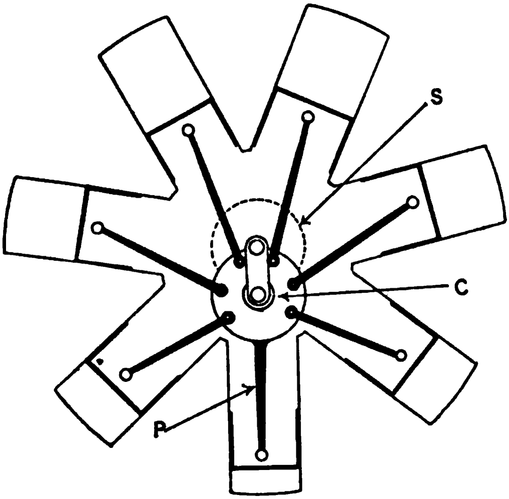

Fig. 224. — The Gnome gasoline engine, widely used in aëroplanes.

One of the most remarkable engines ever designed is the Gnome motor shown in cross section in Fig. 224. Here the cylinders instead of being fixed are free to rotate about a fixed crank-shaft. The cylinders are seven in number and rigidly connected. The section of the crank-shaft is shown in dotted line at S. The piston rod of one cylinder — the one marked P — ends in a large disk. This disk is capable of rotating about the fixed crank-pin C. It has therefore a center of rotation which is different from the center of rotation of the cylinders. All the piston rods, except P, are linked to this disk by means of pins. The crank-case contains the explosive mixture, which is led in through the hollow crank-shaft.

The propeller blades of the aëroplane are rigidly attached to the rotating set of cylinders. The standard speed is about 1200 revolutions per minute. The rapid air-cooling produced by the rotation of the cylinders gives to this motor an enormous advantage over the type having fixed cylinders. Engines of the Gnome type are made to weigh not more than 5 lb. per horse-power.

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Problems

306. The average reading of the barometer at the top of Mount Hamilton (the Lick Observatory) is 65 cm. At what temperature, therefore, does water boil on Mount Hamilton? *Ans.* 95° C. approximately.

307. At the top of Pikes Peak water boils at a temperature of 86°. What, therefore, is the average reading of the barometer on Pikes Peak? *Ans.* 45.68 cm.

308. The pressure of the atmosphere is approximately 15 lb. weight to the square inch. The boiler of an Atlantic liner carries a stream pressure of 300 lb. to the square inch. What is the temperature of the water in this boiler? See Regnault's table. *Ans.* 212°. 3 C.

309. At a place where the acceleration of gravity is 980, a bullet whose mass is 8 g. is allowed to fall from a height of 1000 cm. What will be the kinetic energy of the bullet in ergs just before it strikes the ground? To what fraction of a calorie will this mechanical energy be equivalent? *Ans.* 7,840,000 ergs = 0.187 calorie.

310. Suppose, in the preceding problem, that when the bullet strikes the ground one half its kinetic energy goes to heat the bullet, the other half to heat the ground. If the specific heat of lead is $\frac{1}{30}$, by how much will the temperature of the bullet be raised? *Ans.* 0°.34 C.

311. (*a*) The height of the American Falls at Niagara is 50 m. How much kinetic energy will a gram of water acquire in falling through this distance? ($g = 981$.) *Ans.* 4,905,000 ergs.

(*b*) What will be the equivalent of this energy in heat units? *Ans.* 0.117 calorie.

(*c*) By how much then will the temperature of the water at the bottom be increased over that at the top? *Ans.* 0°.117 centigrade.

312. To raise a mass of 10 lb. through a vertical distance of 8 ft. (80 foot-pounds) requires how many ergs of work? *Ans.* 1079 million ergs.

313. Assuming that the specific heat of iron is 0.12, find the number of heat units required to raise the temperature of 300 g. of iron from 20° C. to 360° C.

314. Why is it that alcohol or ether poured on one's hand gives him the sensation of cold?

315. How much heat must be given to 12 g. of water at 40° C., in order to make it boil?

316. Why are damp clothes apt to make one chilly?

317. Ten lbs. of water at 0° C. are mixed with 40 lbs. of water at 50° C. Find the temperature of the mixture.

318. Give five illustrations of the transformation of some form of energy into heat.

319. What is meant by a "land breeze"? By a "sea breeze"? How do you explain these?

320. Write an examination paper of five questions on the subject of heat.

321. A harvesting machine, made partly of wood and partly of iron, is allowed to stand overnight in an open field. In the morning, which parts will be covered with dew, the iron or the wood? Why?

322. What are clouds and how are they produced?

323. Why is it that in winter ice which is partly covered with dirt or ashes melts sooner than clean ice?

324. Why are teapots often provided with wooden handles?

325. Fifty cubic centimeters of water are heated from a temperature 15° C. to 35° C. Find the change in volume thus produced.

326. A steel rail 10 m. long will change its length by how many millimeters when heated from 0° C. to 50° C.?

327. Why do you pour hot water on the neck of a bottle when you wish to loosen the glass stopper?

328. Explain how the temperature of a greenhouse, not heated artificially, rises above that of the surrounding air.

329. What is the theoretical efficiency of a steam engine whose boiler is at a temperature of 160° C. and whose condenser is at 40° C.?

330. Suppose the temperature of combustion in the cylinder of a gasoline engine to be 1600° C. and the temperature of the exhaust 350° C. Find the highest possible efficiency.

331. How would you proceed to graduate an electrical resistance thermometer?

332. Sketch the isothermals of carbon dioxide and show how they differ in form above and below the critical point.

333. Sketch on the blackboard the four steps of the Otto cycle used in gas engines.

334. Place on the blackboard a diagram of the Gnome motor and explain its action.

> "But truth is one: it has first to be discovered, then justly and exactly uttered."
>
> ROBERT LOUIS STEVENSON,
> *Virginibus Puerisque.*

CHAPTER VIII

MAGNETISM

309. The properties of magnets and electric currents are so intimately connected that no intelligent study of one can be made without some acquaintance with the other. Accordingly the subject of magnetism is here taken up partly on its own merits and partly as a study preparatory to that of electric currents.

Most of the elementary facts concerning magnetism were known in the time of Queen Elizabeth, some two hundred years before the discovery of electric currents. The first great treatise on magnetism was Dr. Gilbert's *De Magnete*, published in 1600; while the discovery of the voltaic cell by Volta was announced to the Royal Society of London as recently as the year 1800.

310. Any body which attracts iron filings is said to be magnetized, and is called a magnet. Those parts of the body which attract the iron filings most strongly are called **magnetic poles,** a name given by Peter Peregrinus, a Picard knight, in 1269. The earliest magnets known were pieces of iron ore which we now call *magnetite*. In earlier times, they were called *lodestones*. But at present practically all of our magnets are made from iron or steel. In its magnetic property, no other substance approaches iron: nickel and cobalt can, however, be made into weak permanent magnets.

LEADING FACTS OF MAGNETISM

311. The following eight experimental facts, presented in essentially the order in which they were discovered, are perhaps the most important phenomena connected with this subject: —

1. **If a piece of magnetized iron be freely suspended at any point, it will always set itself so that a certain direction in it makes a fixed angle with the north-and-south line at that point.**

This fact is illustrated by the ordinary pocket compass and indeed constitutes the fundamental principle of that instrument.

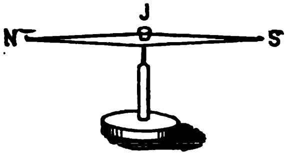


FIG. 225. — The ordinary compass needle.

To make a compass one takes a slender piece of steel, magnetizes it by drawing it over another magnet, and mounts it on a sharp pivot bearing in a jewel cap, *J*, Fig. 225. The needle, *NS*, is now free to turn in any horizontal direction. This idea of the compass, which dates from a time certainly not much later than 1200 A.D., is the *first* important discovery in the science of magnetism.

The mariner's compass devised by Lord Kelvin in 1876 is practically the only one in use on sea-going vessels to-day. As indicated in Fig. 226, the needle is not made out of a single piece of sheet steel, but is composed of eight slender magnets, *NN* in the figure, supported rigidly on a sort of spider-web of 32 radial threads running from the jewel cap to the rim of the compass-card. This rim, *A*, is made of aluminium, which gives lightness and rigidity to the moving system, which is made up of the card, the threads, the magnets, and the pivot bearing. Most of the mass being in the rim, the moment of inertia of the card is relatively large, in spite of the fact that the total weight of the moving parts is generally less than 15 grams. The result of this is that the period of vibration of the card is long; its motions are slow and steady.

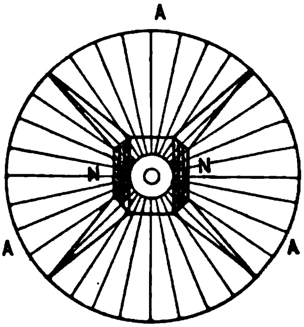


FIG. 226. — The ship's compass.

SOME DEFINITIONS

312. The pole which turns toward the north when the magnet is free to rotate, is called the north-seeking pole, or simply the north pole; that pole which turns toward the south is called the south pole.

The direction of the line joining the poles is called the magnetic axis of the magnet. This definition of "axis" will later need a slight revision, since the poles have not yet been defined as mathematical points.

The direction assumed by the axis of a freely suspended magnet is called the magnetic meridian.

The angle between the magnetic and geographical meridians is called the magnetic declination. It is generally expressed in degrees, minutes, and seconds. In making an accurate determination of the magnetic declination, one must be careful not to assume that the magnetic and geometric axes of the needle coincide. In the Kelvin compass used by mariners, these two axes are made to coincide very nearly by use of long slender needles. In measurements on land any uncertainty as to the position of the pole in the magnet is eliminated by using a thin flat compass needle and by observing first with one face up, then with the other. Thus in Fig. 227, if *GG* be the first position of the compass needle and *G'G'* its second position, we shall find that while the magnetic axis remains fixed in space whichever face of the needle is up, the geometric axis does not. But the directions of the geometric axis *GG* and *G'G'* are those which are actually read upon the graduated circle. Hence the *difference* of the two readings, before and after the needle is reversed in its bearings, will give us twice the angle by which the magnetic and geometric axes diverge. Not only so but the *mean* of these two readings will give us the correct direction of the magnetic meridian. One thus avoids the necessity of locating

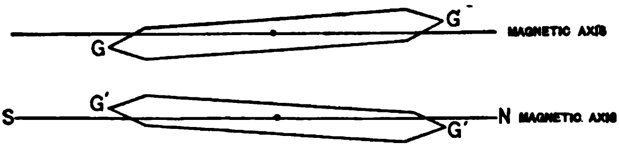


FIG. 227. — Illustrating the difference in direction between geometric and magnetic axes.

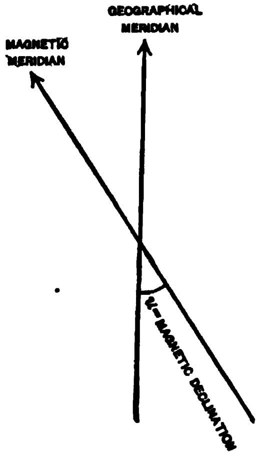


FIG. 228.

the magnetic axis as a mathematical line in the body of the needle.

A little later it will be clear how we may avoid the necessity of locating the magnetic *pole* as a mathematical point, situated on the magnetic axis at a certain distance from the axis of rotation.

313. 2. The Discovery of Columbus. Columbus used the compass on his first voyage to America. Up to this date it had been supposed by European navigators that while the suspended magnet in the compass did not always point exactly north and south, it did always point in the same direction; *i.e.* they supposed the angle between the geographical and magnetic meridians to be constant. But Columbus, much to his own astonishment, and much to the dismay of his sailors, found that as he proceeded west, the needle pointed to a part of the sky which lay more and more to the west. He thus **discovered that the magnetic declination varies from one point to another on the earth's surface.** This we may call the *second* important discovery in magnetism.

Another way of putting it would be to say that what Columbus discovered is that the general statement contained in (§ 311) is true only for a single geographical location; or more strictly, for a single line on the earth's surface.

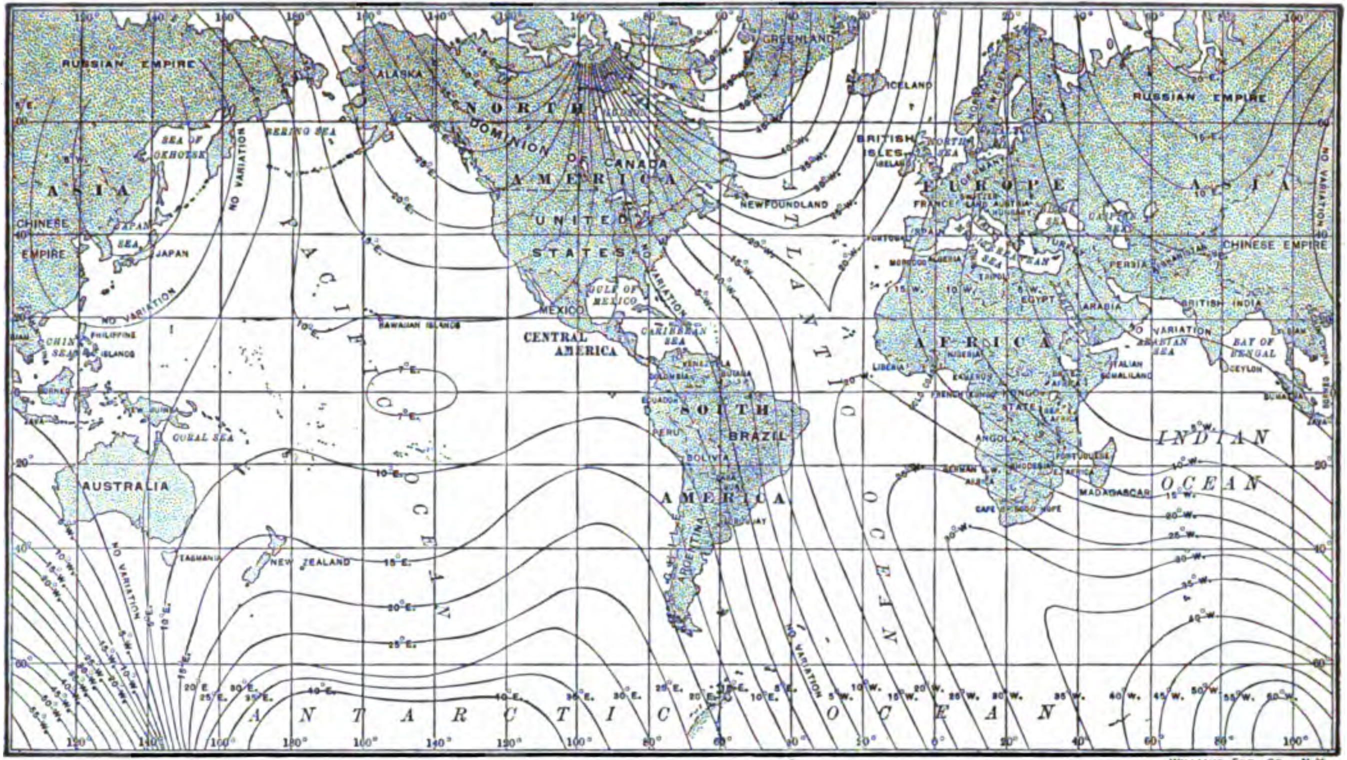
Curves drawn in such a way that the magnetic declination is constant for every point on them are called "isogonic curves." The accompanying map will show the trend of these isogonic lines in various parts of the world. Note especially the line of "no variation" — the so-called *agonic line* — which passes through Knoxville, Tenn.

MAGNETIC DECLINATION (1900)

Augusta, Me.	16° W.	New Orleans	5° E.
Boston	12° W.	Kansas City	8° E.
New York	9° W.	Omaha	10° E.
Cleveland	2° W.	Denver	14° E.
Knoxville, Tenn.	0°	San Francisco	17° E.
Chicago	3° E.	Tacoma, Wash.	23° E.

The preceding table shows the values of the declination in different parts of the United States, to the nearest degree. "W" indicates that the north pole of the compass needle takes up a

THE DECLINATION OF THE COMPASS, FOR THE YEAR 1910.



WILLIAMS ENG. CO., N Y.

FIG. 229. — Map showing the direction in which the compass needle points at various parts of the earth's surface.

position on the west side of the geographical meridian passing through the center of the needle. "E" has a corresponding meaning for the east side.

314. 3. The **magnetic dip.** Not long after the time of Columbus it was found by George Hartmann (b. 1489, d. 1564) of Nuremberg, that if an iron needle be suspended so as to lie *horizontal* before magnetization, it takes up a different position after magnetization. Imagine the needle before it is magnetized to be horizontal. After magnetization it is found that the north end "dips" down as if it had become heavier than the south end. In 1576, Robert Norman, a compass-maker of London, invented the dipping needle shown in Fig. 230. He also weighed the iron needle and found that it was no heavier after magnetization than before. As we shall see later, this dip is due to the fact that the earth itself is a great magnet. **Only the direction, therefore, and not the weight of the needle is changed by magnetization.**

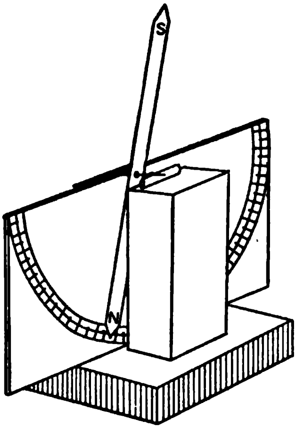


FIG. 230. — Norman's dipping needle.

In the southern hemisphere it is the south end of the magnet which "dips." In the United States it is customary to attach a small weight to the south end of a surveyor's compass needle in order to counteract the effect of the dip.

The phenomenon of magnetic dip might be called the *third* important discovery in this subject.

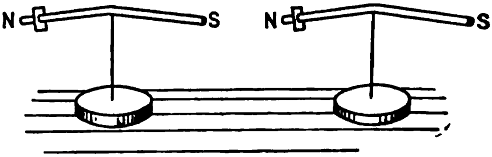


FIG. 231. — Showing attraction between north and south poles.

315. 4. **Like poles repel each other; unlike poles attract each other.** The evidence for this may be shown in a variety of ways; perhaps the most convenient method is the following: —

Mount two magnets, as shown in Fig. 231. Determine the north end of each, and slip bits of paper over them as indicated.

When one magnet is brought near the other, the repulsion between the two north poles or between the two south poles is very evident; likewise the attraction between a north and a south pole.

Norman magnetized a sewing needle, and floated it on water. By bringing into the neighborhood another magnet, he had a delicate means of showing attraction and repulsion of the poles.

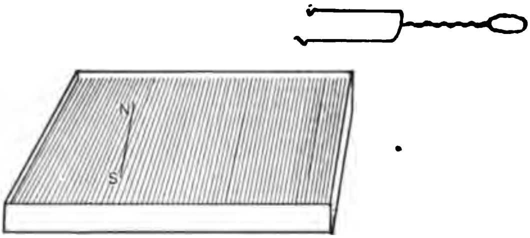


FIG. 232. — Floating magnet, and wire for placing it on the water.

Concerning the *amount* of this attraction or repulsion, it was first shown in 1800 by the French electrician, C. A. Coulomb, and afterwards accurately verified by the German astronomer Gauss (1777–1855), that the attraction of one pole for another, *in air*, varies inversely as the square of the distance separating them and directly as the products of the pole strengths — a quantity which will be defined a couple of pages hence. If we denote this distance between poles by r, and the pole strengths by m and m', respectively, we may describe Coulomb's result by writing: —

$$F = \frac{kmm'}{r^2}, \quad \text{Coulomb's Law.} \qquad \text{Eq. 117}$$

where, for air, k is a mere constant of proportionality which may be determined later. Coulomb established this law by the use of long slender magnets so arranged that the effect of the distant poles was negligible.

The Magnetic Field. Lines of Force

316. If a sheet of white paper be laid over a magnet, and iron filings be sprinkled over the paper, one obtains very striking evidence that the region about the magnet is different from other parts of space. The iron filings arrange themselves in curved lines; and these curved lines remain the same, however many times we repeat the experiment. Any region, such as the neighborhood of a magnet, where a freely suspended compass-needle tends to point in a definite direction is called a **magnetic field.**

" We next introduce a method of conceiving and describing magnetic actions which was invented and much used by Faraday. Since a magnet acts upon a magnetic needle placed anywhere in the surrounding space, we call that space the magnetic field of the magnet. Neglecting the earth's magnetism, we may map out this field as follows: Conceive any plane drawn through the axis of the magnet and place it so that this plane shall be horizontal. Then, at any point in this plane, place a very small magnetic needle, and note the direction which its axis assumes under the action of the magnet; then proceed to move the center of the needle in the direction in which its north pole points, and continue the motion so that, at each point, the center is following the direction indicated by the north pole.

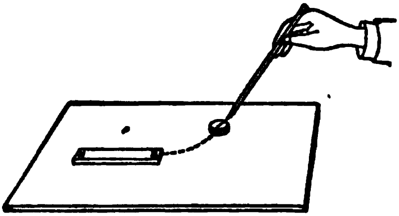


FIG. 233. — Method of mapping magnetic fields.

" The line thus traced will at last cut the surface of the magnet at some point lying toward the south pole; and if we continue the line backward by following the direction continually indicated by the south pole of the needle, it will cut the surface of the magnet at some point lying toward the north pole. Such a line is called a **line of magnetic force**; and since one such line can be drawn through every point in the plane, and any number of planes can be drawn through the axis of the magnet, we can conceive the whole magnetic field filled with such lines." — CHRYSTAL, *Ency. Brit.*, art. " Magnetism." (9th Ed.)

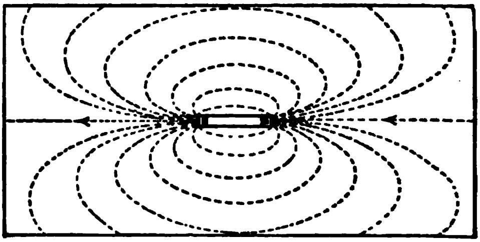


FIG. 234. — Field about a straight magnet.

Figure 233 indicates the process of mapping these fields.

Figure 234 shows the field about a strong bar magnet. The effect of the earth's field is here negligible.

Figure 235 represents a small portion of the field due to the earth alone.

A magnetic field as actually mapped in the laboratory will always be the resultant of the earth's field and the field of the magnet.

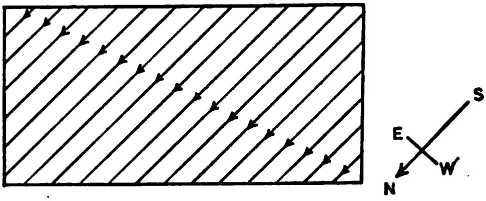


FIG. 235. — Uniform magnetic field due to earth alone.

Fairly uniform magnetic fields can also be produced by placing opposite each other a north and a south pole as shown in Fig. 236. The region through which any artificial magnetic field may be considered uniform is, of course, very limited.

Intensity of Magnetic Field

317. So far we have considered only the fact that a magnetic field has, at every point, a certain definite direction. If, however, a small horseshoe magnet be brought into the field of a large one, and held between the thumb and forefinger, the observer will easily "feel" that the little magnet experiences a *force* driving it toward one pole or the other of the larger magnet, always tending to move into the strongest part of the field. Observe that in those parts of the field where the lines of force are nearly parallel to each other (*i.e.* in a uniform field), the force is very minute.

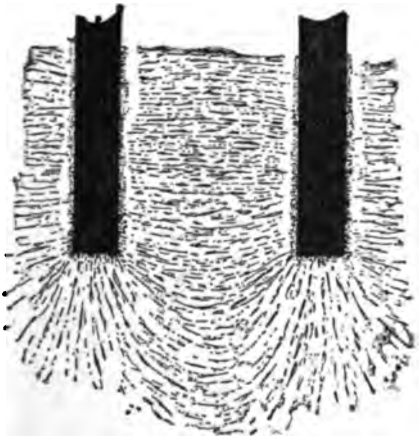


FIG. 236. — Showing a fairly uniform artificial magnetic field.

Suppose now that the little magnet be mounted upon a sewing needle as a vertical axis after the manner shown in Fig. 238.

The needle may well be placed in a block of wood so that the rotating system may be fixed at various heights above the poles of the larger magnet. Now the *force* of translation is counteracted by the reaction of the needle, and one can feel in a very convincing manner the *torque* which a magnetic field in general exerts upon a magnet placed anywhere within it.

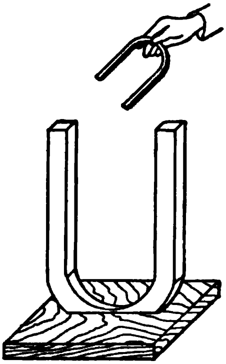


FIG. 237. — Illustrating the force which a magnetic field exerts upon a magnet.

Observe also that the little magnet in Fig. 238 vibrates much more rapidly as its poles are brought more and more on a level with the poles of the big magnet. And just as the period of a clock pendulum diminishes when that pendulum is carried from a weaker to a stronger gravitational field, so this small magnet speeds up in coming from a weaker to a stronger magnetic field. Such a device *might* be used for measuring (comparing) the intensities of magnetic fields, and, indeed, frequently is. But for the measurement of a magnetic field in absolute units, say in centimeter-gram-second units, the following method of approach is simpler.

Definition of Unit Magnet Pole

318. To begin with we assume that we have two equal magnetic poles, situated in air and separated from each other by a distance of one centimeter. **If under these conditions the two equal poles be chosen of such a size that they attract or repel each other with a force of one dyne, they are said to be unit magnet poles, or poles of unit strength.**

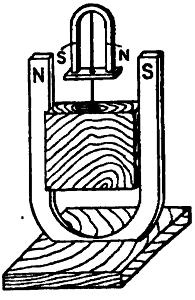


FIG. 238.— Illustrating the torque which a magnetic field exerts upon a magnet.

The chief advantage of this definition, which may at first glance appear rather arbitrary, is that it makes the constant k disappear from Coulomb's Eq. 117. For having assumed m to be unity when $F = r = 1$, and $m' = m$, it follows that in air k must always be unity, a unit vector, indeed; whence follows

$$F = \frac{mm'}{r^2}, \qquad \text{Eq. 118}$$

which now becomes not only the law of Coulomb, for air, but also the defining equation for the unit magnet pole.

This law of attraction and repulsion of magnet poles we may call the *fourth* fundamental discovery in the science of magnetism.

Measure of Magnetic Intensity

319. We are now, for the first time, in a position to define what is meant by **the intensity of a magnetic field, namely, the ratio of the force which a magnet pole experiences in that field to the strength of the pole.**

Let R denote the intensity of a magnetic field at any point, and F the force with which the pole m is urged at that point. Then

$$R = \frac{F}{m}. \qquad \text{Defining equation for intensity of magnetic field.} \qquad \text{Eq. 119}$$

Field intensity, being a ratio of a vector to a scalar quantity, is itself a vector quantity, and is therefore to be resolved and compounded according to the rules for vector addition. From Eq. 118 it follows that **a field of unit intensity is one which will act upon a unit magnet pole with a force of one dyne.** By international agreement (Paris, 1900) this unit is called, after the eminent German astronomer, a **gauss.**

We are now provided with a clear and simple definition for the intensity of field; but since we have as yet no practical method for measuring pole strengths, Eq. 119 cannot be called a laboratory equation. Just how this difficulty is avoided will be cleared up in the following sections when we come to consider the earth's magnetic field.

320. 5. **The earth itself a great magnet.** We have seen that at any one point on the earth's surface the compass needle points in a definite direction, generally not far from north. We have seen that, in the northern hemisphere, the north end of the needle dips; while in the southern hemisphere, the south end dips.

But this is exactly the manner in which a small compass needle would behave in the presence of a large iron sphere which had been magnetized. Accordingly Dr. Gilbert* was led to suggest that the earth itself was a large magnet. And since his time, the theory has been confirmed by navigators who have located the north magnetic pole of the earth. In 1831, Sir James Ross found a place in the extreme north of Canada where he observed the dip of the compass needle to be 89° 59′. This has ever since been accepted as the north magnetic pole of the earth.

Its location is in latitude 70° 5′ and almost due north of Omaha, Nebraska. The south magnetic pole lies between Australia and the south geographical pole not very far from latitude 72° 41′ S.

If we map the magnetic field of the earth by means of compass and pencil as indicated in a preceding paragraph, we shall find that **the lines of force due to the earth are, in any limited region, practically parallel to each other.**

321. 6. The earth's action a couple. In the first chapter of this volume we have considered the distinction between a motion of translation and one of rotation. To a system of forces which produce rotation only the name **couple** has been given. **Norman offered the following three experiments to show that the earth does not exert any force of translation upon a magnet,** or, as we might say, three reasons for thinking that the force which the earth *does* exert upon a magnet is a *couple.*

" First, he weighed several small pieces of steel in a delicate balance and then magnetized them, but could not detect the slightest alteration in their weight, ' though every one of them had received virtue sufficient to lift up his fellow.'

" Secondly, he pushed a steel wire through a spherical piece of cork, and carefully pared the latter so that the whole sank to a certain depth in a vessel of water and remained there, taking up any position about the center indifferently. After the wire was magnetized very carefully, without disturbing its position in the cork, it sank to the same depth as before, neither more nor less, the only difference being that now the wire set itself persistently in a definite fixed direction parallel to the magnetic meridian, the north end dipping about 71° or 72° below the horizon.

---

* *Dr. William Gilbert* (b. 1540, d. 1603), the leading man of science in England during the reign of Queen Elizabeth, also the first advocate of Copernican views in England.

"Thirdly, he arranged a magnetized needle on a cork so as to float on the surface of water, and found that, although it set in the magnetic meridian, there was not the slightest tendency to translation in any direction.

"He concludes that there is no force of translation on the magnet, either vertical or horizontal." — Chrystal, *Ency. Brit.*, art. "Magnetism." (9th Ed.)

Recalling the fact that a couple is made up of two parallel and equal forces acting in opposite senses, but not in the same straight line, we conclude that the earth's action is a couple precisely because its lines of force, throughout the limited region of any single experiment, are parallel.

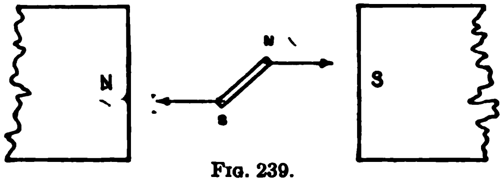


Fig. 239.

The effect of the earth's field upon a magnet is shown in Fig. 239, where the small magnet is supposed to be placed in a uniform field such as that shown in Fig. 236.

If the dish of water, shown in Fig. 240, be supported over a rather strong permanent magnet, and if the magnetic needle be floated on a small cork, it will be seen that this artificial field exerts both a couple and a force upon the floating magnet, continually drawing the magnet back toward the strongest part of the field. The explanation of this lies in the fact that here the force acting upon the *north* pole of the floating needle is *not* exactly opposite in direction to that acting upon the south pole. Hence the resultant of these two forces cannot be zero. If, however, the magnet be removed from underneath the dish of water it will be observed that the floating magnet behaves

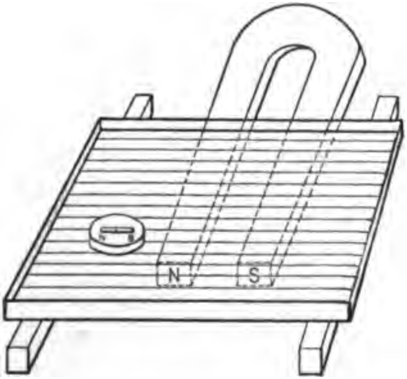


Fig. 240.

precisely in the manner described by Norman. The advanced student will find that the floating magnet here simply moves in such a way as to make its potential energy a minimum.

Digression on the Measurement of the Earth's Magnetic Field

322. On both historical and scientific grounds the following problem is one worthy of rather careful study; for not only is it the first problem in which the absolute system of units was introduced into the science of electricity and magnetism, but it is a thoroughly typical example of the manner in which physical science deals quantitatively with phenomena which are very imperfectly comprehended.

As a preliminary it is necessary to define

Magnetic Moment,

a quantity of frequent use in magnetic measurements. **The magnetic moment of any magnet may be defined as the product of its pole strength m by the distance l between its poles.** This quantity is generally denoted by M, and is defined by the following equation: —

$$M = ml. \quad \text{Defining equation for magnetic moment.} \quad \text{Eq. 120}$$

The laboratory equation for measuring M is Eq. 127, p. 370.

Horizontal Component of the Earth's Field

323. We have already learned, in a general way at least, how the declination and the dip of the compass needle may be measured. In order to complete the determination of the earth's field, it remains only to measure the horizontal component H of the total intensity T. See Fig. 241.

Confining our attention now to the parallel field which belongs to the earth at any point, let us imagine a magnet suspended in this field by means of a delicate fiber, so slender that it exerts practically no directive force on the magnetic needle. The magnet will oscillate to and fro about its position of equilibrium and will then come to rest, pointing along the magnetic meridian. If the magnet lies in a horizontal plane, the only directive force acting upon it is the horizontal intensity H.

324. To determine the numerical value of H the three following steps are needed: —

(i) *To obtain the torque, acting upon the needle in terms of its period.* When the suspended magnet is rotated through an angle θ about its suspending fiber as an axis, there will be a force $+mH$ acting upon the north pole of the magnet, and a force $-mH$ acting upon its south pole.

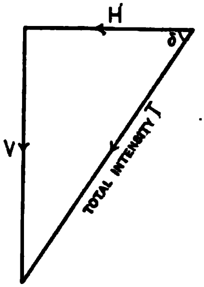


FIG. 241. — Illustrating the relation between the four magnetic elements, namely, horizontal, vertical, and total intensities, and angle of dip δ.

If we denote by l the distance between the two poles, $l \sin\theta$ will be the perpendicular distance between the lines of action of the two forces $+mH$ and $-mH$. The couple L acting upon the needle will therefore be $mHl \sin\theta$. And since the sign of the couple which tends to restore the needle to its position of equilibrium is always opposite to that of the angular displacement θ, we may write

$$L = -ml \cdot H \sin\theta = -MH \sin\theta. \qquad \text{Eq. 121}$$

When θ is very small, we have with sufficient accuracy

$$L = -MH\theta.$$

Dividing each side of this equation by the rotational inertia I of the magnet, we obtain for the angular acceleration A,

$$A = \frac{L}{I} = -\left(\frac{MH}{I}\right)\theta. \qquad \text{Eq. 122}$$

It thus appears that the angular acceleration of the vibrating system is, at every instant, proportional to the angular displacement and opposite in sense. The behavior of the magnet therefore satisfies the criterion (§ 53) for simple harmonic motion. And since the period of any simple harmonic motion is $2\pi\sqrt{-\left(\frac{\text{Displacement}}{\text{Acceleration}}\right)}$, it follows that the period of vibration of this magnet is

$$T = 2\pi\sqrt{\theta \Big/ \frac{MH\theta}{I}} = 2\pi\sqrt{\frac{I}{MH}}. \qquad \text{Eq. 123}$$

Observe that the action of a uniform magnetic field upon a magnet depends only upon the magnetic moment of that magnet, M, not upon the pole strength, not upon the length.

Now T and I are easily measurable quantities; but these data do not enable us to determine either of the two unknown quantities M and H. Eq. 123 merely gives us the product MH in terms of T and I. Hence the necessity for the second step, namely.

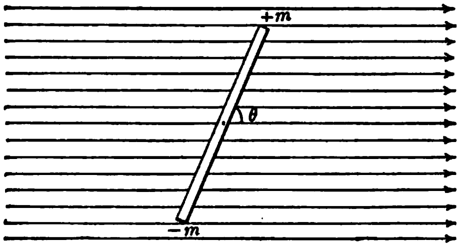


FIG. 242. — Illustrating the couple which acts upon a magnet suspended in the earth's field.

(ii) *To obtain the deflection of the needle in terms of M and H.* Gauss has shown how this relation may be obtained by a very simple experiment.

Having suspended another, and much smaller magnet with a small mirror rigidly attached to it, so that the direction of the magnetic field in which it is placed may be at once indicated, one takes the magnet NS (Fig. 243) whose period he has just measured, and fixes it as nearly as possible at right angles to the magnetic meridian and at a distance r east or west of the suspended needle O.

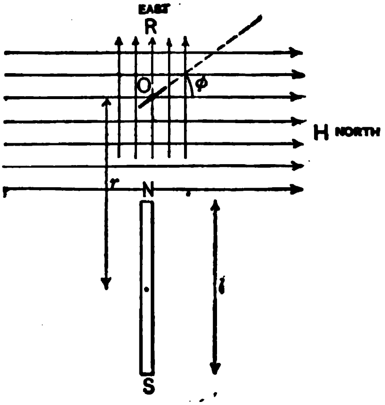


FIG. 243. — Showing the superposition of an artificial field upon the earth's field, and the consequent deflection of the suspended needle.

Before the deflecting magnet NS was brought into the neighborhood, the suspended needle pointed north and south along the magnetic meridian. If there were no magnetic field connected with the earth, the suspended needle would now point east and west owing to the field R of the deflecting magnet; but since both fields are present, the suspended needle sets itself along the *resultant* of the two fields as indicated in the figure. Now if we knew the intensity of each of these fields, we could easily compute the direction of the resultant; for, since these two fields are at right angles to each other,

$$\frac{R}{H} = \tan \phi, \qquad \text{Eq. 124}$$

where ϕ is the deflection, *i.e.* the direction of the resultant referred to the earth's field.

Conversely, if we know the direction ϕ and the intensity R we can compute H. This latter we now proceed to do. The magnetic field at O, due to the pole N alone, is by Coulomb's Law (§ 318) $\frac{+m}{\left(r-\frac{l}{2}\right)^2}$; while the field at O due to S alone is $\frac{-m}{\left(r+\frac{l}{2}\right)^2}$.

Hence the total field R due to the entire magnet NS is given by

$$R = m\left[\frac{1}{\left(r-\frac{l}{2}\right)^2} - \frac{1}{\left(r+\frac{l}{2}\right)^2}\right] = \frac{m \cdot 2\,lr}{\left[r^2 - \left(\frac{l}{2}\right)^2\right]^2}.$$

Or, if we choose r so large that l^2 may be neglected in comparison with r^2,

$$R = \frac{2\,Mr}{r^4} = 2\frac{M}{r^3}.$$

It is important to observe here that the action of a magnet at a distance which is large compared with the length of the magnet depends only upon its moment, M.

Substituting this value of R in Eq. 124 and solving for H/M, one has

$$\frac{H}{M} = \frac{2}{r^3 \tan \phi}. \qquad \text{Eq. 125}$$

Since r and ϕ are easily measured, we have here a laboratory equation for the ratio H/M. Our second step is now complete.

(iii) *Elimination of M.* Between Eqs. 123 and 125 it is now possible to eliminate M and obtain the horizontal intensity of the earth's field, thus

$$(MH)\left(\frac{H}{M}\right) = H^2 = \frac{8\pi^2 I}{T^2 r^3 \tan \phi}; \qquad \text{Eq. 126}$$

or to eliminate H, and obtain the moment of the deflecting magnet, thus

$$(MH)\left(\frac{M}{H}\right) = M^2 = \frac{2\pi^2 I r^3 \tan\phi}{T^2}. \qquad \text{Eq. 127}$$

It should be observed that this method of Gauss avoids the necessity of measuring either the pole strength m or the distance between the poles l. The neat manner in which this is done is to combine these two quantities into a product ml which we have denoted by M and then eliminate M by means of a second equation which is obtained, of course, by means of a second (deflection) experiment.

The advanced student will find that some of the approximations here introduced are not permissible in work of high accuracy, hence certain corrections must be made to Eqs. 126 and 127; but the philosophy of the subject is not thereby changed.

Returning from this digression, we now proceed to the seventh fundamental fact in the science of magnetism, which is that of

Magnetic Induction

325. 7. Magnetization by induction. Up to the time when Columbus first sailed for America only three fundamental facts concerning magnetism were known. One of these is the principle of the compass and another the attraction and repulsion of poles; the third one is the fact that if a piece of soft iron be brought near a magnet, the iron acquires magnetic poles, *i.e.* it becomes magnetized. It is not even necessary that the iron should touch the magnet; one needs only to bring it into the neighborhood of the magnet. **This process is called magnetic induction, more properly magnetization by induction.** It is important to note that the original magnet loses none of its own magnetism by inducing magnetism in another body. The one essential of this process is to bring the body which is to be magnetized into a magnetic field. The field does the rest.

(i) The phenomenon is simply shown by tethering an iron nail to the table by a short thread, as shown in Fig. 244. A strong permanent magnet brought near, and a little above, the nail will keep it suspended in mid air and magnetized. Observe that the end of the nail nearest the south pole of the magnet is itself always a north pole.

(ii) Another excellent illustration of magnetic induction in the earth's field is the following: —

Take a strip of "tin plate" so called (which is really sheet iron plated with tin), having dimensions something like 1 inch by 12.

If this strip has not been magnetized, either end of it will attract the north pole of the needle in a magnetoscope. Next hold the strip with one end down and bend it to and fro sharply between your thumb and forefinger at various points throughout its length. On examining it with the magnetoscope it is observed that the end which has been held down in the earth's field now repels the north pole of the needle. The bending apparently gives freedom to the particles of iron, and they probably arrange themselves in a new way under the influence of the earth's field. If now the other end of the strip be held down during the bending process, the poles of the strip will be reversed. Explain this by a diagram.

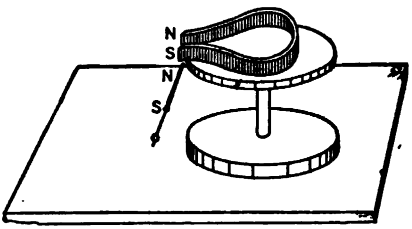


FIG. 244. — A wire nail becomes a magnet when brought into a magnetic field.

(iii) The most remarkable and interesting case of magnetic induction is that which occurs when a piece of iron is brought into the neighborhood of a conductor carrying an electric current. It is a strange fact that every electric current surrounds itself with a magnetic field of its own. And this magnetic field has the same properties as any field produced by a permanent magnet. Accordingly a piece of iron brought into the neighborhood of an electric current will in general become magnetized by induction, just as it would if brought near a permanent magnet. An arrangement of this kind is called an "electromagnet." Economic applications of the electromagnet, such as occur in the telegraph, telephone, dynamo, and motor, form an important chapter in the engineering achievements of the nineteenth century. Some of these will be studied in a later chapter.

Diamagnetism

326. 8. In 1778, Brugmans of Leyden discovered what we may call the eighth fundamental fact, namely, that a lump of bismuth is repelled when brought near either the north or south pole of a magnet. The phenomenon is most easily seen by suspending a small bismuth rod — say one inch long — between the poles of an electromagnet, *i.e.* in a strong magnetic field.

If the suspension thread of the bismuth has a directive force just barely sufficient to hold the little rod along the line joining the *N* and *S* poles of the magnet before the current is put on, it will be seen that as soon as the magnetic field is established, the bismuth rod will turn through 90°. For each end of the rod tries to get as far away as possible from each pole of the electromagnet. The consequence is that a bismuth rod always tries to set itself *across* the lines of force of the magnetic field in which it is placed. Hence the name *diamagnetism.*

That the phenomenon is one of induction is clearly evinced by the fact that the amount of repulsion experienced by the bismuth is proportional to the intensity of the magnetic field in which it is placed.

In 1845, Faraday examined a large number of substances and found that practically all substances possess magnetic quality, and also that the great majority of them are diamagnetic.

Thus copper, lead, silver, gold, and carbon rods all set themselves *across* the lines of force when placed in a magnetic field. Some substances, such as oxygen, platinum, and titanium, behave like iron and set themselves along the lines of force. These Faraday called *paramagnetic.* In most cases paramagnetic substances exhibit very feeble magnetism; but in some four or five substances, such as iron, nickel, and cobalt, the magnetization evoked by induction increases so rapidly with the field and is so permanent that these substances stand in a class by themselves. They are said to be *ferro-magnetic.* All bodies may therefore be grouped in three classes, namely, *diamagnetic*, *paramagnetic*, and *ferromagnetic*.

Measurement of Magnetic Quality

327. It is perfectly fair to ask, When is one substance more magnetic than another? The simplest answer to this question comes by first defining two new terms. A magnetized body has at any instant a definite magnetic moment which we have indicated by M. Let V denote the volume of the magnet. If now one takes the ratio of M to V, he will obtain a number which represents **the magnetic moment of unit volume of the magnet.** This quantity is called **intensity of magnetization** and is everywhere indicated by I.

$$I = \frac{M}{V}. \quad \text{Defining equation for intensity of magnetization.} \qquad \text{Eq. 128}$$

Next suppose a substance to be placed in a magnetic field whose intensity is denoted by H. This substance will at once acquire a certain intensity of magnetization, I. The ratio of this intensity of magnetization to the field strength which produces it is called **magnetic susceptibility**, and is denoted by k.

$$k = \frac{I}{H}. \quad \text{Defining equation for magnetic susceptibility.} \qquad \text{Eq. 129}$$

This quantity, magnetic susceptibility, may be used as a measure of magnetic quality.

When any paramagnetic body, such as a piece of tungsten, is placed along the lines of force between the poles of a magnet, it is observed that the paramagnetic body acquires a *north* pole on the end which lies next to the *south* pole of the magnet; and if a diamagnetic body, such as a rod of gold, be placed between the poles of the same magnet one might be inclined to think that the north pole acquired by the gold would lie next to the north pole of the magnet, since these two *appear* to repel each other. But such a conclusion would be just as unwarranted as it would be for one to conclude that, since a cork rises in water and a hydrogen-filled balloon rises in air, these bodies do not possess weight. All that one can infer from the rising of the balloon is that it has less weight than the air which it displaces; in like manner, all that one can infer from the gold setting itself across the lines of force is that the gold is less magnetic than the medium in which it is placed. And since diamagnetic bodies take up the equatorial position even in a vacuum, this means that these substances are less magnetic than free space which we think of as filled with ether.

That this view is the correct one is indicated by the following experiment. A small tube filled with a weak solution of ferric chloride and suspended in air behaves as a paramagnetic body; but when the suspended tube is surrounded by a stronger solution of ferric chloride it sets itself *across* the lines of force, *i.e.* it behaves as a diamagnetic substance.

Accordingly the susceptibility of paramagnetic bodies, being greater than that of free space, is said to be positive; while the susceptibility of a diamagnetic substance, being less than that of a vacuum, is said to be negative. This opposition of signs is illustrated in the following table.

MAGNETIC SUSCEPTIBILITIES

PARAMAGNETIC		DIAMAGNETIC SUBSTANCES	
Aluminium	$+ 0.65 \times 10^{-6}$	Antimony	$- 0.95 \times 10^{-6}$
Chromium	$+ 3.7$	Argon	$- 0.010$
Magnesium	$+ 0.55$	Bismuth	$- 1.4$
Manganese	$+ 10.6$ (?)	Cadmium	$- 0.17$
Nitrogen	$+ 0.024$	Copper	$- 0.087$
Oxygen	$+ 0.123$	Gold	$- 0.15$
Platinum	$+ 1.32$	Hydrogen	$- 0.008$
Tin	$+ 0.025$	Iodine	$- 0.36$
Titanium	$+ 2.$ *Circa.*	Sulphur	$- 0.5$
Tungsten	$+ 33$	Mercury	$- 0.19$

— From KAYE AND LABY'S *Constants.*

Another viewpoint from which the meaning of this negative sign can be easily read is the following. When a paramagnetic body is free to move in a magnetic field its motion is always toward a stronger portion of the field; but diamagnetics always move, when free, from the stronger towards the weaker part of the field. Here is essentially the same opposition of sign which we have met above. The advanced student will be interested in explaining this latter phenomenon in terms of the fact that the potential energy of a system at rest tends to a minimum.

Magnetic Permeability

328. Still another point of view from which to consider paramagnetics and diamagnetics is the following: —

Let an electric current be passed through a spiral of wire; it will be found that the interior of the coil is a magnetic field. The lines of force will pass from one end of the helix to the other. But if an iron rod be slipped into the helix, the number of lines of force passing through will be multiplied many times. In this case many new lines of force are created by the introduction of the iron as is indicated in Fig. 245.

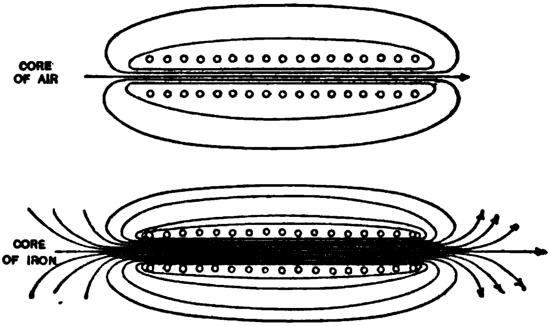


Fig. 245. — Illustrating the large permeability of iron.

If B is the number of lines of force passing through each unit of area of the cross section of the iron rod and H the number of lines passing through the same unit of area when the helix was filled with air, then we may describe the preceding facts by writing

$$B = \mu H, \qquad \text{Eq. 130}$$

where μ is a proportionality factor for which Lord Kelvin proposed the name *magnetic permeability.* This factor μ is, however, not a constant, but a variable depending upon H, and in the case of iron, ranges in value from 1 to 2000 when measured on the C.G.S. system. Engineers generally speak of B as the *flux-density* and of H as the *field.*

Paramagnetic bodies are now distinguished by the fact that they possess a permeability always greater than unity; while the permeability of a diamagnetic body is always slightly less than unity, and the permeability of free space is exactly unity.

The remarkable ability of iron to transmit lines of force is well shown by taking a magnetic field such as that shown in Fig. 246 and placing between the magnet poles a block of soft iron such as indicated in Fig. 247. The lines of force seem to have a decided preference for the iron and even go out of their way for the sake of passing through it. The advanced student will be interested in discovering that susceptibility and permeability are related in the following simple manner, $\mu = 1 + 4\pi k$.

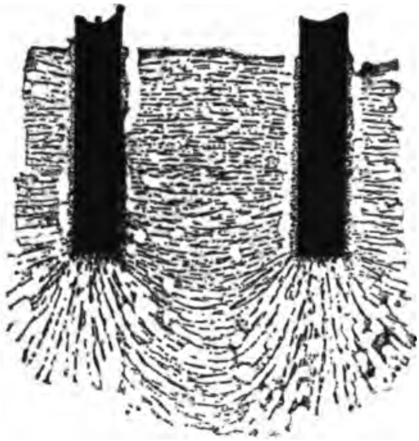


FIG. 246.

Magnetic Theory

329. The several steps which have led up to the modern view concerning the nature of magnetism are something like the following: —

(i) It was early suggested that *magnetization is a molecular phenomenon.* The fact that a magnet may be broken into an almost unlimited number of pieces, each of which remains a magnet with a north and south pole of its own, looks very strongly in this direction. See Fig. 248. Each student should make this experiment for himself. This view is also confirmed by the fact that a tube of iron filings, such as shown in Fig. 249, when placed along the poles of a strong magnet becomes magnetized, and when

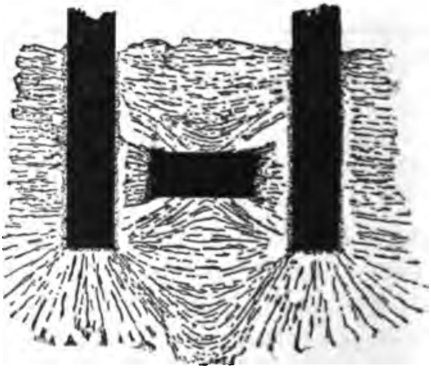


FIG. 247.

removed remains magnetized, until by shaking or jarring it is demagnetized.

More convincing still is the fact that a long wire of soft iron — say 70 centimeters in length and 1 or 2 millimeters in diameter — may be strongly magnetized and by a slight jar lose nearly all of its magnetization.

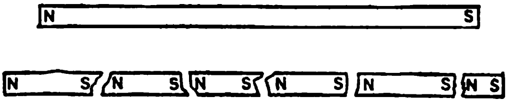


FIG. 248. — A piece of magnetized watch spring broken into fragments, each fragment remaining a complete magnet.

Most striking of all, however, is the fact that **a piece of iron when raised to red heat loses its magnetic quality.** This behavior of iron is easily demonstrated by mounting a compass needle five or six inches long on a single pivot, as shown in Fig. 250, and then bringing near it a piece of unmagnetized iron wire, say six inches long, held in a wood clamp. The compass needle will magnetize the wire and will be deflected from the magnetic meridian, so that one end of it will swing about and touch the wire. If now a Bunsen flame be brought underneath the iron wire until it becomes a bright red, the compass needle will "let go" and will swing back into the meridian, showing that the hot iron has lost its magnetic quality. On removing the flame, the iron will be seen to cool and one end of the compass needle to swing around into contact with the iron, showing that the wire on cooling has regained its magnetic quality.

FIG. 249. — A tube of iron filings magnetized.

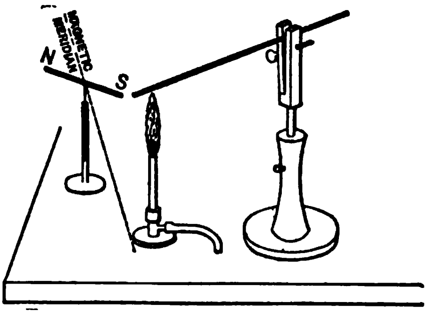


FIG. 250. — Illustrating the disappearance of magnetic quality at red heat.

All of these facts lead us to think that magnetization is not a property of the surface of a body or of the body as a whole, but of its innermost structure. The evidence is indeed overwhelmingly in favor of the view that magnetism is a molecular or atomic phenomenon.

(ii) The second step was made by W. E. Weber, who explained ferromagnetism by assuming that the molecules of iron are themselves small permanent magnets, arranged in no particular order, arranged indeed in the utmost disorder, as indicated in Fig. 251.

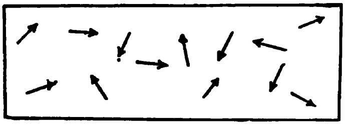


FIG. 251. — Probable arrangement of particles in a piece of ordinary iron.

If these molecular magnets were perfectly free to turn, it is clear that any magnetic force acting upon them would be sufficient to bring them all into line and thus produce at once the maximum intensity of magnetization possible. In order to avoid this, Weber imagined each molecule to be acted upon by a certain force tending to hold the molecule in its original direction. The effect of this restraining force is represented in Fig. 252. The magnetic field does not bring all the tiny magnets into perfect parallelism, but merely brings them more or less into line, as there shown.

In 1890, however, J. A. Ewing, an eminent British engineer, proved by means of a model that it is quite unnecessary to assume any such restraining force as that which Weber imagined to hold the molecules in place. The forces which determine the positions of these molecular magnets are the attractions and repulsions acting between the poles of the magnets. A model made up of little compass needles pivoted freely behaves almost exactly as a piece of iron, so far as its magnetic properties are concerned; that is, it exhibits permanent magnetism, magnetic saturation, etc., which characterize ferromagnetism.

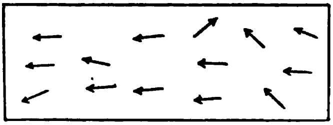


FIG. 252. — Probable arrangement of particles in a piece of magnetized iron.

(iii) The third step is to develop this theory so as to enable one to include in his explanation the facts of paramagnetism and diamagnetism. This extension of the theory we owe to Rowland, J. J. Thomson, and Langevin. A bare outline of it is something like the following. It is well known that a small circular electric current behaves like a small magnet. In 1876, Professor Rowland showed by a justly celebrated and difficult experiment that a revolving electric charge produces the same magnetic effects as an electric current. In 1897, J. J. Thomson demonstrated the existence of small electric charges, having a mass about $\frac{1}{1700}$ that of a hydrogen atom. These small charges are negative in sign and can be obtained from a great variety of sources. They are called **electrons.** The present view (1915) of an atom is that it is built up, in part at least, of a number of electrons in rapid rotation about (or in) a central nucleus of positive electrification. Each of these electrons moving in its orbit is equivalent to a minute magnet. The advanced student will find this view supported by strong experimental evidence, such as that of the Zeeman effect.

Assuming an atomic structure of this kind, P. Langevin in France has pointed out the possibility that these electronic orbits, *in any one atom,* may be oriented in a great variety of ways.

In the case of diamagnetic substances, these orbits are supposed to be distributed in such a symmetrical way that they produce no magnetic field outside of the atom, *i.e.* the diamagnetic atom has no magnetic moment of its own. If, however, this diamagnetic substance be placed in a magnetic field, lines of magnetic force will be thrust through any of these orbits which are not exactly parallel to the field, and as one looks along the lines of force, those electrons which are revolving in a clockwise sense will be accelerated, those revolving in a counter-clockwise sense will be retarded; so that, in either event, the magnetic moment of the atom will be increased in a sense opposite to that of the field. In other words, an atom with a perfectly symmetrical distribution of electronic orbits when placed in a magnetic field behaves as a diamagnetic substance.

In the case of paramagnetic substances, it is supposed that the orbits of the electrons are distributed in such a way that they do not exactly cancel the effects, one of another; that is, these atoms have a magnetic moment of their own, and being little magnets will fall into alignment, more or less, when acted upon by an external field. The turning of these tiny magnets will, of course, always be in such a sense as to bring the S pole of the atom to face the N pole of the field-producing magnet. Atoms which are asymmetric will therefore behave in a paramagnetic way. But superposed upon this effect will be the diamagnetic effect discussed in the preceding paragraph. In most cases, however, these diamagnetic effects are small in comparison with the effect of rotating the orbits. What sort of resistance these orbits offer to rotation is entirely unknown.

In the case of ferromagnetic substances, the experimental facts and Ewing's model lead one to think that there are mutual actions between the atoms or molecules. When an external field is applied to a ferromagnetic substance, the natural groupings of the molecules are changed; these elementary magnets fall into new positions of equilibrium, one after another; and we have the effects known as permanent magnetism, etc. The key then to ferromagnetism is the attraction and repulsion of one molecular magnet upon another.

Summarizing, then, the theory of magnetism is that the diamagnetic effect is probably present in all substances; but in many cases it is masked by the paramagnetic turning of the molecule, and when in addition to these two effects there is also present a mutual action between molecules, we have ferromagnetism.

By going one step farther and assuming that diamagnetism is an *atomic* phenomenon while paramagnetism is a *molecular* property, one may explain an important fact discovered by P. Curie, namely, that the susceptibility of paramagnetics varies inversely as the absolute temperature, while that of diamagnetics is independent of temperature. On the other hand K. T. Compton and E. A. Trousdale offer experimental evidence, obtained by X-ray photography, for thinking that paramagnetism is not a molecular, but an atomic, phenomenon. *Science,* 23 Apr., 1915.

Magnetic Variations

330. Besides that mentioned (in § 313) there is another exception to the general statement that the direction of the compass needle remains fixed. There are several distinct changes constantly taking place. These " variations," as they are called, may fairly be reckoned among the most instructive cosmical phenomena with which we are acquainted. The two most important of them are perhaps the following: —

1. *The diurnal variation.* In the northern magnetic hemisphere during the early morning hours the north-seeking end of the needle is found to be swinging to the eastward; this it continues to do until 7 to 9 A.M., some time during which interval it has in general reached its most easterly position. After hovering near this extreme for a while, it turns about, and begins to march westward. About 10 to 11 A.M. it passes through the average position for the whole 24 hours, and by 1 to 3 P.M. has reached its extreme westerly position. Then it once more turns to the eastward, recrosses the mean position in general some time between 6 P.M. and midnight, and gradually returns to the position from which it started in the morning. The average total change in the course of a year between the morning and afternoon extremes is greatest in the United States in northern Minnesota, where it amounts to as much as 14′, while in southern Florida it is as low as 4′; these quantities vary with the seasons, being about one third greater in midsummer and a like amount less in midwinter.

2. *The secular variation.* In addition to this daily oscillation, the *mean position* about which the needle swings is itself slowly changing. The annual amount of this secular change varies not only for different localities, but also with time. In this country the north-seeking end has *at the present time* (1915) practically no secular variation for points along a line passing approximately through Lake Superior and the West Indies. For places east of this line-of-no-change the mean position of the north-seeking end of the needle is now moving to the eastward. The *present* annual rate of secular change in the region of the Pacific Coast states is about 4′ and in New England about 3′. This change is of great importance to surveyors and map makers. If suitable corrections are not made for the secular variation, a boundary line whose direction has been determined by means of the compass needle will, when resurveyed some years later, have a direction *apparently* different from that previously found.

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Problems

335. Draw on the board a sketch showing how one might place a very long magnet so that one of its poles would deflect a compass while the other would produce little or no effect.

336. A magnet pole whose strength is 9 C.G.S. units acts with a force of 2 dynes upon another pole at a distance of 6 cm. Find the strength of this other pole.

337. Imagine a magnetic needle suspended with an exceedingly fine fiber inside a lamp chimney. In room A of the laboratory the period of vibration of this needle is 3 seconds; at a certain point in room B (where an electric generator is running) the period of the needle is $\frac{1}{17}$ of a second. How much stronger is the magnetic field at the position in room B than in room A?

338. A magnet whose moment of inertia is 100 C.G.S. units vibrates in a magnetic field whose horizontal intensity is 0.2 of a gauss, with a period of $\frac{\pi}{10}$ seconds. Find the magnetic moment of the magnet.

339. A compass needle is freely suspended at rest in a field whose horizontal intensity is $\frac{1}{5}$ of a gauss. A torque of 15 dyne-centimeters is required to twist the needle through $\frac{1}{10}$ of a radian. Find the magnetic moment of the needle.

340. Suppose the dip in a certain locality to be 60° and the horizontal intensity of the earth's field to be 0.25 of a gauss. Find the total intensity of the earth's field.

341. Prove that the " dimensions " of magnetic moment are $[M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}]$.

342. Two straight magnets, *A* and *B*, are freely suspended in the same uniform field. The magnet *A* has a pole strength of 8 and a length of 5 centimeters; the magnet *B* has a pole strength of 7 and a length of 6 centimeters. Suppose the poles to be located exactly at the ends of the magnets, and find which of the two magnets will vibrate more rapidly.

343. A long slender iron bar whose permeability is 1200 C.G.S. units is placed lengthwise in a magnetic field whose intensity is 40 gausses. How many lines of force per square centimeter of cross-section will pass through the iron?

344. Imagine a circular ring of wire 80 cm. in diameter lying horizontally on a table in a locality where the total intensity of the earth's magnetic field is 0.6 gauss, and the dip is 30°. How many lines of force pass through the ring? *Ans.* 480π

> **"Rather doing things worthy to bee written, then writing things fitte to bee done."**
>
> **Sir Philip Sidney.**

CHAPTER IX

ELECTROSTATICS

The subject which we now approach is taken up not only on its own merits but also because it is an essential preliminary to the study of electric currents.

331. If a sheet of dry writing paper be held against the wall and given a few quick flaps with a piece of flannel, the paper will adhere to the wall quite strongly. The rubber handle of a fountain pen, when rubbed over the sleeve of one's woolen coat, acquires the property of attracting small bits of paper or wood; glass rubbed on silk behaves in the same way.

The substance earliest found to have this property was a fossil resin — ordinary amber — found on the shores of Greece.

The Greeks called this resin *electron* (ἤλεκτρον). And hence later Dr. Gilbert called this phenomenon *electric*. The noun *electricity* is first used in 1646 by Sir Thomas Browne, the author of *Religio Medici*.

These facts have been known ever since the time of Thales, 640-546 B.C.; and for two thousand years following him they constituted practically the whole of electric science, barring some knowledge of lightning and of the electric eel, which no one then imagined to have any connection with the amber phenomenon.

Bodies that have acquired this property of attracting small bits of paper are said to be **electrified** or to have an **electric charge**. Bodies which are not charged are said to be **neutral**.

The modern view concerning electricity is that it consists of enormous numbers of exceedingly minute electric charges, each exactly like the other and each called an **electron**. The properties of the electron will be discussed in later sections.

The Electroscope and Proof Plane

332. By many experiments, it has been established that *any* two different substances when rubbed together, or even when brought into contact, become electrified.

But before we can examine the evidence for this statement it will be necessary for us to devise some means by which we can detect the presence of an electric charge. One method is that indicated above, viz. the attraction of bits of thread or straw, but this is not a very sensitive test; much smaller charges can be detected by the gold leaf **electroscope**, an instrument which we shall now describe.

This device is essentially a metal rod supported on a stopper of sealing wax, or, better still, of sulphur, and carrying two leaves of aluminium or gold foil suspended from its lower end, the leaves being inclosed in a glass vessel, as shown in Fig. 253.

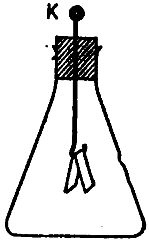

FIG. 253.—Electroscope made of Erlenmeyer flask and aluminium foil.

If, now, any electrified body be brought into contact with the knob *K* or, as we shall see presently, even into the near neighborhood of *K*, the leaves of gold foil will diverge. If the body is not electrified, the leaves will not diverge.

We have thus a simple and exceedingly delicate means for detecting electric charges. The theory of the electroscope will be considered presently.

But we sometimes wish to examine the electrification of a body which is too large to carry to the electroscope, or of a body which we do not wish to move. In such a case, it is most convenient to wax a penny to the end of a glass or rubber rod (see Fig. 254). We may use the glass rod as a handle, and then by touching the penny alternately to the body and to the electroscope, we may determine whether or not the body is charged. Such a device is called a **proof plane.**

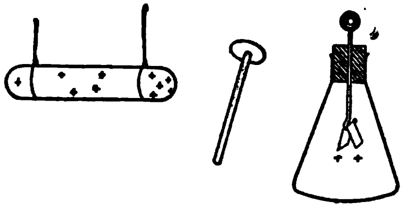

FIG. 254.—Proof plane.

Sensitive as is the form described above, the gold leaf electroscope was modified by C. T. R. Wilson of Cambridge in 1901 so as to be much more efficient.

He supports a rigid strip of brass *B* from a small block of sulphur *S*. On the side of this flat brass rod a narrow strip of gold leaf is attached by its upper end. The whole system of sulphur, brass strip, and gold leaf is suspended by a rod *T*, which is rigidly attached to the containing vessel by means of a block of ebonite.

Sulphur is such a superb nonconductor that, in reasonably dry air, an electroscope of this kind will hold its charge for several hours. In order to communicate a charge to the suspended system, a small wire *C* is passed through the hard rubber block on top of the containing vessel and then bent twice at right angles. When the gold leaf is once charged, the contact between it and this wire may be broken by simply rotating the upper end of *C*.

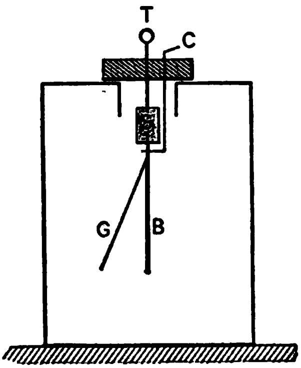

FIG. 255. — C. T. R. Wilson's gold leaf electroscope.

The advanced student will be interested in the lecture electroscope devised by Professor John Zeleny and described in the *Physical Review*, June, 1911. It is a gold leaf instrument, but is useful for a much wider range of purposes than either of those described above.

333. Try touching the tip of a camel's-hair brush on the table top and then to the electroscope. Rub the brush gently over your own hair and again bring it to the electroscope. **Since electrification is produced by such slight means, we are led to think that any two different substances brought into contact become electrified.** This is possibly the most fundamental fact of electrostatics, and may be called the *first* important discovery in this subject.

Just what goes on during the contact, or friction, between two different substances is not yet known. It is certain, however, that the electrification produced depends upon the chemical composition and physical state of the two bodies brought together. In the case of glass rubbed with silk, the electrons — negative electrifications as we shall see later — are transferred *from* the glass to the silk; but when a glass rod is rubbed with catskin, the electrons are transferred *to* the glass from the catskin.

CONDUCTORS AND NONCONDUCTORS

334. A *second* great discovery in electrostatics was made by Stephen Gray in London (d. 1736). He found that when a glass rod is electrified by rubbing, the electrification "leaks off" if the rod is held in the hand, or if the rod is supported by cotton thread, or metal, or a moistened string of any kind. By connecting an electrified glass rod with a distant point by means of a wet string, he was able to transfer the electrification through a distance of many meters.

He found also that if a body containing an electric charge was supported on glass or rubber, or suspended by a dry silk thread, the charge disappeared very slowly. **Those substances which lead off the charge quickly are called conductors; those which prevent the charge from escaping are called nonconductors or insulators.** A conductor supported upon a nonconductor is said to be "insulated." But there are some substances, such as carbon and most acids, which occupy an intermediate position, and can hardly be called either conductors or nonconductors. The following list of substances is arranged in the order of their ability to conduct electricity.

SUBSTANCE	CONDUCTIVITY
Copper	614000.
Mercury	105300.
Carbon of lamp filament	250.
" Conductivity " water	.000001
Sulphur	.00000000000000025

Data from KAYE and LABY'S *Constants.*

There are several explanations of conduction in terms of electrons, but the simplest is that which supposes certain electrons inside of metals and other conductors to have much freedom of motion, while the electrons in insulators are practically all attached to the atoms of matter.

TWO KINDS OF ELECTRIFICATION

335. A *third* great discovery was made in electrostatics by a French soldier and electrician, Dufay (b. 1698, d. 1739). He found that if a light electrified body, say a pith ball, be suspended by a silk thread, other electrified bodies behave always in one of two ways when they are brought near the suspended body. Some charged bodies will attract the pith ball, and some will repel it. The electrification which appears on glass when rubbed with silk behaves in one way; the electrification which appears on gutta-percha (or a rod of sealing wax) when rubbed with flannel behaves in the opposite way. Dufay called these two kinds of electrification *vitreous* and *resinous*, respectively; but the following names, suggested by Benjamin Franklin, are in some respects happier, and have been adopted by the whole world: —

DEFINITIONS. 1. **That kind of electrification which makes its appearance on glass when rubbed with silk is called positive.**

2. **That kind of electrification which appears on sealing wax or gutta-percha when rubbed with flannel is therefore called negative.**

It will be observed that the first of these definitions is a purely arbitrary one. The fact of nature is that there are two kinds of electrification, but which one we shall call positive and which one negative is a matter of choice. The scientific world has, however, agreed upon the definitions given above. If the nomenclature of electrostatics had been postponed until the present time these two names, positive and negative, would doubtless have been interchanged. The reason for this opinion is that all electrons are negative charges and that negative electricity is the only one that has ever been separated from matter. Positive electricity is known only in association with ordinary matter.

By universal consent, the word *negative* has been used for more than a century to denote resinous electrification, that is, the kind which appears on a body when it contains *more* electrons than it does when neutral.

Figure 256 may aid the student to bear in mind not only the present convention, but also Dufay's evidence for thinking there *are* two kinds of electricity.

It is important here to observe that the pith ball, if charged from the glass rod, is always repelled by the glass rod, while it is always repelled by the sealing wax if it has been charged from the sealing wax; in other words, like electrifications always repel each other.

We now see why it is that the gold leaves of the electroscope always diverge when they are charged. For whichever of the two electrifications is given to the electroscope, the leaves will each be charged in the same way, and will, therefore, repel each other. It is important also to observe that the uncharged electroscope gives the same indication, *i.e.* a divergence, for a positive charge as for a negative one. How, then, can we determine when we are dealing with a positively charged body and when with a negatively charged body?

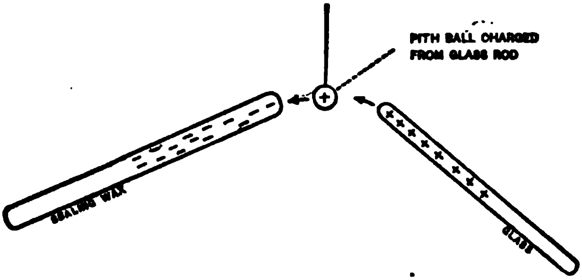

FIG. 256. — Differences in behavior of the two electrifications.

Experimental Distinction between Positive and Negative Charges

336. First charge the electroscope with a known kind of electrification, say positive. Then, if the positive charge on the electroscope be increased, the divergence of the leaves will increase; if, however, we give to the electroscope a negative charge, the divergence of the leaves will diminish.

If we know the original charge on the leaves to be positive, we will then have no difficulty in saying just what the sign of any other charge is; for if the unknown charge increases the divergence, it must be positive; but if it diminishes the divergence, it must be negative.

Coulomb's Law of Attraction and Repulsion

337. If we assume now that two different substances when brought into contact become electrified, and that bodies having charges of the same sign repel each other, while bodies having charges of opposite signs attract, the next inquiry is *How* do these bodies attract (or repel) each other? This question, it will be observed, is, for electrical charges, exactly the same one which Newton asked concerning the behavior of pieces of ordinary matter.

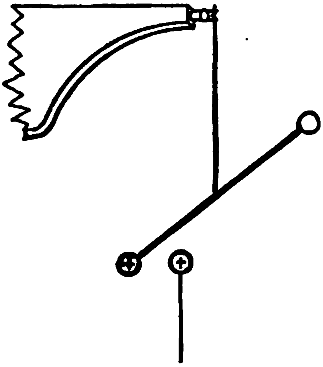

FIG. 257.

It is important to realize that these forces between electrified bodies are ordinary mechanical forces, and are measured sometimes in terms of weight, sometimes by the mass acceleration which takes place, and sometimes by the torque they produce.

The French physicist, Coulomb (1736–1806), discovered the law of attraction for electrified bodies by a series of experiments which are somewhat difficult to repeat, though very simple in idea. He showed that the experimental facts are as follows:—

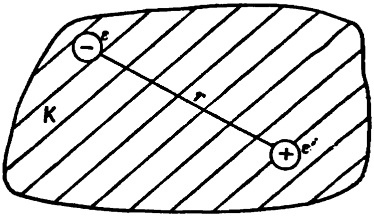

FIG. 258.

(i) The repulsive (or attractive) force between two quantities of electrification (Fig. 257) is directly proportional to the product of the two quantities. Equal quantities of electricity can always be obtained by exactly duplicating any process which produces electrification.

(ii) The force which one charged particle (or a body acting as a particle) exerts upon another varies inversely as the square of the distance separating the particles.

(iii) In addition to the above two facts discovered by Coulomb, it was shown by Cavendish and Faraday that the force which one charged particle exerts upon another depends also upon the material of the nonconductor which separates them. **This nonconducting material is generally called the medium or the dielectric.**

A summary of these three facts is more elegantly made in terms of algebra, as follows: —

Let e and e' (Fig. 258) denote the electric charges of the two particles, r the distance which separates them, and K a quantity which expresses (measures) the effect of the nonconducting medium; then we know, by experiment, that if F denote the force of attraction between the two particles

$$F \propto e;\ F \propto e';\ F \propto \frac{1}{r^2};\ \text{and } F \propto \frac{1}{K}.$$

Hence

$$F \propto \frac{ee'}{Kr^2},$$

or

$$F = C\frac{ee'}{Kr^2}, \qquad \text{Eq. 131}$$

where C is a constant of proportionality whose value will depend upon our choice of units.

The quantity K is known as the *dielectric constant*, and is assumed arbitrarily to have the value of unity when the dielectric is a vacuum. But its value for air (1.0005) differs so little from unity, that for all ordinary purposes we may put $K = 1$ in air. For various kinds of glass K ranges from 5 to 10; while for ebonite and paraffin its value generally lies between 2 and 3. Just how this dielectric constant for any one substance is to be measured can be explained only in a later section after the student has discovered the meaning of "electrical capacity."

The essential thing here is to understand that the force which one charged particle exerts upon another depends upon the medium in which the particles are situated, as well as upon the distance between the particles.

Definition of Unit Charge

338. Before Eq. 131, which describes the law of inverse squares, can be of any value for purposes of computation, it will, of course, be necessary to evaluate the constant C. This is done as follows: Let the two charges indicated in Fig. 257 be taken equal to each other, and let them be placed at a distance of one centimeter apart *in air*. The size of either charge we have still left undetermined. In terms of algebra, we have assumed only the following: —

$$e = e'; \; r = 1; \text{ and } K = 1.$$

If now, working under these conditions, we agree to call e unity when $F = 1$, it follows, by substituting these values in Eq. 131, that $C = 1$. And this is precisely the reason for choosing a unit charge of this size, namely, that the constant of proportionality is thus made to disappear from the equation, giving us the working form

$$F = \frac{ee'}{Kr^2}. \qquad \text{Eq. 132}$$

A unit charge of electricity is, then, one of such a size that it will repel with a force of one dyne an equal charge of the same sign placed at a distance of one centimeter in air.

This is called the electrostatic unit of electrical charge and is generally written "E.S. unit," sometimes E.S.U.

When the consideration of electric currents is taken up we shall find that the electromagnetic unit of quantity — the *E.M.* unit — is 30,000 million times as large as the E.S. unit. Experiment shows that the negative charge on an electron is very nearly 4.774×10^{-10} E.S. units, or 1.591×10^{-20} E.M. units. The electrical engineer employs a unit of quantity which is one tenth of an E.M. unit and which is called a *Coulomb*. The reason for this choice will be clear when we come to study electric currents.

The inverse square law discovered by Coulomb may be considered the *fourth* important discovery in electrostatics. The fundamental importance of this law will be realized when it is understood that this is the first bridge which connects the two great domains of electricity and mechanics. The connection is, however, only a quantitative one; no light is shed upon the nature of an electric charge by thus defining, in terms of a force and a distance, a certain charge with which other charges are to be compared. At the same time, this is a most essential step toward discovering various facts which do shed light upon the nature of electrical phenomena.

Definition of an Electric Field.

339. Since, in general, a force is exerted upon one positively charged body in the presence of another positively charged body, it is clear that we shall have to do work upon one of these charged bodies if we move it nearer to the other one. And, in general, if we move either body in any direction, we shall have to do work upon it, or it will do work for us.

A region in which work must be done to move an electric charge from one point to another is called an electric field.

The intensity of the field in any direction, at any point, is measured by the amount of work, in ergs, which must be done to move a unit quantity of electricity through unit distance along the given direction in the neighborhood of that point.

Electric intensity is, of course, to be measured in the same fundamental units — centimeters, grams, and seconds — as are mechanical forces and mechanical work.

In accordance with this definition, a unit electric field is one in which a unit charge is acted upon with a force of one dyne.

If, therefore, a body carries a charge of e units in an electric field whose intensity is R, the force F acting upon this body will be given by

$$R = \frac{F}{e}, \qquad \text{Defining equation for intensity of electric field.} \qquad \text{Eq. 133}$$

which is the defining equation for R, and which is a strict analogue of Eq. 119 defining the intensity of a magnetic field.

Since F is a vector quantity and e a scalar, it follows that R is a vector. Hence in order to specify an electric field it is quite as necessary to state its direction as its intensity.

Lines of Electric Force

340. This fact led Faraday to introduce into electrical science the conception of "lines of force," a most helpful mode of description. He imagines that in any region where there are charged bodies these "lines of force" run from the positive charges to the negative ones. At any point in space the line of force takes the direction of the electric field at that point. Not only so, but the closeness (density, if you like) with which the lines of force are drawn at any point in the field is proportional to the intensity of the field at that point. In this manner, Faraday and Maxwell gave a simple, lucid, and accurate graphical description of an electric field.

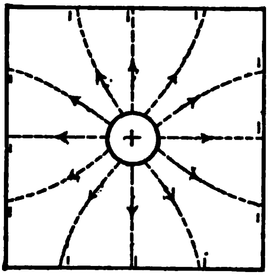

FIG. 259. — Field about a charged sphere.

As one of the simplest possible illustrations consider a positively charged metal sphere placed on a table in a cubical room. Here the *negative* charge lies, as we shall presently learn, upon the walls of the room. If now the region about the charged sphere be tested by a bit of linen thread, say 2 inches long, supported at its middle point by a silk fiber, it will be found that the bit of linen acts as a compass needle for the electric field and that the electric field is radially distributed about the sphere. Hence the lines of force are to be drawn in a radial direction as in the accompanying figure.

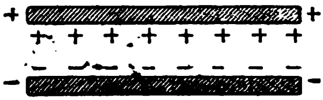

FIG. 260. — Showing distribution of charges over parallel plates.

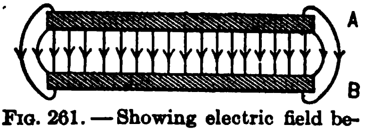

FIG. 261. — Showing electric field between parallel plates in terms of lines of force.

As another illustration consider two equal parallel plates, *A* and *B* (Fig. 260), one charged positively, the other holding an equal negative charge. In such an arrangement, the electric field is limited almost exclusively to the region between the plates; hence also are the lines of force. And since the field is also uniform, it follows that its representation must be very like that indicated in Fig. 261. The analogy between electric and magnetic lines of force is obvious. The space inclosed by a bundle of lines of force forms a **tube of force.**

ELECTROSTATIC INDUCTION

341. For charging bodies there are other methods besides friction. One of the most interesting of these is the following, which appears to have been discovered by Stephen Gray.* He found that one may charge a body, provided this body be a conductor, by simply bringing it into the field of another charged body.

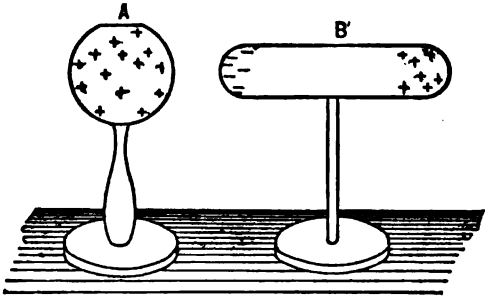

FIG. 262. — The phenomenon of electrostatic induction.

Let *A* (Fig. 262) be a charged body. A tin can on a block of paraffin or a hollow brass sphere supported on a glass stem will answer well. Let *B* indicate an insulated conductor which is not charged. A block of wood covered with lead foil and supported on a glass or rubber stem will answer well. Bring *B* into the neighborhood of *A* — say within three or four centimeters of it — and examine *B* with the proof plane and electroscope as follows: —

1. Touch the proof plane to the charged body *A*. Carry this charge to the electroscope. You will get a distinct separation of the gold leaves.

2. Touch the proof plane to the end of *B* which is nearest *A*, and carry the charge to the electroscope while the leaves still diverge. The divergence of the leaves diminishes, showing that the charge on this part of *B* is opposite to that on *A*.

Repeat the process until the leaves have completely collapsed and have again diverged. They are now charged with the same kind of electrification that is on the near end of *B*.

3. Touch the proof plane to the end of *B* that is most remote

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* Poggendorff, *Geschichte der Physik*, p. 839: but Professor S. P. Thompson assigns the discovery to John Canton, of London, in 1753.

from *A*. The leaves begin to collapse again, showing that the two ends of *B* have charges of different signs.

This experiment should always be repeated until the evidence is perfectly clear for thinking that when an uncharged conductor is brought into the field of a charged body, the uncharged body becomes electrified exactly as indicated in Fig. 262 above. **The charges on *B* are said to be induced, and the phenomenon is called electrostatic induction, or influence.**

At this point it is all-important to note that the induced charge on *B* does not at all diminish the charge on *A*. It is produced by means of the charge on *A*, but not at the expense of the charge on *A*. Nor does a glass-plate or other insulator thrust in between *A* and *B* interfere in the slightest with the induction or influence of *A* on *B*.

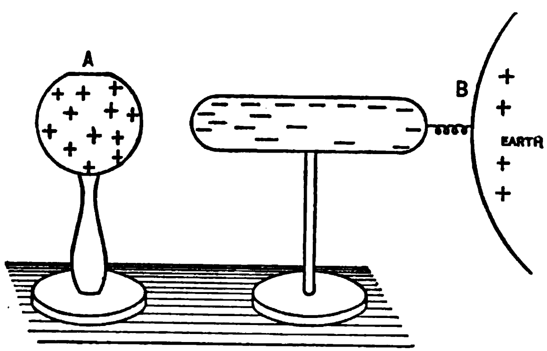

Fig. 263. — Charging by induction.

In terms of the electron theory, this process of charging by influence is very simply explained. Every conductor is supposed to contain within it a large number of more or less free electrons. As soon as a positively charged body, *A*, is brought near this conductor, *B*, these free electrons are attracted, just as any other negatively charged bodies would be, from all parts of the conductor to the end nearest the positively charged body. Hence that end shows a negative charge. The electrons are drawn away from the far end of the conductor; hence it has less than its normal number of electrons and is therefore positively charged.

342. Up to this point we have considered each of the conductors, *A* and *B*, insulated. Imagine now the body *B* to be connected with the earth, which we know to be a good conductor. We have then an interesting special case.

Our conductor *B* (Fig. 263) has now been very much enlarged;

indeed, it includes the earth itself, so that B is at least 8000 miles long. There is, therefore, only one end of B which is near enough for us to examine, and at this end we find, as before, an electrification which is opposite that on A.

Suppose now we remove the wire. The body B is again reduced to its former size; but it will now be charged negatively while A is charged positively. **We have thus learned how to charge a body without using friction and without diminishing the original charge on A.** This fact might be called a *fifth* fundamental discovery in electrostatics.

The Electrophorus

343. A simple illustration of the foregoing principle as well as one of the most convenient instruments in the laboratory for producing electric charges quickly and easily is the electrophorus announced by the Italian physicist, Volta, in 1775. It consists of a plate of hard rubber, rosin, or some substance which can be electrified by friction and a metal disk C, which is provided with an insulating handle H, and which is called a "carrier." The hard rubber plate P generally rests upon a sheet of metal S, which is called the "sole plate."

Suppose the plate to have been rubbed with catskin and therefore to be negatively electrified. Then when the carrier is made to approach the plate of ebonite the induced charges will be distributed as shown in Fig. 264. When the carrier is placed upon the plate, it will be supported by slight rugosities so that it will not receive any charge by conduction from the plate. If now the finger be touched to the carrier, the negative charge will run off to earth and the positive charge will remain bound. When, however, the carrier is removed, by the handle, to a distance from the plate, the bound charge becomes free, and available for charging other bodies. Evidently this process may be repeated as many times as one likes.

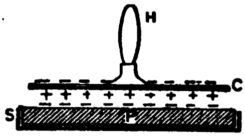

Fig. 264.—Electrophorus of Volta.

Within recent years machines have been invented by which one can carry on this process of induction continuously. Only instead of expending energy in lifting the charged carrier away from the charged plate, one employs his energy in rotating a series of charged disks.

The earliest of these machines was invented by Belli in 1831; the latest and best form is that of Wimshurst. The advanced student will find the action of these *influence machines,* as they are called, described in some of the larger texts, such as that of Poynting and Thomson.

Equal Quantities of Positive and Negative Electrification Always Produced

344. Let us now modify the electroscope, described in § 332, either by fitting it with a hollow metal vessel, as indicated in Fig. 265, or by connecting it, by use of a light wire, to a hollow insulated vessel, as diagrammed in Fig. 266.

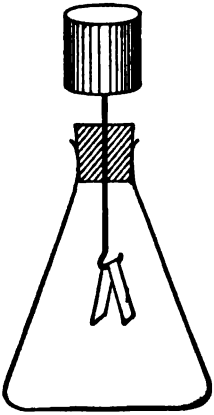

FIG. 265. — Equality of positive and negative charges.

Any charged body introduced into this can will affect the gold leaves even more than if it were brought up near to the electroscope itself; for, indeed, the can is now a part of the electroscope. The can should also be provided with a metal lid, suspended on the same silk thread which carries the charged body. Now what is most remarkable is that it matters not in what part of this closed vessel the charged body is placed, the effect on the gold leaves is just the same.

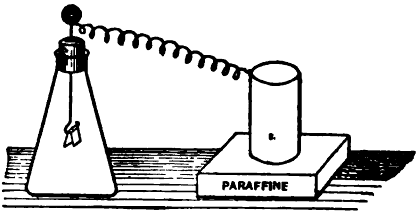

FIG. 266.

We are now prepared to try the following fundamental experiment: Take two nonconducting glass rods, *A* and *B* (Fig. 267). About the end of one of them fasten a short pocket or tube of woolen goods so that it will not easily slip off. On the side of this pocket sew another similar pocket of woolen cloth into which the second glass tube will slide with gentle friction. By merely inserting the glass rod into the pocket and twisting it about, one is easily able to electrify the lower end of the free glass rod positively and the woolen pocket negatively.

Observe then that —

(i) No effect is produced upon the electroscope so long as both the rods are held in the can. When they are electrified by twisting the glass rod in the pocket, no divergence of gold leaves follows.

(ii) But if either the free rod or the one that carries the woolen cloth be removed, the gold leaves diverge, showing that both the cloth and rod are charged.

(iii) The charge on the glass is opposite in sign to that on the woolen cloth.

The same series of experiments may be tried with a piece of silk and a glass rod.

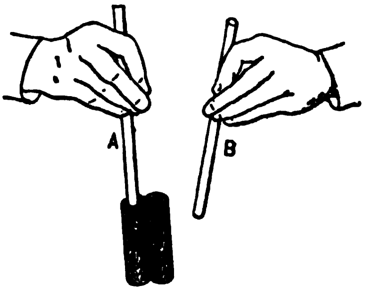

FIG. 267.

In each case we have two oppositely charged bodies such that their combined effect on the electroscope is zero. The same phenomenon is observed, indeed, whatever two bodies are rubbed together. The inference is, therefore, that whenever we produce by friction a certain electric charge, we always produce at the same time an equal charge of opposite sign. This is always true in the case of induction. If we induce two opposite charges on a conductor and then remove the inducing charge, we find that the induced charges will just neutralize each other; all evidence of electrification disappears, showing that the two induced charges are equal and opposite. This general truth, discovered by Faraday and expressed as follows, may be called the *sixth* fundamental fact in electrostatics.

When electrification is produced by friction, by induction, or by any other means, the positive and negative charges so produced are always equal.

Distribution of Electricity over Conducting Surfaces

345. It is also interesting to examine, by means of a proof plane and electroscope, the various parts of the surface of an irregularly shaped conductor, such as that represented in Fig. 268.

The proof plane, after being brought once into contact with a flat portion of the body, does not produce nearly so great a divergence of the gold leaves as when touched once to either end of the body. The sharper the end (*i.e.* the greater the curvature of the surface), the greater the charge which the proof plane carries away.

We see from experiments of this kind that the electrification is not uniformly distributed over a conductor. The sharper any end or point, the more the charge tends to collect there. When we examine the inside and outside of a tin cup, in the same manner, we find that practically all of the charge resides on the outside.

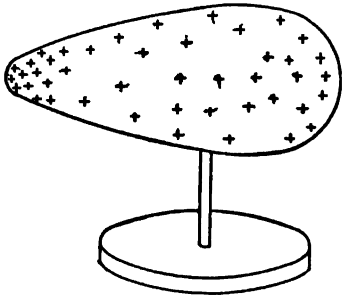
FIG. 268. — Electric charge accumulates near sharp end of body.

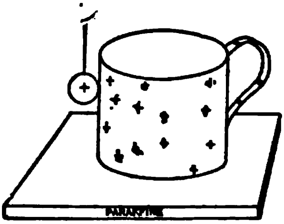

FIG. 269. — Charge resides on outside of conductor.

In general, it has been found that the manner in which electrification distributes itself upon a conductor depends only upon the shape of the body and upon the presence of surrounding charges.

To describe *each* of these various distributions would be an infinite task; but we may always be certain that **the charge on the outside of any closed conductor is distributed in such a way that it will produce no electric field — no electric force — inside the closed conductor.**

Even in the case of a conductor such as a tin pail which is not quite closed, the electrification is very nearly all on the outside, and the intensity of the electric field inside is very small.

Accordingly, if we wish to protect any instrument from electrical disturbances, we have only to place it inside an electrical conductor which is closed or nearly closed.

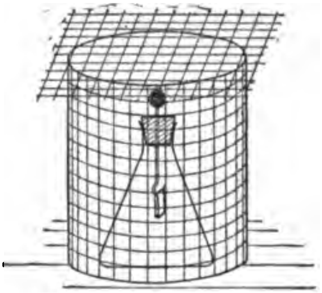

FIG. 270.—Electroscope protected by a network of wires.

Take a small electroscope and place it inside a roll of wire screen, as shown in Fig. 270. If the meshes of the screen are reasonably fine, there is scarcely any electrical disturbance you can produce outside the screen which will affect the sensitive gold leaves inside.

This great fact that any electrical disturbance outside a closed conductor has no effect on the region inside the closed conductor was thoroughly established by Faraday. Let us put it down as *seventh* on our list of fundamental discoveries in electrostatics.

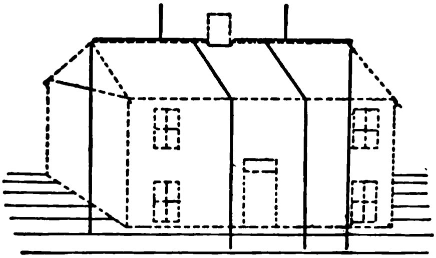

FIG. 271. — Maxwell's method of protection against lightning.

With this principle in mind, Clerk Maxwell suggested that the best method for protecting houses from lightning is to place them in a network of conductors (Fig. 271), which shall be as nearly as possible equivalent to a closed conductor. Benjamin Franklin, in 1760, first suggested the use of single metallic rods, connected with the earth, as a means of protection against lightning. Franklin's discovery of lightning as an electrical discharge and his invention of the lightning rod immediately suggests the question, What is the

Origin of Atmospheric Electricity

346. Up to the year 1910, this phenomenon had remained a profound mystery. Previous to that date many experiments had been made on friction of the air, evaporation, freezing, etc., in hopes that some one of these processes would be accompanied by electrification; but all in vain. Between 1908 and 1910, however, Dr. G. C. Simpson of the British Meterological Service, working at Simla in India, found that the electrification brought down by rain and snow is sometimes positive and sometimes negative; but that the total quantity of positive electricity brought down by rain is 3½ times greater than the total quantity of negative electricity.

Any satisfactory explanation must cover these facts as well as those of lightning.

The German physicist, Lenard, had already shown that raindrops cannot fall through the atmosphere with a speed greater than about 8 meters a second, however large the drop; for in the case of smaller drops the limiting velocity is less than 8 m./sec. and in the case of larger drops it is found that they will go to pieces and break up into smaller drops as soon as a speed of 8 m./sec. is exceeded.

Then Simpson found by experiment that, when drops of distilled water are allowed to fall through a current of air which is moving vertically upward and which has sufficient strength to produce spray, some of the drops become electrified positively and some negatively; but that the negative drops produced are three times as numerous as the positive drops.

Since any process always sets free equal quantities of positive and negative electricity, it follows that the negative charges are confined mainly to the small, slowly moving drops; the positive charges to the larger drops.

The essential feature in the production of atmospheric electricity is then the electrification — or separation of electrons from neutral atoms — which takes place when a raindrop breaks or splashes.

The following remarkably clear account of the electricity which accompanies thunderstorms — as also much of what has been said above — is taken from a lecture given by Professor W. J. Humphreys of our own Weather Bureau before the Franklin Institute in 1914.

" A strong upward current of air is one of the most conspicuous features of the thunderstorm. It is always evident in the turbulent cauliflower heads of the cumulus cloud, the parent, presumably, of all thunderstorms. Besides, its inference is compelled by the occurrence of hail, a frequent thunderstorm phenomenon, whose formation requires the carrying of raindrops and the growing hailstones repeatedly to cold and therefore high altitudes. And from the existence of hail it is further inferred that an updraft of at least 8 metres per second must often occur within the body of the storm, since, as experiment shows, it requires approximately this velocity to support the larger drops, and even a greater velocity to support the average hailstone.

" Experiment also shows that rain cannot fall through air of ordinary density whose upward velocity is greater than about 8 metres per second, or itself fall with greater velocity through still air; that in such a current, or with such a velocity, drops large enough, if kept intact, to force their way down, or, through the action of gravity, to attain a greater velocity than 8 metres per second with reference to the air, whether still or in motion, are so blown to pieces that the increased ratio of supporting area to total mass causes the resulting spray to be carried aloft or left behind, together with, of course, all original smaller drops. Clearly, then, the updrafts within a cumulus cloud frequently must break up at about the same level innumerable drops which, through coalescence, have grown beyond the critical size, and thereby, according to Simpson's experiments, produce electrical separation within the cloud itself. Obviously, under the turmoil of a thunderstorm, its choppy surges and pulses, such drops may be forced through the cycle of union (facilitated by any charges they may carry) and division, of coalescence and disruption, from one to many times, with the formation on each at every disruption, again *according to experiment*, of a correspondingly increased electrical charge. The turmoil compels mechanical contact between the drops, whereupon the charges break down the surface tension and insure coalescence. Hence, once started, the electricity of a thunderstorm rapidly grows to a considerable maximum.

" After a time the larger drops reach, here and there, places below which the updraft is small — the air cannot be rushing up everywhere — and then fall as positively charged rain, because of the processes just explained. The negative electrons in the meantime are carried up into the higher portions of the cumulus, where they unite with the cloud particles and thereby facilitate their coalescence into negatively charged drops. Hence, the heavy rain of a thunderstorm should be positively charged, as it almost always is, and the gentler portions negatively charged, which also usually is the case.

" Such in brief is Dr. Simpson's theory of the origin of the electricity in thunderstorms, a theory that fully accounts for the facts of observation and in turn is itself abundantly supported by laboratory tests and simulative experiments.

" If this theory is correct, and it seems well founded, it must follow that the one essential to the formation of the giant cumulus cloud, namely, the rapid uprush of moist air, is also the one essential to the generation of the electricity of thunderstorms. Hence, the reason why lightning seldom if ever occurs except in connection with a cumulus cloud is understandable and obvious. It is simply because the only process that can produce the one is also the process that is necessary and sufficient for the production of the other."

The Idea of Electric Potential

347. Few conceptions — perhaps none — have ever been introduced into the treatment of electricity that have been more helpful, in the way of simplification, than that of " potential," an idea which is due to the celebrated French astronomer, Laplace. The name is due to the English physicist, Green. Not only is the idea simple; it is quite independent of the meaning of other electrical terms.

Definition of Potential Difference

The **difference of potential** between any two points in an electric field, say P and S in Fig. 272, is defined as **the work required to carry a unit of positive electricity from P to S against the electrical forces in the field.**

Difference of potential is then a difference of electrical condition which determines the direction of transfer of electricity from one point to another; just as difference of level determines the direction of transfer of water from one point to another.

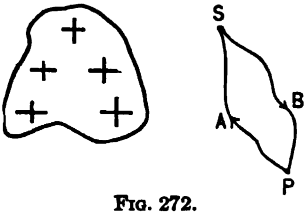

FIG. 272.

It is important to observe that in this definition all forces, such as friction, which are always opposed to the direction of motion are especially ruled out; the only force considered is an electrical one, which, as Coulomb has shown, depends upon the distance only. Since this is true, a little consideration will show that the work done on the unit charge in passing from P to S is independent of the path pursued between these two points. For, suppose that less work is done when the charge is carried along the route A than along the route B; and imagine the charge to be carried along the entire circuit $PASBP$. If the work done along the path PAS is positive, that along the path SBP will be negative; in other words the field will do work upon the charge as it passes from S to P along B. And since more work is done along the path B than A, it follows that we shall, by going through this cycle, get more work out of the field than we have spent upon it; and every time this cycle is repeated we shall gain a definite amount of energy. But since it is impossible to create energy (§ 107), we must conclude that our hypothesis is wrong and that the same amount of work will be required to carry a charge from P to S along any path whatsoever.

Potential Difference a Scalar Quantity

348. From the definition just given it will be seen that difference of potential is a ratio between a certain amount of work and a certain electric charge. Its defining equation, if we denote potential difference by $P.D.$, work by W, and charge by Q, is

$$P.D. = \frac{W}{Q}; \quad \text{Defining equation for difference of potential.} \qquad \text{Eq. 134}$$

and since W and Q are each scalar quantities it follows that potential difference is also a scalar.

Special Case

349. *Find the difference of potential between any two points P and S when the electric field is due to a charge Q on a small sphere at O.*

For the sake of simplicity we may assume (what will not detract from the generality of the result) that the three points O, P, and S all lie in the same straight line, as indicated in Fig. 273. Let us now suppose the distance PS to be divided into a large number, say n, of small equal parts, PR_1, R_1R_2, R_2R_3, . . . $R_{n-1}S$.

Denote the distance OS by r_S, the distance OP by r_P, and the distances OR_1, OR_2, OR_3, etc., by r_1, r_2, r_3, etc. Then remembering that electric intensity is the force on unit positive charge, we have

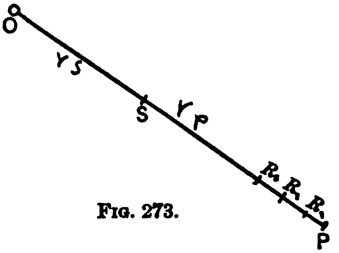

FIG. 273.

$$\text{Electric intensity at } P = \frac{Q}{r_P^2},$$

$$\text{Electric intensity at } R_1 = \frac{Q}{r_1^2},$$

$$\text{Electric intensity at } R_2 = \frac{Q}{r_2^2},$$

etc.,

$$\text{Electric intensity at } S = \frac{Q}{r_S^2}.$$

Hence

$$\text{Average intensity over distance } \overline{PR_1} = \frac{Q}{r_P r_1} \text{ very approximately,}$$

$$\text{Average intensity over distance } \overline{R_1R_2} = \frac{Q}{r_1 r_2} \text{ very approximately,}$$

etc.

Hence the work in carrying the positive unit

$$\text{from } P \text{ to } R_1 = \frac{Q}{r_P r_1}(r_P - r_1) = Q\left(\frac{1}{r_1} - \frac{1}{r_P}\right),$$

$$\text{from } R_1 \text{ to } R_2 = \frac{Q}{r_1 r_2}(r_1 - r_2) = Q\left(\frac{1}{r_2} - \frac{1}{r_1}\right),$$

etc.,

$$\text{from } R_{n-1} \text{ to } S = \frac{Q}{r_{n-1} r_S}(r_{n-1} - r_S) = Q\left(\frac{1}{r_S} - \frac{1}{r_{n-1}}\right).$$

Let us now take the sum of these small quantities of work, and it will be observed that all the intermediate terms cancel out, and we have for the total work done in carrying a unit charge from P to S, the quantity

$$Q\left(\frac{1}{r_S} - \frac{1}{r_P}\right),$$

which is the solution of the problem.

Definition of Potential at Point

350. Let the initial point P be chosen at such a great distance (say at infinity) from the charge Q that $\frac{1}{r_P} = 0$; then the intensity at P is also zero, and we have an expression for the amount of work done in bringing the positive unit from an infinite distance to the point S, namely, $\frac{Q}{r_S}$.

The potential at any point is accordingly defined as the amount of work which must be done in carrying a positive unit charge from an infinite distance to that point. This quantity may be denoted by V_S, and when the field is produced by a single charged particle, may be defined algebraically as follows: —

$$V_S = \frac{Q}{r_S}. \qquad \text{Eq. 135}$$

It is important to observe that the theorem just proved applies not only to a field produced by a charged particle, but also to a body which acts as if its charge were all concentrated in a single point. A charged sphere acts in this way, namely, as if its entire charge were concentrated at the center of the sphere. Hence Eq. 135 gives us the potential of a spherical conductor of radius r_S.

If there be several charges in the field, then the total potential at any point S is obtained by adding the separate potentials at the point S due to each separate charge. Potentials, it must be remembered, are scalar quantities. Hence follows the general theorem that **the potential at any point S, in an electric field, is obtained by adding the quotients of each charge divided by its own distance from the point S.**

For purposes of computation, the following algebraic expression of this theorem is convenient: —

$$V_S = \Sigma\left(\frac{Q}{r_S}\right). \qquad \text{Eq. 136}$$

When this expression is positive, it means that work must be done *upon* the positive unit to bring it from infinity to the point

S; but when negative, the meaning is that work will be done *by* the positive unit in coming from an infinite distance to the point S. In speaking of the potential at a point, it is not necessary to think of any electricity at that point; just as when we speak of a certain height above the sea level we do not need to think of a heavy body at that height.

Zero of Potential

351. Ordinarily it is not necessary to go to infinity in order to reach a point where the electric intensity is zero. For the earth is such a tremendous body that its potential is not appreciably affected by any operations which we may carry on in our laboratories and factories. **Hence the earth is chosen as a body of zero or standard potential on exactly the same principle that the mean sea level is chosen as a surface of zero or standard height.** The potential of any body is, by universal agreement, merely the excess or defect of its own potential above or below that of the earth.

Equipotential Surfaces

352. One of the best methods for describing an electric field is simply to join together all those points in it which have the same potential as any selected point P. But so long as two points are at the same potential, it is evident that no work will be done in carrying electricity from one of these points to the other. Nor will there be any tendency of electricity to flow from one such point to another. In general the locus of these points will be a surface; and this surface is said to be "equipotential."

Accordingly an equipotential surface is defined as one on which the potential has the same value at every point.

Thus if the charge which produces the field is situated on a single particle, and if the charge remains constant, the potential at any point S will depend only upon the distance of S from the particle; and hence, for all points which lie on the surface of a sphere drawn about the charged particle as a center,

$$\frac{Q}{r} = \text{constant}.$$

Any one of the circles shown in Fig. 274 represents, therefore, a section, by the plane of the paper, of an equipotential surface (a sphere) due to a point charge at Q.

Since there is no change of potential over any one of these surfaces, it follows, from the definition of potential, that no work will be done in carrying a unit charge from any one point to any other point on the same equipotential surface. But in taking a positive unit from the surface passing through P to the surface passing through S, one must do $V_S - V_P$ units of work. The simplicity of this expression for work shows the convenience and importance of the idea of potential.

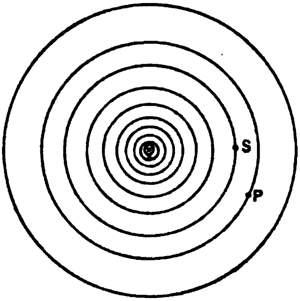

FIG. 274. — Equipotential surfaces due to a point charge.

When the charge resides upon a body other than a particle or a sphere, the equipotential surfaces become, of course, more complicated than those shown in Fig. 274.

The work done in carrying a unit charge from one point to another in an electric field may also be regarded as the product of the average intensity of field R and the distance s between the two points.

Therefore

$$V_S - V_P = Rs, \qquad \text{Eq. 137}$$

where s is the distance between P and S. See Fig. 274.

From this it follows that between any two equipotential surfaces the intensity R will be greatest in the direction in which the distance s is least.

353. And this leads us back again to the idea of

Lines of Force

For " the direction of an electric field " at any point is merely the direction in which the electric intensity is greatest, that is, the direction in which the distance between any two equipotential surfaces drawn near that point is least. **The direction of the electric intensity at any point is therefore normal to the equi-**

potential surface passing through that point. This result would also follow immediately from the definition of an equipotential surface as one along which no work is required to carry an electric charge.

When, therefore, a field has once been described by drawing its equipotential surfaces, it becomes a matter of great ease to also describe it in terms of lines of force. For we have merely to draw the normals to these surfaces at every point. Thus in Fig. 275 the equipotential surfaces are indicated by dotted lines; the normals which are shown solid are, therefore, the lines of force.

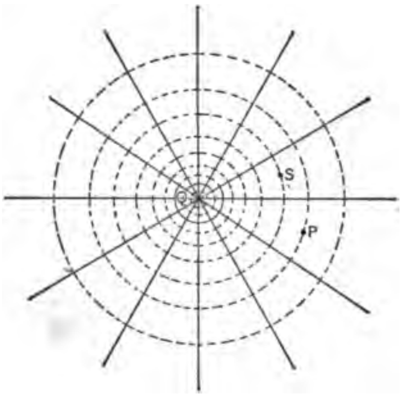

FIG. 275. — Illustrating the relation between lines of force and equipotential surfaces.

Surfaces of Conductors are Equipotential

354. A little consideration will show that when a conductor is insulated and charged it will be an equipotential surface as soon as the electric charge comes to rest, which it does almost instantaneously. For if the component of the electric intensity along the surface of the conductor had, at any point, a value other than zero, the electricity at this point would move in the direction of the force exerted upon it. But, by hypothesis, the charge is at rest. **Any conductor holding a charge at rest is therefore an equipotential surface.** Indeed, this is the one practical method for "manufacturing" equipotential surfaces in the laboratory.

At this point the student will find it helpful to show, by means of a diagram, just how one can arrange a conductor (i) so that it will have a positive charge while it remains at zero potential, and (ii) so that it will have zero charge while it is at positive potential.

The advanced student will discover that the entire science of electrostatics reduces itself almost to a special case of the potential theory.

Measurement of Potential

355. The simplest means for obtaining a quantitative indication of potential difference is perhaps the gold leaf electroscope. But why, some one asks, does not the divergence of the gold leaves measure quantity instead of potential? For it is evident, at once, that the force which drives the gold leaves apart is directly proportional to the amount of charge on the leaves. The reply to this query is that if the electroscope be isolated and entirely disconnected from any other body, the divergence of the leaves *does* measure the *quantity* of charge on the instrument *as well as its potential.* For in the case of any single conductor the charge and the potential increase in the same ratio.

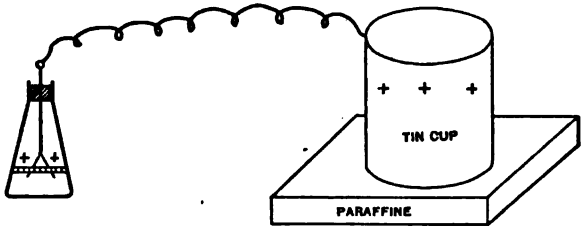

FIG. 276. — Illustrating the use of the electroscope to measure potential.

If, however, the electroscope be connected, *by means of a rather long* wire, with another *and much larger body,* it is clear that the amount of work done in bringing up a unit charge from the earth to the instrument will depend, not at all upon the charge of the larger and distant body, but *only* upon the quantity of electricity *on the electroscope;* in other words, the potential of the system (made up of the electroscope, the wire, and the large body) is shown directly by the divergence of the gold leaves.

Wherever the electroscope be placed, the connecting wire insures that its potential shall be the same as that of the body; for this wire renders the entire system a single metallic conductor. Note that, whatever part of the large body is touched by the connecting wire, the reading of the electroscope remains the same.

If while the electroscope be still connected with, say, the tin cup in Fig. 276, another electrified body or conductor be brought into the immediate neighborhood of the electroscope, its divergence may change very much, but its potential will remain constant. The reason the divergence may change is precisely *because* the potential of the gold leaves does remain constant. For if a unit charge be now carried up to the electroscope from infinity, the charge on the neighboring body *B*, in Fig. 277, will either aid or oppose the motion; and hence the charge on the leaves will need to be either larger or smaller, respectively, than before, in order to maintain their potential the same as that of the large conductor with which they are connected.

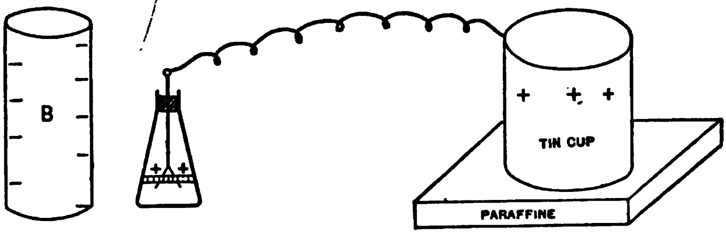

Fig. 277. — Illustrating the effect of a neighboring body on the reading of an electroscope.

Therefore, in order to use an electroscope as an instrument for measuring potentials, it is only necessary to connect the body and the electroscope by a wire which is long in comparison with the dimensions of the body, and to see that no third body influences the gold leaves.

It need hardly be added that the size of the electroscope must be small in comparison with that of the body whose potential is to be measured, otherwise the original charge on the body would be appreciably diminished and its potential thereby reduced.

The divergence of the gold leaves always indicates the difference of potential between the leaves themselves and the conductors immediately surrounding them. For this reason, it is customary to surround the gold leaves with a metallic hood, such as that shown in Fig. 270, which is sufficiently open for one to view the leaves. This metal cover is then connected to earth and one is thus always sure that the difference of potential which he is measuring is that between the body connected to the leaves and the earth. An instrument of this kind provided with a scale is known as an **electrometer.** The advanced student will be interested in understanding and using the quadrant electrometer devised by Lord Kelvin and perfected by Dolazalek, a fundamental measuring instrument in electrostatics.

The Idea of Electrical Capacity

356. Imagine the potential of some one conductor to be increased while all the surrounding conductors are connected to earth so that their potential remains zero. Then two things must happen; the charge on this one conductor must increase and the charges induced by it on the surrounding conductors must increase in the same proportion.

Consequently the value of the potential at any point on this conductor, which we have already found to be $\Sigma\left(\frac{q}{r}\right)$, will increase in the same ratio as the charge on this one conductor. This constant **ratio between the charge and potential when all other conductors are connected to earth** is called the **capacity** of the conductor.

The capacity of a conductor is *not,* therefore, "the amount of electricity it will hold when it is full," but its numerical value is the same as the amount of electricity which the conductor holds when its potential is unity; in other words, it is the constant ratio which experiment shows to exist between the charge and the potential of the conductor.

If capacity be denoted by C, its definition may be given in algebra as follows: —

$$C = \frac{Q}{V}. \qquad \text{Defining equation for electrical capacity.} \qquad \text{Eq. 138}$$

This equation will be found exceedingly useful in computing any one of the three quantities involved, and may indeed be regarded as the fundamental equation of electrostatics.

Capacity of a Sphere

357. It is evident from the outset that the capacity of a body must depend upon its size. Take, for example, two spheres of different radii and place the same charge upon each of them. We have already found (§ 350) that the potential of a sphere is $\frac{Q}{R}$. Hence, in this case, the charge Q will raise the larger sphere to a lower potential than it will the small sphere. Hence by the definition of capacity (Eq. 138) it is clear that the larger sphere will have the larger capacity. Imagine a sphere to be placed in a room which is large compared with the sphere. We may at once find the capacity of the sphere by computing the potential at the center of the sphere produced by a positive charge Q. If we denote any small portion of the charge on the surface by q, and the radius by R, then for the potential at the center we may write

$$V = \Sigma\left(\frac{q}{R}\right) = \frac{1}{R}\Sigma(q) = \frac{Q}{R}, \text{ which is Eq. 135.}$$

If the room is large the potential at the center of the sphere will not be appreciably affected by the negative charges induced on the walls of the room. Hence the expression $\frac{Q}{R}$ gives the potential at the center of the sphere and also for the metal sphere itself, since the potential throughout such a closed conductor is uniform. And since in all cases we have, by definition, $V = \frac{Q}{C}$, it follows that in the case of a sphere, in air,

$$C = R. \qquad \text{Eq. 139}$$

In other words, the capacity of a sphere is equal to its radius, which is a purely geometrical quantity.

But suppose now that the positively charged sphere instead of standing alone in a large room has other conductors standing near it, and connected to earth. The induced charges on these other conductors are of opposite sign to that on the sphere. The result is that less work will be required than before to bring a positive unit up to the sphere, the charge on the sphere remaining the same; for now the induced charge which is negative in sign will help by attracting the positive unit. In other words, the potential of the sphere will be diminished by the presence of neighboring conductors; and since the charge remains the same, the capacity of the sphere in the presence of its neighbors is increased in the same ratio in which its potential is diminished.

358. In illustration of this fact, we proceed to compute the

Capacity of Two Concentric Spheres

or, more properly, the capacity of a sphere surrounded by another concentric sphere, which is connected to earth. The arrangement is shown in Fig. 278, where the two shaded blocks represent insulation. The charging wire passes through a small hole in the outer sphere. Such a device is frequently called a "condenser."

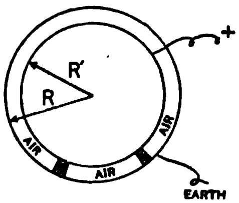

FIG. 278. — Condenser made up of two concentric spheres.

As before the potential of the inner sphere is uniform and equal to that at the center. And since every part of the positive charge is at a distance R' from the center, and every part of the equal induced negative charge at a distance R, we have, for the potential of the inner sphere,

$$V = \Sigma\left(\frac{q}{r}\right) = \frac{Q}{R'} - \frac{Q}{R}.$$

Hence the capacity of the inner sphere is

$$C = \frac{Q}{V} = \frac{Q}{\dfrac{Q}{R'} - \dfrac{Q}{R}} = \frac{RR'}{R - R'}. \qquad \text{Eq. 140}$$

From Eq. 140, it will be seen that the mere presence of the outside sphere, connected to earth, increases the capacity of the inner sphere in the ratio of R to $R - R'$.

Capacity of Two Parallel Plates

359. Still considering the pair of concentric spheres, imagine them to increase in size until the inner one is enormous compared with the distance between the two spheres. Then $R - R'$ will be negligible compared with R' and we may write for the capacity of the system

$$C = \frac{4\pi RR'}{4\pi(R-R')} = \frac{4\pi R'^2}{4\pi(R-R')}, \text{ very approximately}$$
$$= \frac{\textit{area of sphere}}{4\pi \times \textit{thickness}} = \frac{A}{4\pi t}, \text{ say.}$$

Now any pair of parallel plane plates at a distance, t, apart may be considered as a portion cut out of an infinitely large pair of concentric spheres separated by the same distance, t.

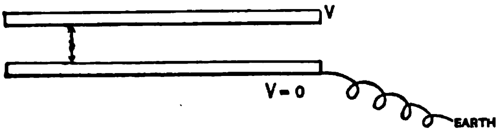

FIG. 279.—Condenser made up of two parallel plates.

By symmetry the capacity of such a portion will be proportional to the area of this portion; and accordingly we have for the capacity of a pair of parallel conducting plane plates, in air,

$$C = \frac{A}{4\pi t}, \qquad \text{Eq. 141}$$

where A is the area of either one of the plates and t is the distance separating them.

Effect of Medium upon Capacity

360. We have just seen that the electrical capacity of a conductor depends partly upon its dimensions, and partly upon the presence of neighboring bodies.

But it was discovered by Faraday in 1837, that there is a *third* factor entering into capacity, namely, the medium which surrounds the conductor. This result came out of the following experiment. Faraday prepared two equal condensers, each consisting of an outer brass sphere, fitted with an inner brass sphere insulated from it. The space between the spheres of one condenser was left filled with air; that between the spheres of the other condenser was filled with shellac. It was then observed that, when the air-filled condenser was charged up to any given potential, it required 1.37 times as much electricity to bring the shellac-filled condenser to the same potential. In other words, a medium of shellac gives to its condenser a capacity 1.37 times greater than does a medium of air. Imagine two condensers, A and B, alike in all respects except that A is filled with air and B with some other medium, then, by definition, the ratio

$$\frac{\textit{Capacity of } B}{\textit{Capacity of } A} = \textbf{Dielectric constant} \text{ of the medium } B.$$

The dielectric constant of a substance is generally denoted by K and is often spoken of as its *specific inductive capacity.* The student will remember that he has already met this same quantity in the discussion of Coulomb's Law (§ 337). Indeed the only reason the capacity of a body is increased by surrounding it with shellac, sulphur, etc., is that by so doing one diminishes the force to be overcome in bringing up to the body a unit charge from the earth.

In the light of Faraday's discovery, which we may call the *eighth* fundamental fact of electrostatics, it now becomes necessary to modify our previous expressions for capacity when any dielectric other than air is employed, and write,

For a sphere alone, $C = KR.$ Eq. 139′

For concentric spheres, $C = \dfrac{KRR'}{R - R'}.$ Eq. 140′

For parallel plates, $C = \dfrac{KA}{4\pi t}.$ Eq. 141′

Summarizing, one may say that three factors determine the capacity of a conductor, namely, (i) *the geometrical dimensions of the body;* (ii) *the presence of other conductors in the neighborhood, and* (iii) *the dielectric constant of the medium in which the conductor is placed.*

The Leyden Jar

361. It is now nearly two hundred years since Pieter van Musschenbroek (1692–1761) at Leyden discovered that a bottle, half filled with water and held in the hand, made a very convenient collector of electric charge.

The student will at once recognize that the water is here the charged conductor, surrounded by a dielectric of glass, this in turn surrounded by another conductor (the hand) connected to earth. For laboratory purposes, it is more convenient to replace the water by an inside coating of tin foil, and the hand by an outside coating of the same metal. Large capacities are made by building up alternate layers of tin foil and thin mica, or glass, sheets.

All of our Atlantic cables and underground wires are really Leyden jars. The central wire is the inner coating, the insulating material is the dielectric, and the sheath of steel wire or lead is the outer coating. One must never forget that, in attempting to telephone or telegraph through such a cable, he is using a Leyden jar whose capacity is proportional to its length.

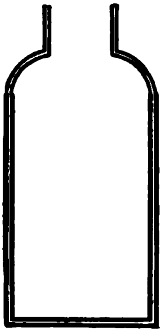

FIG. 280. — Vertical section of Leyden jar.

Energy of a Charged Conductor

362. In view of the fact that the energy of any isolated system — electrical as well as mechanical or thermal — remains constant, the expression for the energy of a Leyden jar becomes a matter of considerable importance.

To obtain the energy of a charged conductor is not a difficult matter; for, by the law of the conservation of energy, we have only to measure the amount of work done in bringing the charge up and putting it on the conductor.

Suppose we start with an insulated conductor whose potential is zero; let it be charged by conduction from some source. We may think of any finite charge which is given the conductor as brought up in very minute charges, one at a time; each one of these will slightly increase the potential of the conductor, so that when the charge is complete the final potential of the conductor will be, say, V.

Had the potential of the conductor remained at its initial value (zero), the work required to bring up any small charge would have been vanishingly small, since the potential of any conductor is the amount of work required to bring up a unit charge from the earth to that conductor.

If, on the other hand, the conductor had been at potential V during the whole of the charging process, then the amount of work required to bring up any small charge q would have been qV.

But, as a matter of fact, the potential of the conductor varied directly as the amount of charge placed upon it, so that **the mean value of the potential during the charging process was $\frac{V}{2}$. If then the total charge be called Q, the amount of work required to charge the conductor is $\frac{1}{2}QV$; and this by the principle of the conservation of energy represents the energy of the charged conductor.** In virtue of the defining equation for capacity, $Q = CV$, this energy expression may take any one of the three following forms, each of which is useful, namely: —

$$\left.\begin{matrix}\text{Energy of}\\ \text{Charged Conductor}\end{matrix}\right\} = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}. \qquad \text{Eq. 142}$$

When C, Q, and V are each measured in C.G.S. units, it follows, of course, that Eq. 142 gives the energy in terms of ergs.

When a Leyden jar or other conductor is discharged, this expression gives us a measure of the heat which is produced in the spark gap and in the wires which lead to it. The first to measure this heat was Dr. Kinnersley (contemporary and friend of Franklin), who inclosed two knobs between which the discharge took place in a glass bulb which was essentially the bulb of an air thermometer. The lead wires from the two coatings of the jar passed through the glass walls of the bulb.

The Oscillatory Discharge

363. It was first observed by Savary that, when a Leyden jar is discharged through a coil of wire so as to magnetize a sewing needle placed inside the coil, sometimes the needle is magnetized in one way, sometimes in the opposite way, even though the inner coating of the jar be always charged with electricity of the same sign and the coil connected in the same way. See Fig. 281.

This fact, known as "anomalous magnetization," was first explained by Joseph Henry in 1842, by supposing that the charge oscillated from one coating to the other, but became a little smaller at each oscillation. Accordingly the last discharge which was strong enough to reverse the magnetization of the needle was the one which determined which end of the needle would be the north pole and which the south.

Later (1857) Feddersen examined the spark of a Leyden jar discharge by means of a rotating mirror, such as that which is used for viewing a manometric flame (§ 227), only turning much more rapidly, and found that in this mirror the image of the spark looked something like a string of beads, thus furnishing irresistible evidence that the spark was, under his conditions, a periodic phenomenon.

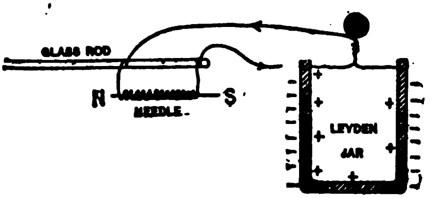

Fig. 281. — Illustrating Henry's experiment.

In more recent times, C. V. Boys and many others have photographed this oscillatory discharge.

The advanced student will be interested in following Lord Kelvin's beautiful prediction of this periodic phenomenon, made in 1853 before Feddersen had proved it by experiment. For Kelvin proves, what has since been verified by experiment, that the period of the discharge is

$$T = 2\pi\sqrt{\frac{L}{1/C}} = 2\pi\sqrt{LC}, \qquad \text{Eq. 143}$$

where C is the capacity of the jar and L a quantity which represents the *electrical* inertia of the circuit and which is known as *self-inductance.* The reciprocal of capacity represents therefore, by analogy with the pendulum, spiral spring, and other vibrating systems, the electrical elasticity of the circuit. But this is a matter which can be cleared up only by more advanced study.

Rowland's Berlin Experiment

364. It is easily shown that an electric charge at rest produces no effect upon a compass needle or upon any delicately suspended magnet that it would not produce upon the same needle before it was magnetized. In other words, magnetism and electrification at rest seem to be entirely independent phenomena.

Maxwell, Kelvin, and others were, however, led to think that an electrically charged particle once put into motion ought to produce a magnetic field in its neighborhood.

But it was not until 1876 that the experiment was tried. In that year, H. A. Rowland of Baltimore, working in Berlin, mounted a vulcanite disk on a vertical axis, gilded the disk on both sides, charged the gilt surface, and put it into rapid rotation. A sensitive magnet near by showed no deflection so long as the disk was at rest; but the magnet was deflected as soon as the disk was set into motion, showing that the translation of the charged gold foil had established a magnetic field. When the sense of rotation was reversed the deflection was reversed; and when the sign of the electricity on the rotating disk was reversed, the deflection was also reversed.

The importance of this result for the electron theory can hardly be overestimated. It furnishes a firm experimental basis for Langevin's theory of magnetism, explained in § 329 above, as well as for the electron theory of conduction, and explains many other phenomena, such as deflection of cathode rays, in which electrons are moving at high speed.

We might call this discovery of Rowland's the *ninth* fundamental fact regarding electric charges.

Problems

345. An insulated conducting sphere whose radius is 7 is charged until its potential is 5. Find the amount of its charge. *Ans.* 35.

346. Two insulated conducting spheres whose radii are 3 and 8 respectively are charged so that their potentials are 5 and 10 respectively. Find the common potential which these two spheres will assume when connected by a wire whose capacity may be neglected. *Ans.* $8\frac{7}{11}$.

347. In the preceding problem find the energy of each of the two charges before the spheres are connected. Then compute the energy of the final charge. Explain the difference.

348. A small sphere of rosin carries a charge of 2 electrostatic C.G.S. units and is placed in an electric field whose intensity is 25 C.G.S. units. Find the force in dynes with which the sphere will be urged.

349. Find the attraction between two small balls, 4 cm. apart in air, one of them having a positive charge of 12 units, the other a negative charge of 5 units.

350. At each of two corners of an equilateral triangle whose sides are 60 cm. is placed a charge of 40 C.G.S. units of electricity. Find the intensity of the electric field at the third corner.

351. A conducting sphere of 20 cm. radius carries a charge of 80 units. If it be made to share its charge with another insulated sphere whose radius is 5 cm., what will be the ratio of the charges on the two spheres?

352. Find the electric potential at a point 40 cm. distant from the center of a hollow sphere of 4 cm. radius charged with 60 C.G.S. units of electricity.

Suppose the radius of the hollow sphere to be 100 cm., how will the result be affected?

353. Find the "dimensions" of electrical quantity, electrical potential, and electrical capacity.

354. Find the capacity of a condenser made up of two concentric spheres having radii of 100 and 98 respectively, and separated by air.

355. Two condensers of the same dimensions as in the preceding problem are exactly alike except that in one case the space between the spheres is filled with an oil whose dielectric constant is 2. How will a charge of 100 units divide itself between these two condensers?

356. Which will require the greater work; to charge a condenser of capacity 10 to a potential 5, or a condenser of capacity 30 to a potential 3?

357. A charged insulated disk is connected with a gold leaf electroscope. Describe and explain the change of divergence in the leaves as a second disk held in the hand is made to approach the first.

358. Explain the difference between joining up Leyden jars "in series" and "in parallel."

359. A parallel plate condenser in air has its plates separated by a distance of ½ mm. What change in its capacity will be produced if the plates are separated to a distance of 2 mm. and the intervening space filled with an oil whose dielectric constant is 3?

360. Draw the lines of force which represent the distribution of electricity on a Leyden jar, the outer coating of which is connected to earth.

361. How much power would be required to charge a condenser having 3000 C.G.S. units of capacity to a potential 200 C.G.S. units in ⅛ second?

362. A condenser whose capacity is C is charged with a quantity Q. What diminution of energy will occur when one half this charge is led off to earth?

363. What must be the radius of a sphere in order that a charge of 80 units may raise its potential 16 units?

364. How would you proceed to find the difference of potential between two points, knowing their distance apart and the average intensity of the field between them?

365. Prove that an equipotential surface can intersect neither itself nor any other equipotential surface.

366. What is the curve which exhibits the relation between the charge and the potential of an insulated conductor?

367. What is the intensity of field between two plane condenser plates which are $\frac{1}{10}$ cm. apart and differ in potential by 200 C.G.S. electrostatic units?

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"Many men who have no experience are ready to express an opinion."

THEOGNIS.

CHAPTER X

ELECTRIC CURRENTS

365. Electricity has been frequently spoken of as flowing from one end of a conductor to another, and as redistributing itself on a conductor. In the Leyden jar discharge, electricity is spoken of as oscillating from one coating to the other. Rowland succeeded in impressing a speed of 100 ft./sec. upon an electric charge.

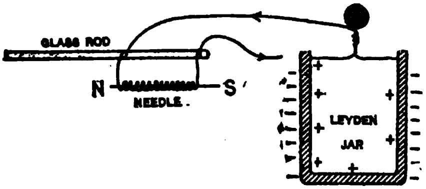

FIG. 282. — An electric charge passing along a coiled wire magnetises a sewing needle.

In all these cases where electricity is put into motion the phenomenon is spoken of as an **electric current**, and the conductor along which the electricity moves, or with which it moves, becomes endowed with two very remarkable properties, namely: —

(i) Every part of the conductor is surrounded with a magnetic field, *i.e.* every part of the conductor affects a magnetic needle when brought near it.

(ii) Heat is developed in every part of the conductor through which the current flows.

Any wire exhibiting these two properties is said to have an electric current flowing through it. Thus if a Leyden jar be discharged through a spiral of copper wire, as indicated in Fig. 282, one finds that at the instant of discharge this little coil of copper wire acquires the ability to magnetize a sewing needle placed inside of it.

An even simpler illustration of the current to be obtained from a charged body is indicated in Fig. 283. Here the two wires of a current-indicator — called a galvanometer — are connected one to the soleplate, the other to the carrier of a Volta electrophorus.

The galvanometer consists merely of a coil of wire with a suspended magnet at the center or of a magnet with a suspended coil of wire between its poles. Either device will indicate the presence of a magnetic field about the coil of wire.

In trying the experiment it is convenient to follow these five steps: —

(i) Charge the electrophorus by friction.

(ii) Connect the soleplate to one terminal of the galvanometer.

(iii) Place the carrier, *by means of its handle,* upon the plate. The carrier now contains two charges as shown in Fig. 264, p. 397. The negative charge, on top, is free to run off; the positive charge on the underside is held bound by the negatively charged ebonite plate.

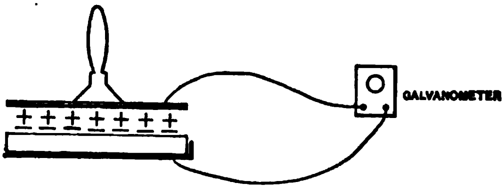

FIG. 283. — Showing the current which is produced when an electrophorus is discharged.

(iv) Now connect the *second* terminal of the galvanometer to the carrier. The negative charge passes to the soleplate, *via* the galvanometer, and produces a deflection in the galvanometer.

(v) Now lift the carrier off the plate, keeping the galvanometer terminal attached to it. The *positive* charge on the carrier now becomes free and flows off through the galvanometer, producing a deflection in the opposite sense.

This experiment serves merely to illustrate the connection between the subject we have just left and the one we are just approaching. The first great step in the science of electric currents was taken by the man who first showed us how to produce a permanent flow of electricity, a continuous, as distinguished from an intermittent or impulsive, current. This man was he whose acquaintance we have already made as the inventor of the electrophorus.

The Production of Electric Currents

366. There are three principal methods of producing electric currents. In chronological order, these are: —

(i) The method of Galvani and Volta, discovered by Galvani at Bologna in 1786, perfected by Volta at Pavia in 1799.

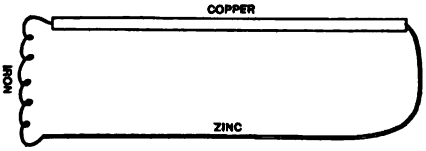

Fig. 284. — Circuit made up entirely of conductors of the first class.

(ii) The method of Seebeck — thermoelectric currents — discovered in 1821.

(iii) The method of Faraday, employing magnets, discovered in 1831.

In this chapter we shall consider only the first two of these methods, postponing the third until we take up the subject of electromagnetism of which it is the very foundation.

The Voltaic Cell

367. At the very close of the eighteenth century it was discovered by the Italian physicist Volta that all conductors of electricity can be divided into two classes. This division is based upon the following experiments: —

If we make a closed circuit out of several different metals, *i.e.* if we make an endless chain in which each link is composed of a different simple substance, such as zinc, copper, or iron, we see that no electric current is produced. **All substances which when joined together at the same temperature in any order, as in Fig. 284, produce no current, are called conductors of the first class.**

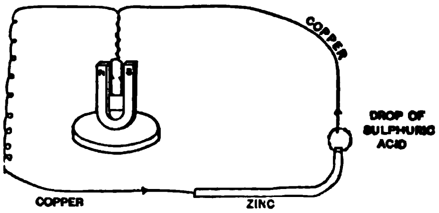

Fig. 285. — Combination of conductors of first and second class producing a current.

Volta, found, however, that if into a circuit such as the above he introduced one link composed of a *compound* substance, such as a solution of table salt or dilute sulphuric acid or copper sulphate, he then always obtained an electric current. **Conductors of this kind, which Volta called conductors of the second class, always undergo chemical decomposition when introduced into a circuit containing two different metals and always yield a current.** The modern name for a conductor of the second class is **electrolyte,** *i.e.* any substance which is decomposed when a current passes through it. Conductors of the first class practically include only carbon and the various metals.

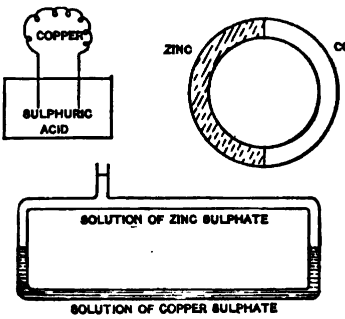

FIG. 286. — No two conductors can act as a Voltaic cell.

368. It has been found by experiment that no two conductors when joined together will produce a current so long as they are at the same temperature. We may use any of the three combinations indicated in Fig. 286, but no current will thus be obtained. **But three or more conductors in series, each conductor being made of a different substance, and not all belonging to the same class will always give a current.** Any such combination is called a *Voltaic cell.*

One of the simplest of these cells is made up as follows: take a strip of sheet copper, say one centimeter by ten; solder it, end to end, to a similar strip of zinc. Take a small beaker of dilute sulphuric acid, say one part strong sulphuric acid to ten parts of water. Complete the circuit by bending the copper-zinc strip into a U-shape and dipping the ends into the acid, as shown in Fig. 287. Bubbles of gas are given off from each metal, but much more freely from the copper.

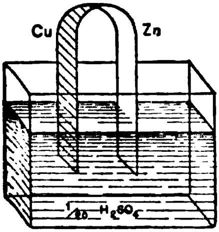

FIG. 287. — Typical Voltaic cell.

Here the zinc and copper are conductors of the first class, the dilute sulphuric acid is a conductor of the second class. The acid undergoes decomposition because the oxygen of the water has a greater affinity for the zinc than for hydrogen. The zinc is said to be "eaten up" by the acid. Zinc sulphate is formed and dissolved in the solution which becomes less and less acid.

If the copper or zinc is cut at any point outside of the solution and if a galvanometer be inserted, it will be found that an electric current flows from the copper to the zinc as long as this chemical action continues.

Some Definitions

369. Closed circuit. A series of conductors joined end to end so as to form an endless chain is said to be a closed circuit. It may or may not have an electric current passing through it.

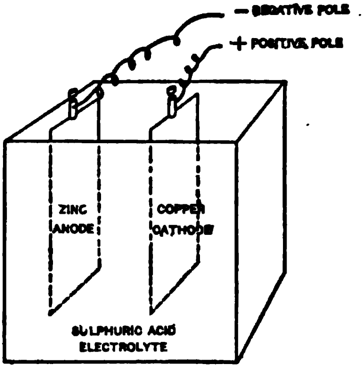

FIG. 288. — Various parts of a Voltaic cell.

Open circuit. When a closed circuit is cut or interrupted at any point, it is then called an open circuit.

Poles. If there be a Voltaic cell in the circuit, the two ends thus left free by opening the circuit are called the poles of the cell. The pole which is connected with the copper plate, or with the plate whose electrical behavior is like that of copper toward zinc, is called the **positive pole.** That pole which is connected with the zinc plate, or with the plate whose electrical behavior is like that of zinc toward copper, is called the **negative pole.**

Electrodes. We have already seen that at least one member in the chain of conductors which makes up the circuit of a Voltaic cell must be an electrolyte, practically always a solution. The two conductors which connect with this liquid are called **electrodes.**

The electrode by which the current enters the electrolyte is called the **anode**; and that electrode by which the current leaves the electrolyte is called the **cathode.** In the typical cell which we have just studied the zinc plate is the anode and the copper plate the cathode. Etymology of these two words? Very often a current from some outside source is passed through a cell containing an electrolyte; in such a case both electrodes may be made of the same substance.

Battery. Two or more Voltaic cells connected together are called a **battery.**

The eight definitions just given are so frequently and so continually used in all departments of electrical science, that the student should master them here and now, once for all.

370. Direction of Current. When the circuit is closed by a wire joining the positive and negative poles, the direction of the current in this wire is said, by universal agreement, to be from the positive to the negative pole. We may hereafter, therefore, speak of the current as leaving the positive pole and entering the negative pole. It may be shown by the following beautiful experiment, due to Volta, that the direction of the current as above defined is the direction in which the positive electrification *may be considered* to travel. As a matter of fact there is no evidence whatever for thinking that positive electricity ever does travel through a wire or through any other body. Experiment indicates that it is the motion of the negatively charged electrons which produces those effects which we ascribe to an electric current. The direction of an electric current is then just the opposite of that in which the electrons travel.

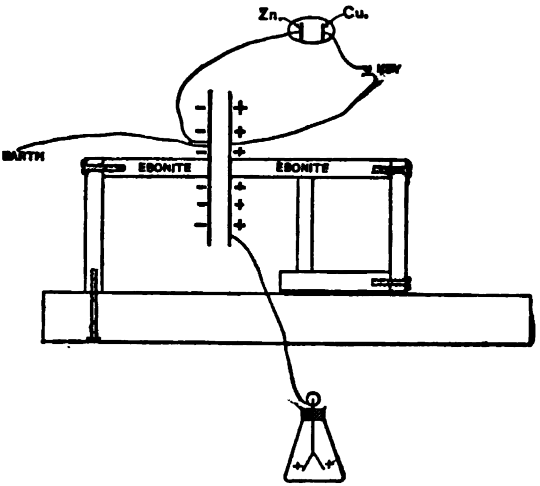

FIG. 289. — Illustrating the reason for calling the copper pole of a cell positive.

Volta's experiment was as follows: Take an adjustable parallel plate condenser such as that shown in Fig. 289 and join the positive pole of a Voltaic cell to one plate, the negative pole to the other plate. Connect one pole, say the negative, to earth also. Place an electric key between the copper pole and the movable plate of the condenser. Then, if an electroscope be joined to the movable plate, it will be observed that this plate always acquires a positive charge when the key is depressed. In order to make the charge as large as possible the condenser plates should be brought very close together before the key is depressed (in order to increase the capacity of the system) and then separated after the key has been released. In like manner, it is found that when the poles of the cell are reversed, the charge on the electroscope is also reversed.*

Electromotive Force of the Voltaic Cell

371. Anything which can move electricity from one place to another is called an **electromotive force,** a long word which we shall sometimes contract into E.M.F. One method of moving electricity from one point to another is to produce a difference of potential between these points; just as one can start a flow of water between two points by producing a difference of water level between those points.

As we shall see presently (§ 420) difference of potential is not the only kind of electromotive force we meet in nature.

In the Voltaic cell it is evident that some kind of E.M.F. is continually at work, else the current would cease; and experiment shows that between the poles of such a cell there exists a difference of potential. There appears to be a definite and measurable difference of potential produced by the contact of any conductor with an electrolyte; and the amount of this difference of potential appears to be characteristic and different for each metal immersed in the electrolyte. The whole question of these contact differences of potential is an extremely complicated and difficult one. Some differences of opinion still exist among scholars. But this much at least appears to be clear, that a strip of copper, partly immersed in a dilute solution of sulphuric acid, stands at a higher potential than a strip of zinc partly immersed in the same acid. Part of this difference of potential occurs at the surface between the zinc and the electrolyte; practically all of the remainder is located in the surface between the electrolyte and the copper. Now these three substances constitute the typical Voltaic cell; and hence when the poles of such a cell are joined by a conducting wire, the end of this wire which is connected to the copper pole will be maintained at a higher potential than the end connected with the zinc pole. Consequently the current flows from the copper pole to the zinc pole, outside of the cell; and from the zinc electrode to the copper electrode inside the cell. This process continues as long as the chemical action between the zinc and the electrolyte is kept up.

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* In demonstrating this phenomenon, it is important to remember that if the two shellacked surfaces of the condenser plates come into contact they are almost certain to produce troublesome electrification. Three tiny bits of paper pasted on one disk, 120° apart, will prevent this contact without seriously diminishing the capacity of the condenser.

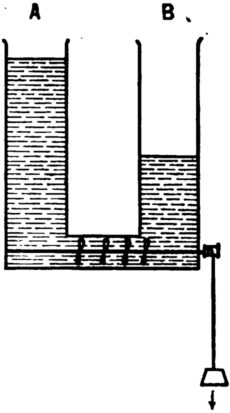

FIG. 290. — Analogue of an open circuit.

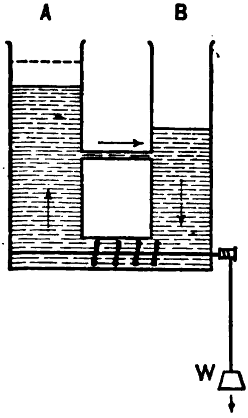

FIG. 291. — Analogue of a closed circuit.

An Analogy

372. Very helpful is the following hydraulic analogy devised by Sir Oliver Lodge. He likens a Voltaic cell to two tall vessels of water, connected at the bottom by means of a pipe in which there is at work a water pump, such as the propeller blades shown in Fig. 290. The pump is driven by a weight, *W*, and will lift the water in the tank, *A*, until the difference of level between *A* and *B* furnishes a back pressure just equal to the forward pressure of the pump. The difference of pressure here represents a hydro-motive force just as the difference of potential in the cell represents an electro-motive force.

Now imagine the tanks *A* and *B* to be connected by a cross-pipe as indicated in Fig. 291. It is evident that the level in *A* will fall to some extent and that a current of water will be maintained in the cross-tube as long as the pump works. So in the case of the cell, as soon as the poles are joined by a conductor, their potential difference diminishes slightly and a current of electricity is maintained as long as there is any zinc unconsumed in the acid.

Just as the work done by Lodge's pump is at the expense of the potential energy of its driving weight, so the work done by a voltaic cell is at the expense of the chemical energy of the cell.

SOME PRACTICAL FORMS OF THE VOLTAIC CELL

373. 1. The Gravity Cell. This cell is composed of a copper plate immersed in a saturated solution of copper sulphate, and a zinc plate immersed in a solution of zinc sulphate, generally arranged essentially as indicated in Fig. 292.

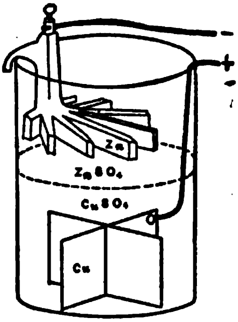

FIG. 292. — A gravity cell.

The copper sulphate is somewhat denser than zinc sulphate, and hence remains in the lower half of the jar, while the zinc sulphate, when carefully poured in, floats on top. Hence the name, "gravity" cell.

One of the chief merits of this cell is, that so long as the circuit is open there is not much waste of material taking place in the cell. When, however, the circuit is closed and the electric current begins to flow, we observe that the zinc plate grows smaller, that the copper sulphate grows less and less, while the zinc sulphate increases and the copper electrode becomes covered with a coating of fresh metallic copper.

In the shorthand of chemistry these phenomena are described quantitatively as follows: —

$$Zn + ZnSO_4 + CuSO_4 + Cu = 2\ ZnSO_4 + 2\ Cu.$$

The metallic zinc has disappeared; the metallic copper has increased, at the expense of the copper sulphate. Accordingly, if such a battery is to furnish a current continuously, it must now and then be supplied with a new zinc plate and with a fresh solution of copper sulphate.

The source of energy is here evidently the zinc, which is oxidized — "burned" if you like — to form zinc sulphate.

This cell is well adapted to work in which small currents are needed very frequently or for a long while at a time.

374. 2. The Leclanché Cell. In this cell the two conductors of the first class are zinc and carbon; the electrolyte is a solution of ammonium chloride, sometimes called "sal ammoniac." With the carbon is mixed some dioxide of manganese, and both are held in a porous jar or small canvas bag. The current easily passes through the pores of the jar or the meshes of the bag. The carbon plate here evidently corresponds to the copper plate of the gravity cell. Here again the zinc is used up whenever the poles are joined and the current flows. The sal ammoniac also diminishes. Accordingly, from time to time, the zinc and sal ammoniac must be renewed.

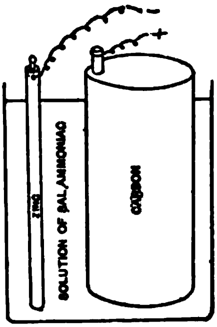

Fig. 293. — A Leclanché cell.

The student has probably already asked himself why the oxide of manganese is mixed with the carbon. The answer is found in the fact that, if carbon alone is used, the carbon becomes covered with a layer of hydrogen gas soon after the circuit is closed; for the ammonium chloride (NH_4Cl) breaks up, giving off ammonia (NH_3) and depositing free hydrogen (H) at the carbon electrode. The chlorine unites with the zinc.

This coating of hydrogen makes the carbon plate cease to act as a carbon plate and to behave like a metallic plate; it also offers more resistance than carbon to the passage of the current. **This phenomenon is called polarization, which may be defined as the establishment of an E.M.F. in a direction opposite to that furnished by the cell.**

The effectiveness of a cell is always diminished when polarization occurs. The manganese dioxide, however, uses up the hydrogen and keeps the carbon plate clean. The purpose of the manganese dioxide is, then, to prevent polarization. It does this by furnishing oxygen which unites with the hydrogen to form water; and once in union with oxygen the hydrogen is harmless.

In the language of chemistry, the action of the Leclanché cell is described as follows: —

$$Zn + 2\,NH_4Cl. + 2\,MnO_2 = ZnCl_2 + 2\,NH_3 + Mn_2O_3 + H_2O.$$

This same equation also describes what goes on in the so-called "dry cell" of which some 50 million a year are used in the United States.

The Leclanché cell is excellently adapted to furnishing a current for a small time, or for intermittent use, as in the case of a door bell or call bell. It will remain on open circuit with still less waste than occurs in the gravity cell.

375. 3. The Storage Cell. In this cell the electrodes are of lead and lead peroxide, respectively; the electrolyte is dilute sulphuric acid. The lead plate is the negative one; the lead oxide is the positive one. When the circuit is closed, the lead peroxide plate gives up a part of its oxygen, while the lead plate, the negative electrode, becomes oxidized, until finally the two electrodes become very much alike, and the current therefore becomes less and less, other things being the same. The battery is now said to be discharged. For, as we have already seen (§ 368), two electrodes which are alike never give a current when immersed in any one electrolyte.

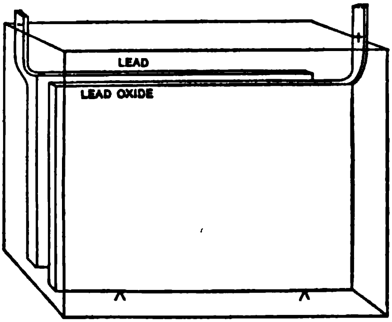

FIG. 294. — A storage cell.

But in order to put the cell again in good working shape, it is necessary only to pass an electric current through the cell in a direction opposite to that obtained when the cell was in use. **This process of regeneration is known as charging.** The charging simply restores the oxygen to the positive plate, and removes the oxygen from the negative plate, leaving it spongy metallic lead. **When the cell is used as an electric generator, the process is known as discharging.**

The following two features distinguish the storage battery from most other Voltaic cells: —

(*a*) The fact that the plates are prepared, *i.e.* made chemically different, by electrical means.

(*b*) The fact that they are capable of yielding currents which are enormous compared with those from other forms of batteries.

The chemical behavior of the lead storage cell is essentially as follows: —

(i) During the charge, at the anode or "positive grid,"

$$PbSO_4 + \overset{-}{SO_4} + 2\,H_2O = PbO_2 + 2\,H_2SO_4.$$

(ii) During the charge, at the cathode or "negative grid,"

$$PbSO_4 + \overset{+}{H_2} = Pb + H_2SO_4.$$

The ion $\overset{-}{SO_4}$ of the first equation and the ion $\overset{+}{H_2}$ of the second result from the decomposition of the sulphuric acid by the charging current.

(iii) During the discharge, we have at the positive plate,

$$PbO_2 + H_2SO_4 + \overset{+}{H_2} = PbSO_4 + 2\,H_2O.$$

(iv) During the discharge, at the negative plate,

$$Pb + \overset{-}{SO_4} = PbSO_4.$$

Observe that while the effect of the charging current is to deliver hydrogen at the negative plate, the effect of the discharging current is to produce hydrogen at the *positive* plate.

Edison has recently devised a storage cell in which the anode is spongy iron, the cathode nickel peroxide, and the electrolyte caustic potash.

In charging, the iron salt is reduced; in discharging, oxidized.

A storage cell is often spoken of as an accumulator, not because it stores up electricity but because it stores up chemical energy, just as the mainspring of your watch stores up mechanical energy in order to give it out again as needed.

It is this type of cell which is employed in practically all electric vehicles for propulsion and in all gas cars for starting and lighting.

376. 4. The Standard Cell. There is one cell which is distinguished above all others for producing an extraordinarily constant E.M.F. So independent is its E.M.F. of all local conditions and changes of many kinds that it is now used the world over as a standard by which to measure other E.M.F.s. The elements and construction of this cell are both shown in Fig. 295. The positive electrode is mercury in mercurous sulphate, the negative is cadmium amalgam in cadmium sulphate. This cell was invented and patented by Edward Weston of Newark, N.J., but many of the ideas involved in it had been already developed by Latimer Clark and Lord Rayleigh in England. By following the specifications prescribed by the last International Electrical Conference (London, 1908) it is possible for two individuals, using different material, to set up Weston cells whose E.M.F.s do not differ by more than two or three parts in 100,000.

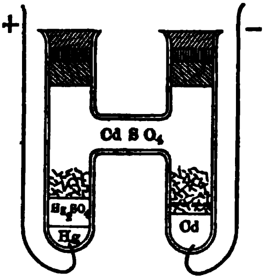

FIG. 295. — Weston normal cell.

It will be observed that all four forms of cells just described have this one common feature, viz., they consist of two different conductors of the first class, joined in series with one or more different conductors of the second class.

Having learned how to produce electric currents, we pass now to a quantitative study of them.

THE MEASUREMENT OF ELECTRIC CURRENTS

377. Unit of electromotive force. In the preceding chapter we learned that a difference of potential between two points is measured, in electrostatic units, by the work required to carry unit positive charge from one of these points to the other. Unit difference of potential would then exist between two points when one erg of work is required to carry unit charge from one point to the other.

The " practical unit " of potential difference employed in the measurement of currents is almost exactly $\frac{1}{300}$ as large as the E.S. unit just defined. This practical unit of E.M.F is called the " volt " in honor of Volta. It can be defined intelligently only after we have studied the subject of electromagnetism. In the meantime, its value is easily realized by remembering that an ordinary dry cell (Leclanché) has an E.M.F. of very nearly 1.5 volts; while that of a gravity cell is 1.08 volts. It is a very common practice to speak of the difference of potential between two points as a difference of " electrical pressure " or as the " voltage " between those points.

As indicated above, the Weston cell is employed everywhere as a standard of voltage, and its E.M.F. has been found to be 1.0183 volts at 20° C.

378. The galvanometer. As a preliminary to the measurement of currents, it is essential to understand the kind of magnetic field with which every electric current is surrounded.

This magnetic field is such that, if the wire conveying the current be placed parallel to a compass needle, the needle will be deflected so that it will not remain parallel to the wire. This is the fundamental discovery made by the Danish physicist Oersted, in 1820. Such an instrument is the one most commonly used to detect the presence of a current, and is called a **galvanometer.*** Almost immediately after a current ceases to flow in the wire, the magnetic field about the wire ceases to exist, and the wire no longer affects the compass needle. **But practically as soon as the current begins to flow the needle tends to set itself at right angles to the direction of the current. This type of current indicator is therefore called a "moving magnet" galvanometer.**

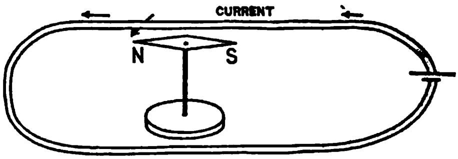

Fig. 296. — A galvanoscope in which the magnet is movable, while the wire conveying the current is fixed.

But if the compass needle is fixed and the wire movable, then the *wire* tends to set itself at right angles to the compass needle. The student will observe that this is merely another instance of the great general fact that action and reaction are equal and opposite.

In practice a compass needle is not so convenient as the U-shaped magnet shown in Fig. 297. The movable wire is a coil which can rotate about a vertical axis. This device, first used by Kelvin and D'Arsonval, is called a "moving coil" galvanometer.

Consider any closed electric circuit, *i.e.* a series of conductors, joined end to end so as to form a closed path.

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* The first galvanometer was a freshly prepared frog's leg, which twitched whenever a current passed along the sciatic nerve and muscles of the hind leg.

(i) The principle of the moving magnet galvanometer, then, is that whenever an electric current flows through any circuit, a compass needle in the neighborhood tends to set itself so that the greatest possible number of its lines of force will pass through the circuit;

(ii) While the principle of the moving coil galvanometer is that the portion of the circuit which is movable sets itself always so as to include as many as possible lines of force from the magnet. In either case, the electric current announces its presence by producing a rotation, *i.e.* an angular displacement.

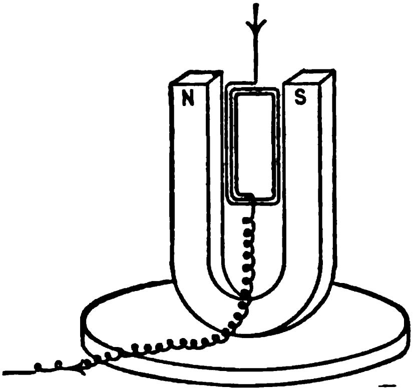

FIG. 297. — A galvanoscope in which a part of the wire conveying the current is movable, while the magnet is fixed.

From what precedes, it will be clear that each of these two forms of galvanometer indicates a current by the fact that, in general, the current exerts a turning moment on the magnet, or, what amounts to the same thing, the magnet exerts a turning moment upon a part of the circuit conveying the current.

The advanced student will discover that this behavior of the compass needle and the moving coil is merely a special case of the general principle that a body at rest tends to move into a position where its potential energy is a minimum.

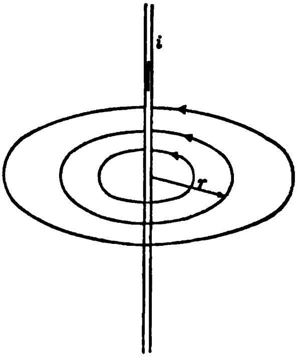

FIG. 298. — Illustrating Biot and Savart's Law.

379. Biot and Savart's Law. The deflected compass needle of Oersted was turned into a measuring instrument through the labor of two French physicists, Biot and Savart.

In the same year in which Oersted found that a current will deflect a magnet, Biot and Savart showed that, in the case of a very long straight wire,

(i) The magnetic field at any point about the wire varies inversely as the distance r of that point from the wire. See Fig. 298.

(ii) The intensity R of this magnetic field varies directly as the current strength i.

(iii) The direction of the current and the direction of the lines of force are related as the direction of translation and rotation in a right-handed screw. See Fig. 298.

The facts are known as Biot and Savart's Law and may be summarized by writing

$$R = K\frac{i}{r}, \qquad \text{Biot and Savart's Law.} \qquad \text{Eq. 144}$$

where K is merely a constant of proportionality, the value of which, if we use the C.G.S. system of units, turns out to be "2."

380. Case of a plane circular circuit. But for experimental purposes a much more convenient form of circuit is one where the wire is bent into the form of a circle. In this case, it can be shown, by the apparatus indicated in Fig. 299, that

(i) the magnetic field R *at the center of the circle* varies directly as the current strength in the wire.

(ii) The magnetic field *at the center of the circle* varies inversely as the radius of the circle.

(iii) The direction of the field *at the center of the circle* and the direction of the current in the wire are related in the manner of the right-handed screw.

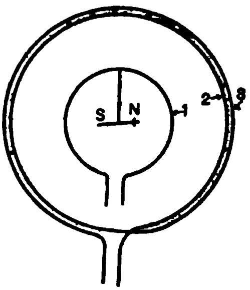

FIG. 299. — Poynting's apparatus.

These facts are illustrated by Poynting as follows: Wind on a frame three circles of insulated wire such as those marked 1, 2, and 3 in Fig. 299. Make the radii of "2" and "3" just twice that of "1." If the same current be passed through each of the three circles, but so that the sense of the current in the small circle is opposed to that in the two larger circles, then there will be no effect upon the magnetic needle suspended at the center.

For the larger radius of the outer circle is just compensated by the fact that there is here just twice as large a current acting upon the needle. Let us denote the current by i, the magnetic field at the center of the coil by R, and the radius of the circle by r, then a summary of the above facts becomes

$$R = k\frac{i}{r}.$$

Now it has been agreed by the scientific world to give this constant k an arbitrary value of 2π, and then call this the defining equation for the electric current, namely, —

$$R = \frac{2\pi i}{r}. \quad \text{Defining equation for current strength.} \qquad \text{Eq. 145}$$

Definition of Unit Current

381. The Ampère. It has been found most convenient to use Eq. 145 for the definition of current. This is equivalent to saying that **unit current is one which when passing about a circle of unit radius will produce at the center of the circle a magnetic field of intensity 2π.** When the centimeter, gram, and second are used as fundamental units, this current is called the "electromagnetic C.G.S. unit of current," or more briefly the E.M. unit of current.

The ampère, which is the practical unit of current, is one tenth of the C.G.S. unit.

382. The standard current. Just as the Weston cell is used for a concrete standard of electromotive force, so it has been found convenient to have a concrete standard of current. This standard is called the "international ampère" and has been specified by the International Electrical Congress at London in 1908 to be a current which when passed through a solution of silver nitrate will deposit silver on the cathode at the rate of 0.00111800 of a gram per second. This description of the standard current will be clear to the student after he has mastered Faraday's *Laws of Electrolysis*. See § 402.

The international ampère is the unit employed by engineers in all countries. Switchboard instruments and other commercial meters which we shall take up later are everywhere graduated in multiples or submultiples of this unit.

OHM'S LAW

383. Assuming that the student is now more or less familiar with the manner in which currents and electromotive forces are measured, let us consider a particular case where a definite and measurable electromotive force E is producing a definite and measurable current i in a circuit such, say, as that represented in Fig. 300. The following query now arises, **How does the strength of the current i in any circuit depend upon the electromotive force in that circuit?**

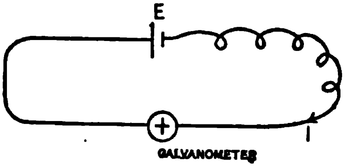

FIG. 300.

The answer to this question is contained in the discovery of the German physicist Ohm, in 1827, namely, **for a large class of bodies, the current varies directly as the total E.M.F. in the circuit.** For this class of bodies the ratio of the E.M.F. to the current is a constant for any given circuit. This general fact is known as Ohm's Law; and the constant ratio of E.M.F. to current is called the **electrical resistance** of the circuit.

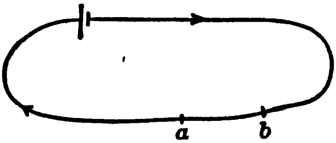

FIG. 301. — Ohm's Law applied to a portion of a current.

Even in the case of bodies such as conducting gases where the ratio of E.M.F. to current is *not* constant, this ratio at any instant is called the resistance at that instant.

Let us denote this resistance by R, then we may obtain an algebraic expression for Ohm's Law by writing

$$\frac{E}{i} = R. \qquad \text{Ohm's Law, defining equation for resistance.} \qquad \text{Eq. 146}$$

But if, instead of considering the entire circuit, we consider only a portion of it, such as that between the points a and b (Fig. 301), then it is found that Ohm's Law holds in the same way for this portion of the circuit as for the whole circuit, *always provided there is between a and b nothing of the nature of a battery or dynamo,* i.e. *no source of electromotive force.*

The Unit of Resistance. The Ohm

384. To obtain the unit of resistance one employs the same method which is used in the other departments of physics, namely, takes the defining equation and gives the other variables such values as will make the desired quantity unity. Thus in Eq. 146, if we make $E = i = 1$, then $R = 1$. **A circuit is therefore said to contain one unit of resistance when one unit of E.M.F. will produce in it one unit of current.**

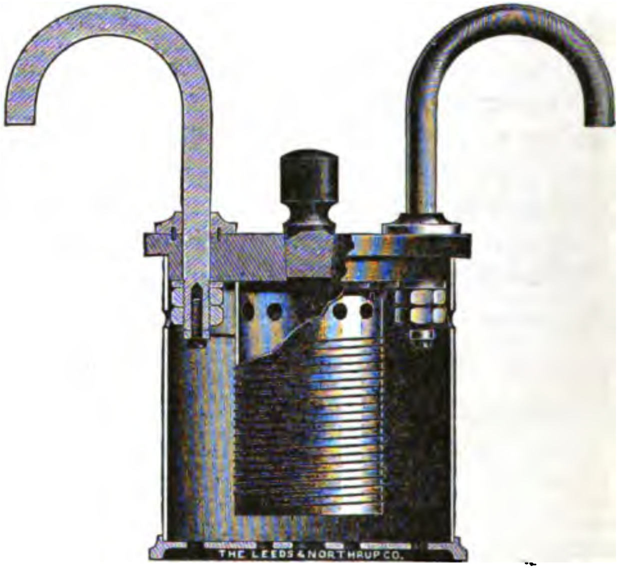

FIG. 302. — A resistance coil.

The electromagnetic C.G.S. unit of resistance thus defined is entirely too small for laboratory use and hence a practical unit, one thousand million times as large, was adopted by the British Association in 1861, and afterward ratified by an International Congress at Paris. This practical unit is called an "ohm."

1 Ohm = 10^9 electromagnetic C.G.S. units of resistance.

Expressed in the practical system of units, Ohm's Law now becomes: —

$$\text{Current in amperes} = \frac{\text{Electromotive force in volts}}{\text{Resistance in ohms}}.$$

Standard of Resistance

385. By international convention agreed upon in London in 1908, the "international ohm" is represented sufficiently well by the resistance of a column of mercury of uniform cross section at 0° C., when its length is 106.300 centimeters and its mass is 14.4521 grams. When a wire is said to have a resistance of three ohms it is meant that a given E.M.F. applied to the ends of this wire will produce exactly one third as much current as when the same E.M.F. is applied to the above described mercury column.

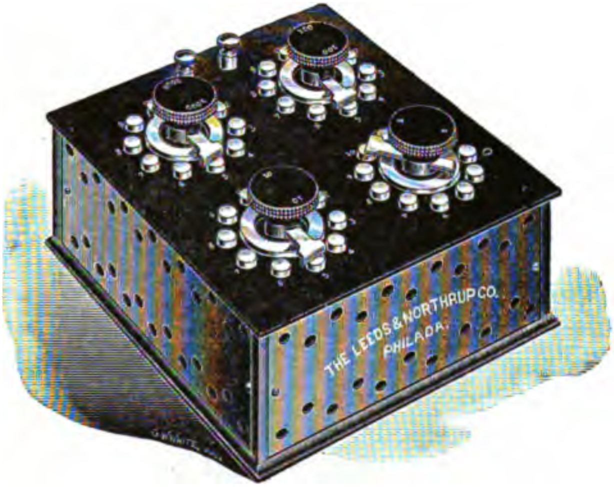

FIG. 303. — A resistance box.

Most standards of resistance are now made of manganin wire coiled on a metal cylinder from which the wire is insulated by silk and shellac. The advantage of manganin is that it changes its resistance scarcely at all when the temperature varies. See Fig. 302.

386. *Resistance Boxes.* In the laboratory there is constant demand for a set of resistances which will enable one quickly to put into a circuit any number of ohms, within the limits of the set, which he may wish. For this purpose, units of resistance are wound on spools; these spools are mounted on the under side of a hard rubber plate; on the upper side of this same plate are various dials by the rotation of which one can join up very quickly as many ohms as he wishes, just as he would pick out from a set of weights any desired mass for use on the pan of a balance. See Fig. 303.

Digression on the Voltaic Cell

387. Fall of Potential. With Ohm's Law in mind it will be profitable to reconsider for a moment the fall of potential in a circuit containing a voltaic cell. It is practically impossible to measure the contact difference of potential of two substances alone. But it is now generally assumed that the seat of E.M.F. in a cell is located at the two surfaces of separation between the electrolyte and the two electrodes. Fig. 304 is intended to represent the case of an ordinary gravity cell. Here the potentials are plotted as a function of the resistance. Let the ordinate of the point *A* represent the potential of the zinc electrode; *AB* will represent the rise of potential in passing from the zinc to the zinc sulphate solution. The difference between the ordinates of *B* and *C* will measure the fall of potential in passing through the electrolyte; *CD* is the rise in potential in passing from the solution to the copper electrode; while *DF* is the fall of potential in passing through the external resistance *R*.

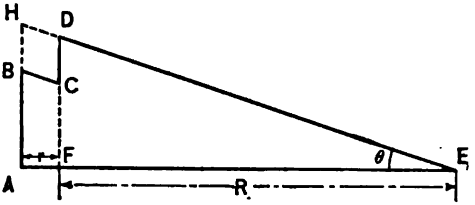

Fig. 304. — Representing the fall of potential in a battery circuit.

If we denote the resistance of the cell itself — the so-called internal resistance — by *r*, and the current by *i*, then

$$i = \frac{\text{Potential Difference } DF}{R} = \frac{\text{Potential Difference } AH}{R + r}$$

$$= \frac{\text{E.M.F. of cell}}{R + r}.$$

The potential difference *AH* which is obtained by prolonging *DE* backwards until it intersects *AB* produced is the E.M.F. of the cell. Note that *DF*, which is called the terminal potential difference, is always less than the E.M.F. of the cell, except when the circuit is open. The resistance is then infinite and the two are equal. The current, it will be observed, is numerically equal to the tangent of the angle θ in the diagram.

388. Grouping of Cells. In order to obtain higher voltages than are given by single cells, one joins the zinc of one cell to the copper of another, and so on, as shown in Fig. 305. If all the cells are joined up in tandem as indicated they are said to be *in series*. In this case the internal resistances are added as well as the E.M.F.s. Hence n cells, each furnishing E volts, will give a current i defined by the following equation

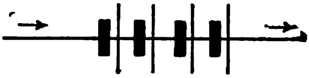

FIG. 305. — Cells in series.

$$i = \frac{nE}{nr + R}. \qquad \text{Eq. 147}$$

The second principal method of joining cells is to connect all the zinc poles to one end of the external resistance R, and all the copper poles to the other end of R. This arrangement simply makes one big cell out of the n separate cells. The internal resistance of the combination is only $1/n$ that of a single cell; but the E.M.F. is exactly the same as that of a single cell. Applying Ohm's Law, one obtains for the current,

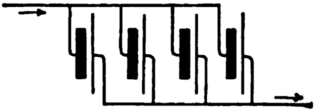

FIG. 306. — Cells in parallel.

$$i = \frac{E}{\frac{r}{n} + R} = \frac{nE}{r + nR}. \qquad \text{Eq. 148}$$

Ohm's Law (continued)

389. The International Volt. Returning from this digression, it is interesting to note that while Ohm's Law has been used (§ 384) to define *resistance* in the electromagnetic C.G.S. system of units, it is employed quite differently in the practical international units. For by convention of the International Electrical Congress in London in 1908, the *ohm* and the *ampere* are taken as the first and second primary standards respectively; and the *international volt* is defined as "*the electrical pressure which when steadily applied to a conductor whose resistance is one international ohm will produce a current of one international ampere.*" This is the quantity which we have already seen to be represented by $\frac{10000}{10183}$ of the E.M.F. of the Weston cell.

Resistivity

390. Having discovered that the ratio between E.M.F. and current is a constant for any given circuit, we now proceed to inquire what physical features of a circuit determine its resistance. **Upon what physical properties of a circuit does its resistance depend?** The most direct experimental answer is obtained, perhaps, by taking a circuit such as that shown in Fig. 307.

Pass through this circuit a small steady current from any source. Now since it has been shown by Faraday that the current i is the same in every part of the circuit, and since $E = iR$, it is clear that we may study the variations of R by observing the variations of E. To measure the E.M.F. between any two points such as C and D on this circuit, one has only to attach to these points the terminals of a sensitive galvanometer whose resistance is high compared with that portion of the battery circuit which lies between C and D. A branch circuit such as that from C to D *via* the galvanometer is generally called a "shunt."

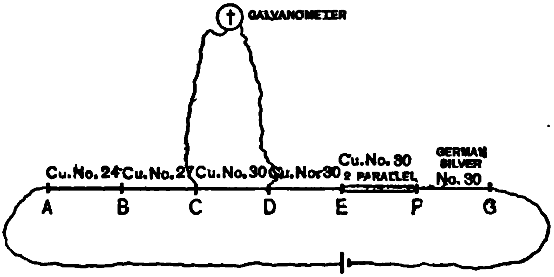

Fig. 307. — Illustrating Ohm's Law.

To each pair of points on the main circuit will correspond a certain deflection of the galvanometer, and this deflection will measure (in arbitrary scale divisions) the E.M.F. between that pair of points.

The length of each section in Fig. 307 is the same; that is, the linear distance between each junction and its next door neighbor is constant.

Now it is observed that the E.M.F. between B and C is just twice that between A and B, No. 24 wire having just twice as large a cross section as No. 27. This illustrates the general fact that **the resistance between any two points on a conductor varies inversely as the area of the cross section of the conductor.**

This rule may again be verified by measuring the E.M.F. between the points C and D, which are connected by No. 30 wire, which has just half the cross section of No. 27. Here also it is found that the deflection corresponding to the interval CD is just twice that due to BC.

Next measure the potential difference between C and E; it will be found just twice that between C and D, which illustrates the general fact that **for any uniform conductor the resistance varies directly as the length.**

The last section of the wire, that between F and G, is the same in length and cross section and is carrying the same current as the section CD. But the material is different, the chemical composition of the wire is not the same. Comparing the deflection between F and G with that between C and D, the former is found to be many times greater; from which one may infer that, other things being equal, the resistance of German silver wire is many times greater than that of copper; and in general, that **the resistance of any conductor depends upon the chemical composition of the substance from which it is made.**

This property of a conductor which determines how its resistance depends upon its chemical composition is called its "*resistivity*," a quantity which we may denote by ρ and may measure as follows: —

Let the cross section of a wire be s, and its length l. We may then summarize the preceding results by writing

$$R \propto \frac{1}{s},$$

$$R \propto l,$$

$$R \propto \rho.$$

Hence $R \propto \frac{\rho l}{s}$ and $R = k\frac{\rho l}{s}$, where k is a proportionality constant.

But it has been agreed to call the resistivity of a substance unity when its length, cross section, and resistance are each unity, from which it follows that k is always unity, and hence

$$R = \frac{\rho l}{s}. \quad \text{Defining equation for resistivity.} \qquad \text{Eq. 149}$$

The resistivity — or, as it is frequently called, specific resistance — **of any substance is therefore numerically equal to the resistance in ohms between two opposite faces of a centimeter cube of that substance.**

The conductivity of any substance is defined as the reciprocal of its resistivity. In like manner the conductance of a body is defined as the reciprocal of its resistance.

Indeed, it has been adopted as a general principle in electrical nomenclature that specific properties, properties of substances as distinguished from bodies, shall be denoted by nouns ending in *-ity;* while properties which belong only to particular bodies shall be described by nouns ending in *-ance.* The advantage of this is that terms such as "impedance" and "inductance," which may be unfamiliar to the reader, carry on their face a part of their meaning.

Measurement of Resistance

391. Wheatstone's Bridge. In 1833, S. H. Christie devised an instrument by means of which one can easily and accurately measure a resistance by comparing it with a known resistance. The device is now known as Wheatstone's Bridge, partly because Wheatstone employed it in his researches and partly because he improved it.

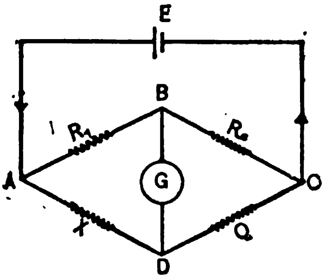

FIG. 308. — The Wheatstone Bridge, invented by Christie.

The instrument consists essentially of six conductors joining four points, A, B, C, D, in such a way that when a voltaic cell is placed in one branch, say AC, and there is no current produced in another branch BD, we are enabled to determine the value of the resistance in one of the remaining branches, say AD.

In Fig. 308, let E denote a voltaic cell and G a galvanometer.

The branches R_1, R_2, Q, and X are called the *arms* of the bridge. We shall suppose X to be the unknown resistance and shall assume that the other three arms are so adjusted that no current is passing through the *bridge*, BD; then the current in R_1 will be the same as that in R_2; also the current in X will be identical with that in Q. The absence of current in G shows that B and D are each at the same potential; let us denote this potential by V. Then if V_A denote the potential of the point A, V_C the potential of the point C, we may express this equality of currents by writing: —

$$\frac{V_A - V}{R_1} = \frac{V - V_C}{R_2},$$

and

$$\frac{V_A - V}{X} = \frac{V - V_C}{Q}.$$

Whence, by division,

$$\frac{X}{R_1} = \frac{Q}{R_2} \text{ or, } X = \left(\frac{R_1}{R_2}\right)Q. \qquad \text{Eq. 149}\textit{bis}$$

From this equation one easily obtains the value of X as soon as he knows Q and the ratio $\frac{R_1}{R_2}$. The two arms R_1 and R_2 are known as the *ratio coils*.

392. Variation of Resistance with Temperature. As a matter of fact the resistance of any conductor depends more or less upon its temperature. This fact is, indeed, the basis of one of the best methods of measuring moderately high temperatures, namely, the electrical resistance thermometer, suggested by Siemens, perfected by Callendar. See § 259, above.

Practically all metals decrease in resistivity as their temperatures fall. But the resistivity of carbon and of all electrolytes *increases* as the temperature falls.

One of the most remarkable discoveries of recent times is that made by H. Kamerlingh-Onnes in his celebrated Cryogenic Laboratory at Leyden. Here he found that at the temperature of boiling helium, − 269° C., the resistivities of certain metals, among them mercury, tin, and lead, fall to less than one millionth part of their values at the temperature of melting ice.

The resistance of a lead conductor under these conditions is so minute that a current induced in a coil of lead wire will persist for hours with almost no diminution. Metals in this condition are called **super-conductors.**

The slight variation of resistance which a bismuth wire suffers when placed in a magnetic field has proven to be important in the measurement of dynamo fields; but this must be left for the advanced student.

393. Electrical and Thermal Conductivity. In 1853, two German physicists, Wiedemann and Franz, discovered that for pure metals, the ratio of the thermal to the electrical conductivity is a constant at any one temperature, and later researches have shown that this ratio increases directly as the absolute temperature.

This parallelism between heat conduction and electric conduction — known as the *Wiedemann-Franz Law* — remained unexplained for nearly half a century, when in 1900 Drude showed that both these processes are harmonized by the electron theory. H. A. Lorentz puts the matter as follows: "The fundamental idea of the modern theory of the thermic and electric properties of metals is that the free electrons in these bodies partake of the heat-motion of the molecules of ordinary matter, travelling in all directions with such velocities that the mean kinetic energy of each of them is equal to that of a gaseous molecule at the same temperature.

"If we further suppose the electrons to strike over and over again against metallic atoms, so that they describe irregular zig-zag lines, we can make clear to ourselves the reason that metals are at the same time good conductors of heat and of electricity, and that, as a general rule, in the series of the metals, the two conductivities change in nearly the same ratio. The larger the number of free electrons, and the longer the time that elapses between two encounters, the greater will be the conductivity for heat as well as that for electricity." *Theory of Electrons* (1909), p. 10.

TABLE OF RESISTIVITIES

SUBSTANCE	RESISTANCE AT 0°C. OF A WIRE 1 CM. LONG 1 CM². IN SECTION	RESISTANCE AT 0°C. OF A WIRE 1 FOOT LONG AND $\frac{1}{1000}$ INCH IN DIAMETER
Aluminium	2.94×10^{-6} ohms	17.69 ohms
Antimony	40.5×10^{-6} ohms	243.8 ohms
Bismuth	119.0×10^{-6} ohms	716.5 ohms
German silver	$16\text{–}40 \times 10^{-6}$ ohms	96–240 ohms
Gold annealed	2.42×10^{-6} ohms	14.57 ohms
Copper annealed	1.59×10^{-6} ohms	9.57 ohms
Copper hard drawn	1.78×10^{-6} ohms	10.71 ohms
Iron	$9\text{–}15 \times 10^{-6}$ ohms	54–90 ohms
Manganin	42.05×10^{-6} ohms	253.14 ohms
Mercury	95.8×10^{-6} ohms	576.7 ohms
Platinum	11.0×10^{-6} ohms	66.2 ohms
Silver	1.6×10^{-6} ohms	9.7 ohms
Tungsten	$5. \times 10^{-6}$ ohms	30.1 ohms

From KAYE AND LABY'S *Tables.*

DEFINITION OF THE ELECTROMAGNETIC UNIT OF QUANTITY

394. Quantities of electricity are measured just as we should measure a quantity of water delivered by a stream of known strength in a given time. Hence

$$Q = it \qquad \text{Eq. 150}$$

becomes the defining equation for electrical quantity Q. If a Leyden jar is charged by a current i flowing for a time t, the charge Q on the jar is it.

The practical unit of quantity is called the coulomb, and is defined as the quantity of electricity transferred by a current of one ampere in one second.

THE ELECTROMAGNETIC UNIT OF CAPACITY

395. The capacity of a condenser having already been defined (§ 356) as the ratio of its charge Q to the potential difference E between its plates, one has

$$Q = CE. \quad \text{Defining equation for capacity.} \qquad \text{Eq. 151}$$

Hence the so-called "practical" unit of capacity is that of a condenser charged to a potential of one volt by one coulomb of electricity.

In honor of Faraday this unit of capacity is called the "farad"; but it is entirely too large for laboratory purposes, and hence standard condensers which are made for the purposes of measuring capacities are generally graduated in terms of a unit which is one millionth of a farad and is called a "microfarad." Sometimes a third of a microfarad is used because this represents approximately the capacity of a mile of Atlantic Cable.

Problems

368. A current of one ampere and an E.M.F. of 50 volts are required to feed an incandescent lamp. What is the resistance of the lamp in ohms?

369. A Leclanché cell is used to ring a door bell. The resistance of the wire in the bell is 2 ohms, the resistance of the line is ½ an ohm, and the resistance of the cell is 1 ohm. The E.M.F. of the cell is 1.5 volts. What current will be produced when the circuit is closed?

Ans. 0.429 ampere.

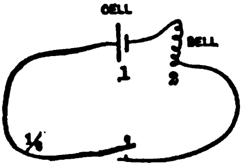

FIG. 309.

370. The E.M.F. of a single gravity cell is 1.1 volts; the resistance of each cell is 3 ohms. If three of these cells be joined up in series as indicated in Fig. 310, and the poles of the battery be connected by a resistance of 1 ohm, what current will flow in the circuit?

Ans. 0.33 ampere.

371. A telegraph sounder has a resistance of 70 ohms, and it requires a current of 0.2 ampere to work it. Will 20 gravity cells such as described in the preceding example be sufficient? Will 40 cells suffice? Be able to describe your reasons at the blackboard.

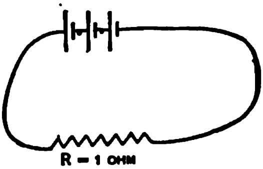

FIG. 310.

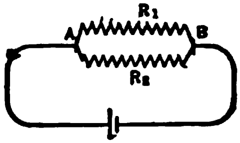

FIG. 311.

372. Two wires R_1 and R_2, alike in all respects, and having each a resistance of 20 ohms, are joined "in parallel" as shown in Fig. 311. What is the sum of their resistance, *i.e.* what is the resistance of that part of the circuit which lies between the points *A* and *B*? Use equation for resistance given in § 390, viz.,

$$R = \frac{\rho l}{s}$$

One might imagine the two wires fused into one of equal length; the numerical value of *s* would then be doubled, but as an electrical conductor the large wire would be equivalent to the two smaller ones.

373. What is the total resistance of the two wires R_1 and R_2 when joined " in series " as indicated in Fig. 312?

374. A gravity cell of 1.1 volts is joined up in opposition to a storage cell of 2.1 volts, *i.e.* their positive poles are connected. What is the resultant E.M.F. of the two cells? *Ans.* 1 volt.

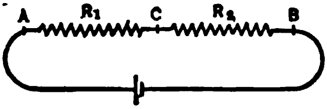

FIG. 312.

375. Three gravity cells, each having a resistance of 3 ohms and an E.M.F. of 1.1 volts, are joined " in parallel " as indicated in Fig. 313, and then connected to a coil of wire having a resistance of 8 ohms. Find the current in the coil. How much current passes through each cell? *Ans.* 0.122 ampere; 0.041 ampere.

376. Siemens proposed a unit of resistance equivalent to that of a column of mercury 1 m. in length and 1 mm.^{2} in cross section. Find the value of this unit in terms of the ohm.

377. What magnetic field will be produced at the center of a circle of wire conveying a current of 2 amperes, when the radius of the circle is 9 cm.?

378. Which will affect a magnetic needle at the center more strongly, a current 8 amperes in a circle of 8 cm. or a current of 100 amperes in a circle of 1 m. radius?

379. A room is illuminated by 40 incandescent lamps joined up in parallel. Each lamp has a resistance of 200 ohms when hot and requires a voltage of 110. Find the current necessary to light the room.

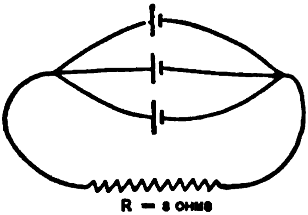

FIG. 313.

380. Taking the resistivity of copper as 1.6×10^{-6}, find the resistance of a pair of copper wires whose diameter is 3 mm. and whose joint length is 40 m.

381. A storage cell whose resistance is $\frac{1}{10}$ ohm and whose electromotive force is 2 volts sends a current through a wire of 3 ohms joined in series with a wire of $\frac{1}{2}$ an ohm. Find the potential difference between the terminals of each of the two wires.

382. The E.M.F. of a gravity cell on open circuit is 1.12 volts. But when the circuit is closed with an external resistance of 20 ohms, the voltage between the terminals of the cell drops to 1.00. Find the internal resistance of the cell.

383. Ten cells are arranged so as to have five in series and two of these series in parallel. The E.M.F. of each cell is 1.5 volts; the resistance 1 ohm. Find the E.M.F. and resistance of the combination.

384. The terminals of a cell are joined by three parallel wires whose resistances are r_1, r_2, r_3, respectively. Prove that the current passing through the cell will be distributed among these three conductors inversely as their resistances.

385. An overhead trolley line has a resistance of half an ohm per mile. The cars running are taking 125 amperes from the line: but all these cars are more than 2 miles away from the power-house. Find the drop in voltage between the power station and a point on the line one mile away.

ELECTROLYSIS

396. Among the most important effects of an electric current must be reckoned that by which skates are plated with nickel, spoons with silver; for this is the same process by which small amounts of gold and silver are separated from large amounts of copper in the great copper refineries. All of these are instances of the general fact, largely established by Sir Humphry Davy, that when a current is passed through a solution, that solution is, in general, broken up into two other substances.

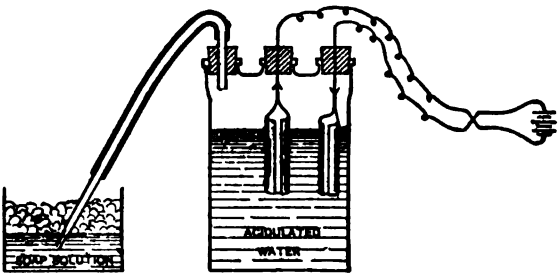

FIG. 314. — The decomposition of water by an electric current.

In the first year of the nineteenth century, the same year in which Volta described the voltaic cell, it was found by Carlisle and Nicholson that a current passing through water breaks the water up into hydrogen and oxygen. Hydrogen is given off at the cathode and oxygen at the anode.

A clear grasp of the following six experiments is perhaps as short a route as any to an understanding of this process, which is called electrolysis.

397. *Experiment* 1. — In order to pass a current through water it is necessary to dissolve some salt or acid in it to make the liquid a good conductor. The most convenient substance to add is 10 or 15 per cent of strong sulphuric acid.

Take a Wolff's bottle half full of the dilute acid; through two of the holes in the bottle insert two strips of sheet platinum soldered to copper wires, as indicated in Fig. 314; connect these to a source of electromotive force which will send five to ten amperes through the acidulated water. Gas will be given off from each electrode, and the mixture may be collected for examination by attaching one end of a rubber tube to the remaining neck of the bottle, and immersing the other end of the tube in a soap solution. The escaping gas will thus blow a series of soap-bubbles which may be exploded by a lighted taper; for it has been shown that the gases are evolved in just the right proportion to explode, without leaving behind any oxygen or any hydrogen, *i.e.* the proportions of hydrogen and oxygen present are chemically equivalent. It is safer, of course, to remove the glass nozzle from the soap solution before applying the taper.

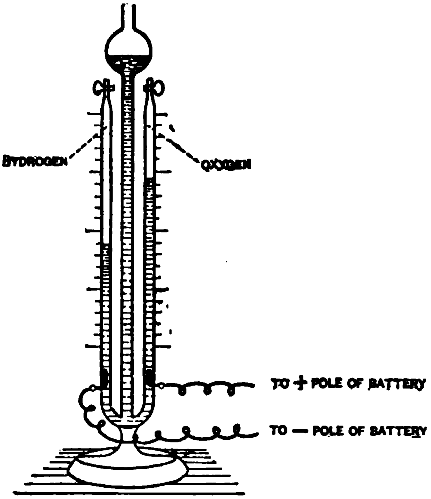

FIG. 315. — Electrolysis of water, in which the hydrogen and oxygen are collected in separate vessels.

398. *Experiment* 2. — By using a U-tube with electrodes, one sealed into each branch, the gases can be collected before they have a chance to mix. If the arms of the tube are graduated, one can measure directly the volume of hydrogen set free, and also the volume of oxygen. Accurate measurements of this kind show that when the gases are collected under equal pressures, the volume of hydrogen is exactly double that of oxygen. But chemists tell us that these are exactly the volumes of hydrogen and oxygen that unite to form water, and that the density of oxygen is sixteen times as great as that of hydrogen. What then will be the *mass* of oxygen set free as compared with the mass of hydrogen?

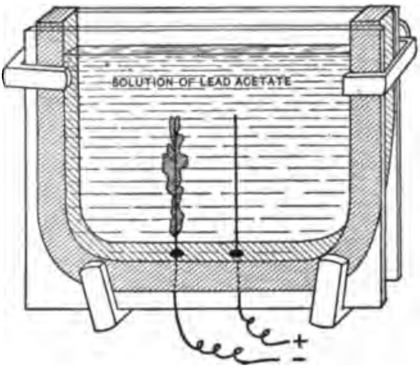

FIG. 316. — Electrolysis of lead acetate.

399. *Experiment* 3. — Instead of acidulated water we may use a solution of lead acetate as an electrolyte. The phenomenon is most conveniently seen or projected when the solution is placed in a flat vessel such as is represented in Fig. 316.

If we use a pair of platinum wires as electrodes, it will be seen that the lead acetate, which is a clear transparent solution, is broken up so as to yield dark metallic lead at the negative pole of the battery, *i.e.* at the same pole where the hydrogen appeared in the preceding experiment. In general, hydrogen and all the metals behave as if they rode through the electrolyte, on the current, *i.e.* they collect at the same pole at which the current leaves the electrolyte.

400. *Experiment* 4. — Moisten a strip of filter paper with a strong solution of phenol phthaleïn * and sodium chloride, table salt. If the positive pole of a battery be connected with one end of the strip, the negative pole will, when drawn over the strip, cause an electric current to pass along the strip, for the moistened paper is a conductor. This current will deposit sodium

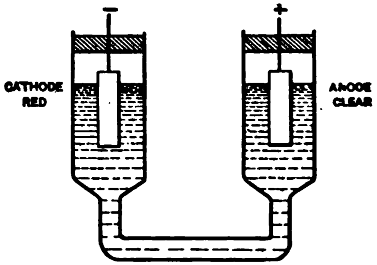

FIG. 317 — Illustrating the chemical reactions which sometimes occur at the electrodes.

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* This is a substance which chemists very commonly use to detect the presence of an alkali. The phenol phthaleïn is colorless; but a drop of it in an alkaline solution will yield a red color.

at the negative electrode, the sodium will unite with the water in the strip and produce caustic soda, *i.e.* a strong alkali, which, in the presence of the phenol phthaleïn, turns a brilliant red. The consequence is that a brilliant red line may be traced when the negative electrode is moved over the strip of filter paper.

A more easily visible method of demonstrating this phenomenon is to take a U-tube as shown in Fig. 317 and provide it with a pair of platinum electrodes. If now the salt solution (NaCl) be placed in this tube and a few drops of phenol phthaleïn be added, it will be found that as soon as the current begins to pass, the solution in the immediate neighborhood of the cathode turns red. This indicates the presence of an alkali in that region and in no other; in other words, the metallic sodium which the current deposits upon the cathode unites at once with the water to form sodium hydroxide. The chlorine which is set free at the anode dissolves in the water and forms a bleaching solution. The result of this is that when the current is reversed there is sufficient chlorine water about what is now the cathode to immediately bleach out the red due to the phenol phthaleïn. Accordingly it is necessary for the current to flow some little while after reversal before the red color remains about the cathode.

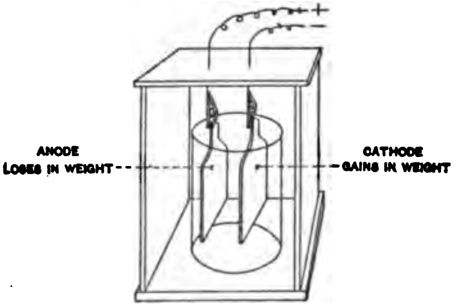

FIG. 318. — The copper coulometer.

401. *Experiment* 5. — If we immerse two copper plates in a solution of copper sulphate and pass a current from one plate to the other, the mass of the plate by which the current enters will diminish, while the mass of the plate by which the current leaves will grow larger; but the copper solution remains practically unchanged. By accurately weighing the plates before and after the passage of the current, and noting the duration of the current, one can determine just how much copper any given current will deposit in one second of time. **A cell fitted with electrodes and electrolyte in this manner is called a coulometer or voltameter.** The copper thus deposited is very pure, and this process is now largely employed for refining copper.

402. *Experiment* 6. — When several voltameters are connected in series, after the manner shown in Fig. 319, the same current will pass through each of them. But on weighing the electrodes it is found that the amount of element deposited upon any particular electrode, by a given current flowing for a given time, varies directly as the atomic weight of the element and inversely as the valency of the element. Thus silver is univalent and has an atomic weight of 108, while copper is bivalent and has an atomic weight of 63. Accordingly in the arrangement described in Fig. 319 the amount of silver deposited will always be $108/1 \div 63/2$, or a little over three times as great as that of copper. The ratio of the atomic weight of an element to its valency is generally known as its "chemical equivalent."

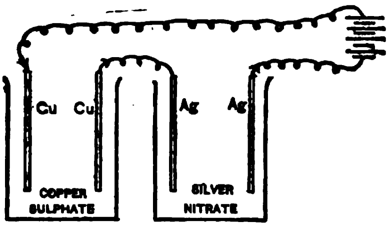

FIG. 319. — The current produces equivalent decomposition in cells containing different electrolytes joined up in series.

Faraday made a very careful study of many phenomena such as we have just been considering, and he summarized his results as follows: —

FARADAY'S LAWS OF ELECTROLYSIS

First Law

The amount by weight (*i.e.* the mass) of an electrolyte which is decomposed by an electric current is proportional to the current flowing and to the time during which it flows.

In order to express this law in terms of algebra let us denote by i the current in amperes, by t the time in seconds, and by m the increase in mass of cathode measured in grams. Then

$$m = eit, \qquad \text{Eq. 152}$$

where ϵ is a proportionality constant which is called the "**electrochemical equivalent**" of the element deposited.

The electrochemical equivalent e is numerically equal to the mass of any element deposited by the passage of one coulomb of electricity through the electrolyte, while the reciprocal of e is the amount of electricity associated with one gram of the element in question. Thus *in the case of hydrogren,* if we denote this quantity of electricity passed by e, we have

$$\frac{e}{m} = \frac{1}{\epsilon} = \frac{96540 \text{ coulombs}}{\text{gram}}.$$

Second Law

When an electrolyte, or a series of electrolytes, is decomposed by an electric current, the components into which it is separated are always chemically equivalent, *i.e.* they are set free in just such amounts as may recombine and form a chemical compound without any left-over material.

For purposes of computation it is often convenient to put this law in terms of symbols as follows: Let subscripts "1" and "2" refer to two different electrolytic cells. Then if atomic weights be denoted by m, valencies by v, and electrochemical equivalents by ϵ, we have

$$\frac{\epsilon_1}{\epsilon_2} = \frac{m_1 v_2}{m_2 v_1}. \qquad \text{Eq. 153}$$

It has recently been shown by H. A. Wilson that these two laws hold not only for liquids but also for salts in the state of vapor produced by heat.

403. Theory of Electrolysis. This experimental evidence seems, then, to indicate that there is associated with each atom of matter a certain definite charge of electricity which is the same for all atoms of the same valency and is quite independent of the "atomic weight." The charge associated with each unit of valency would then appear to be a natural unit of electricity — an "atom of electricity," as suggested by Helmholtz. The atom of matter with its associated charge is called an "ion." But the charge alone, this natural electrical unit, is called an "electron," a name which was given to it in 1891 by the Irish physicist Dr. G. Johnstone Stoney.

Numerous experiments have shown that the charge carried by an atom of unit valency is precisely the same as the charge carried by those electrons which are produced in a variety of other ways. These are the same electrons which give a body a negative charge when present in excess and which render a body positively charged when present in less than the normal number. We have already seen that an electric current in a solid conductor probably consists in a stream of these electrons moving in a direction opposite the E.M.F. In rarefied gases, we shall presently find the electrons moving entirely alone, quite free from matter. But in electrolytes they travel only when attached to matter. Some of the atoms have an excess of electrons; they are called *negative ions* and always travel against the current, that is, towards the anode. Other atoms have less than their normal number of electrons; they are called *positive ions* and always travel with the current towards the cathode.

TABLE OF ELECTROCHEMICAL EQUIVALENTS

SUBSTANCE	RELATIVE ATOMIC WEIGHT	VALENCY	ELECTROCHEMICAL EQUIVALENT IN GRAMS PER COULOMB
Aluminium	26.9	3	0.0000936
Chlorine	35.18	1	0.0003672
Copper	63.1	1	0.0006588
Copper	63.1	2	0.0003290
Hydrogen	1.000	1	0.0000104
Iron	55.5	2	0.0002895
Iron	55.5	3	0.0001930
Oxygen	15.88	2	0.0000829
Silver	107.12	1	0.001118
Zinc	64.9	2	0.0003387

From the *Smithsonian Tables*, Third Edition, p. 272.

An atom from which a single electron has been detached is to be thought of as a *univalent positive ion;* while an atom which carries one extra electron is probably a *univalent negative ion.* A neutral atom, on this view, is one consisting of a number of electrons and an *equal* amount of positive charge kept in equilibrium by the mutual electrical forces acting between these two kinds of charges. The electron theory thus affords us a simple explanation of both of Faraday's laws without the introduction of any hypothesis not already in use in other fields of electricity.

When the E.M. unit of current — 10 amperes — is employed, it is evident that each value in the last column of the above table must be multiplied by 10.

Heating Effects of Electric Currents

404. In these days every one is familiar with the fact that when an electric current is passed through a tungsten filament, the temperature of the filament may be increased to such a point that the tungsten becomes white-hot. Trolley cars are heated by electric currents passing through coils of wire. Iron chains are now made by welding together the two halves of each link by means of an electric current. Heavy steel rails are welded together by the same process. In the manufacture of high grade tool steel electric furnaces are employed which are heated exclusively by electric currents.

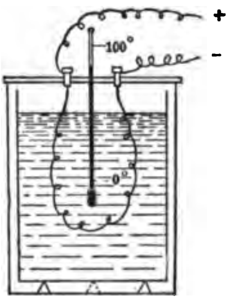

FIG. 320. — Calorimeter for measuring the rate at which heat is produced by an electric current.

Between the years 1840 and 1843 the English physicist Dr. Joule made a very careful study of the heat produced by an electric current. He discovered that *for any one piece of wire* the amount of heat produced by any current depends upon two things only: —

1. Upon the amount of current flowing, *i.e.* upon the number of amperes, and
2. Upon the time during which it flows.

By use of a calorimeter containing a coil of wire, it was shown that when the current in the coil of wire was *doubled,* the heat was developed *four* times as rapidly as before; and when the current was *trebled,* the heat was imparted to the water *nine* times as fast as before.

But if any particular current was allowed to flow first for five minutes, then for ten minutes, it was found that the amounts of heat (calories) developed in these two intervals were in the ratio of 1 : 2. And Joule found that in general he could describe the results of his experiments as follows: —

Joule's Laws of Heating

1. **In any given conductor, the heat developed by an electric current in any given time varies directly as the square of the current.**

2. **In any given conductor, the heat developed by any given current is directly proportional to the time during which the current flows.**

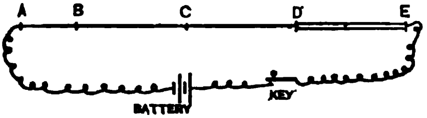

FIG. 321. — Amount of heat produced by an electric current depends upon length, diameter, and material of wire.

405. Hitherto we have been considering the effect of a current upon a single conductor. Suppose, however, we take a series of different conductors and join them end to end, and pass one current through the entire chain. Will the heating effect be the same on each link of the chain? An experimental answer to this question may be obtained as follows: —

Join together four wires as represented in Fig. 321, where

AB is 10 cm. of iron wire;	0.25 mm. diam.
BC is 20 cm. of iron wire;	0.25 mm. diam.
CD is 20 cm. of copper wire;	0.25 mm. diam.
DE is 20 cm. of iron wire;	1.00 mm. diam.

Through this series pass a current sufficient to heat the part *AB* red-hot. This will require about four amperes. Observe, then, —

(1) That the part *BC* (which is exactly similar to *AB* except in length) is just as hot as *AB*, and consequently is giving off twice as much heat as *AB*;

(2) That the copper wire *CD* (which is exactly like *BC* except in chemical composition) is very much cooler than *BC*;

(3) That *DE* (which is exactly like *BC* except that its area of cross section is larger) is also much cooler than *BC*.

From experiments of this kind, in which, however, all the quantities involved were accurately measured, Joule concluded that the amount of heat which any given current develops in any conductor in a given time depends upon the following three factors: —

(1) *The length of the conductor,* the amount of heat being directly proportional to the length;

(2) *The chemical composition, i.e.* the kind of material of which the conductor is made; thus we have seen that the iron wire becomes much hotter than copper wire of the same size; in like manner a carbon filament becomes much hotter than an iron wire of the same size, other things being equal. In a chain made of alternate links of silver and platinum wire each of the same size, a current will heat the platinum red-hot while the silver remains comparatively cool.

(3) *The area of cross section,* the amount of heat being inversely as this area. For this reason the insurance companies require copper wire of not less than a certain diameter to be used in leading a current to a given number of incandescent lamps. If the wire be too small, there is danger of its getting hot and setting fire to the building.

These results we may summarize as follows: —

Let H = heat measured in ergs developed by any current;
i = strength of current in C.G.S. units;
t = duration of current in seconds;
l = length of the conductor in centimeters;
s = area of cross section of conductor in cm.^{2}.
ρ = a constant depending upon the material of which the conductor is made.

Then the most careful experiments by Joule and his successors show that

$$H = \rho\frac{l}{s}i^2t \text{ ergs,} \qquad \text{Eq. 154}$$

provided the constant ρ for each different substance has the same numerical value which we have already found for the resistivity of that substance. This being so, the factor $\rho\frac{l}{s}$ is simply the resistance of the conductor. See Eq. 149. Accordingly Joule's results assume the form,

$$H = Ri^2t \text{ ergs} = \frac{Ri^2t}{10^7} \text{ joules,} \qquad \textbf{Joule's Law.} \qquad \text{Eq. 155}$$

where the resistance R as well as the current i and the time t is measured in C.G.S. units. We may, therefore, write as the third and last of Joule's Laws, —

3. **In any conductor, the heat developed by a given current flowing for a given time is directly proportional to the resistance of the conductor.**

If, however, we use the *practical* units and measure the current in amperes, the resistance in ohms, the time in seconds, and the heat in calories, then we have the following laboratory equation: —

$$H = 0.24\, Ri^2t \text{ calories.} \qquad \text{Eq. 156}$$

In the laboratory it is practicable to measure any three of the four quantities in this equation, so that we can employ the equation to determine the remaining quantity. By measuring H, i, and t we may thus determine R.

The whole story of the heating effect of an electric current is contained in the above equation, whose meaning is that a current of *one ampere flowing through a resistance of one ohm develops in this resistance* 0.24 *calorie per second.*

Other Forms of Joule's Laws

406. In virtue of Ohm's Law it is clear that we may write Eq. 155 in either of the three following forms: —

$$H = i^2Rt = iEt = \frac{E^2}{R}t \text{ ergs,} \qquad \text{Eq. 157}$$

where E is the E.M.F. between the extremities of the resistance R, and H is the heat developed in the resistance R. The first form shows us that when i is a constant, $H \propto R$; the last form shows that when E is a constant, $H \propto \frac{1}{R}$. The importance of clearly distinguishing the conditions under which an experiment is performed and of taking account of them in our equations will be seen from the following: —

Join in series two pieces of wire, about No. 36, having each approximately the same length and diameter, but one made of copper, the other of iron. If now a current be passed through these two wires, the value of i will be the same for each. Hence $H \propto R$, and since the resistance of the iron is much greater than that of copper, the heat is localized mainly in the iron, so that as the current is increased the iron is the first to become red-hot and fuse.

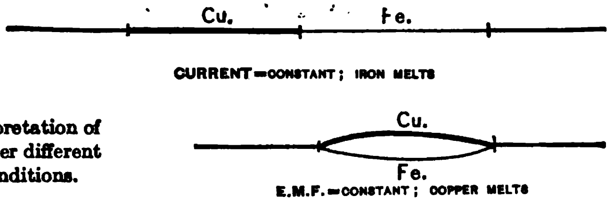
Cu.
Fe.
CURRENT = CONSTANT; IRON MELTS
Cu.
Fe.
E.M.F. = CONSTANT; COPPER MELTS

Fig. 322. — Interpretation of Joule's Law under different experimental conditions.

When, however, these two wires are placed in parallel, the current is no longer the same in each; it is now the E.M.F. that is constant as we pass from one wire to the other. Hence $H \propto \frac{1}{R}$, and we may now expect the heat to be concentrated mainly in the good conductor, that is, in the copper. As the current is increased this expectation is justified, for we now observe that the copper wire is the first to get hot and melt.

In the laboratory it is well to remember this principle; for when a set of resistance coils is joined to a source of constant E.M.F., such as a storage battery, the low resistances are apt to be burned out if put in circuit alone. But if the coils are joined to a source of constant current, it is the high resistances which are in danger.

407. *Definition of the International Watt.* The *watt* has already (§ 114) been defined as a unit of power equal to a *joule per second.* Electrical engineers commonly measure power by use of Eq. 157 which may be written

$$\textit{Power} = \frac{H}{t} = iE \text{ watts,}$$

where H is measured in joules, i in amperes, and E in volts. The "international watt" is defined as "the energy expended per second by an unvarying electric current of one international ampere under an electrical pressure of one international volt."

It is important for the student to note that the *watt* and *kilowatt* are units of power, not of energy. When one pays his electric light bill he pays for so many *kilowatt-hours, i.e.* for electrical energy.

Some Applications of Joule's Law

408. *The Thermogalvanometer.* Joule's equation shows us that, since the amount of heat developed in any given conductor depends upon the square of the current, the heating effect is independent of the direction of the current; the energy transformed into heat is the same for a current $+i$ as for $-i$. Hence the heating effect offers an excellent method for measuring alternating currents. An exquisite illustration of this is to be seen in the thermogalvanometer recently devised by the English physicist Duddell for the measurement of small alternating currents. A minute closed circuit composed of one piece of bismuth and one of antimony is suspended between the poles of a permanent magnet in such a position as to include as few lines of force as possible.

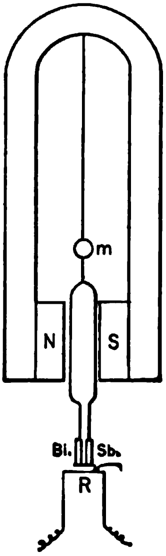

FIG. 323. — Duddell's thermogalvanometer.

Just below the lower junction of the bismuth and antimony is placed a short but exceedingly fine wire, one which has therefore a high resistance. The alternating current to be measured is passed through this wire R, Fig. 323. A certain fraction of the heat generated in R is communicated to the lower junction of the suspended circuit and thus produces a thermoelectric current, and causes the suspended circuit to rotate through a certain angle in the magnetic field. This deflection is read off by means of a mirror m attached to the suspension fiber. The scale is calibrated by passing through the instrument a direct current of known intensity.

409. *The Incandescent Lamp.* We may increase the heat either by increasing the current or by increasing the resistance. Heat is developed wherever resistance of any kind is located. It is for this reason that long slender carbon filaments of high resistance are used in incandescent lamps. The heat is located almost entirely in the filament, scarcely at all in the wires which lead the current to the filament.

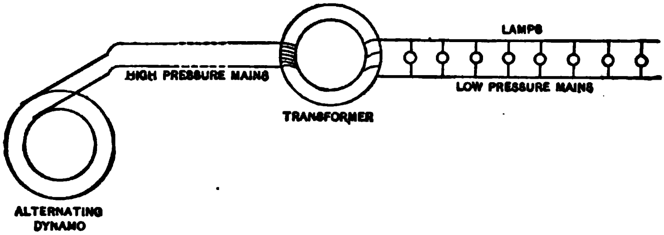

FIG. 324. — The alternating current system.

Note also that it is the heat, not the electric current, which makes the filament luminous. If the carbon filament were raised to the same temperature by any other means, it would be just as bright.

Figures 324 and 325 indicate the manner in which incandescent lamps are usually connected in the alternating and direct current systems respectively.

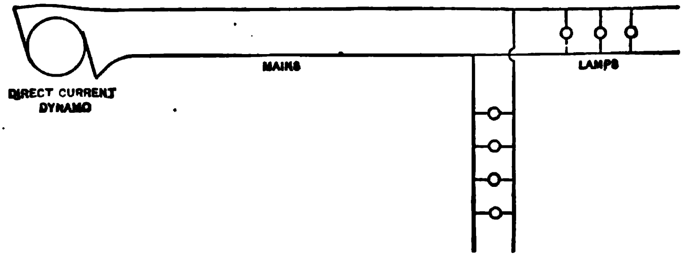

FIG. 325. — The direct current system.

The carbon filament has recently been replaced by one of tungsten, because the carbon filament cannot be heated much above 2000° C. without disintegration which blackens the bulb; the tungsten has a melting point of approximately 3000° C. and can be worked at a higher temperature than carbon; hence, in accordance with Stefan's Law, radiates more powerfully. A 60-watt tungsten lamp when used on a 100-volt circuit requires about

0.6 ampere and has a life of something like 1200 hours. Such a lamp will give 48 candle-power; from which it will be seen that each candle-power calls for the expenditure of $1\frac{1}{4}$ watts.

It has lately been found possible to push the tungsten filament to a still higher temperature — and hence higher efficiency — by filling the bulb with nitrogen. The nitrogen conducts considerable heat from the filament; but this is more than compensated for by the increased radiation at higher temperature. These "nitrogen-filled" lamps as they are called produce as much as 1000 candle-power on an expenditure of 500 watts.

410. *The Arc Lamp.* The phenomenon of the electric arc, first observed by Sir Humphry Davy in 1800, is one which is exceedingly complicated and even yet imperfectly understood.

This much, however, seems to be established; when two carbon rods having between them a potential difference of from 50 to 100 volts are made to approach each other, nothing happens until the distance between them is less than that of a thin sheet of paper, such a perfect nonconductor is ordinary air. But when once the carbons touch each other and the current begins to flow, an enormous amount of heat and a temperature of some 3500° C. is generated right at the point of contact; for it is at this point that most of the resistance of the circuit is located.

Now suppose the carbon poles to be slightly separated — say by $\frac{1}{8}$ inch. If the air between the poles were in the same condition as before, the current would immediately cease; as a matter of fact the entire region between the poles is filled with ionized gases and has now become a good conductor. The reason for this is that any white-hot body will give off both positive and negative ions in great quantity. The negatively charged electrons given off at the cathode appear to bombard the anode and thus produce an intensely hot spot called the "crater."

Most of the light from the carbon arc comes from this positive crater. The essential feature of the arc is a hot cathode and as soon as the current reaches a value too small to keep the cathode hot the arc becomes unstable.

The ordinary carbon arc requires from 5 to 10 amperes, and a voltage of from 45 to 60. Such an arc gives between one and two candles per watt.

The *magnetite arc* now coming into wide use has a copper anode opposed to a cathode made of black oxides of iron mixed with some titanium oxide. The light in this arc does not come from a crater but from the intensely brilliant flame — the arc itself — between the poles. The efficiency of the magnetite arc is considerably higher than that of the carbon poles.

The *mercury arc* and various other forms must be left for advanced study.

The Theory of Electric Heating

411. Before leaving this brief outline of the heating effect it is well for the student to see that Joule's Law follows as an immediate consequence of Ohm's Law and the definition of potential difference. Joule might therefore have predicted his result, which as a matter of fact he discovered from experiment.

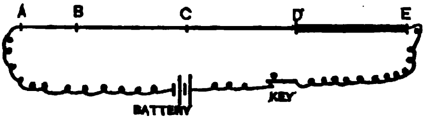

FIG. 321. — Amount of heat produced by an electric current depends upon length, diameter, and material of wire.

Consider any electric circuit, say that represented in Fig. 321. Between any two points such as B and C which include between them no source of E.M.F. there is a difference of potential E which is given by Ohm's Law, namely, $E = iR$, where R is the resistance between B and C. If the current i flows for a time t, a quantity of electricity it will pass from B to C. By definition of potential difference, E (or iR) is the amount of work done in carrying one unit of electricity from B to C. The amount of work done in carrying it units will therefore be i^2Rt. If we assume that all this work which is done in transferring electricity from B to C is spent in heating the conductor, then it follows that

$$H = i^2Rt,$$

which is Joule's Law. Now since experiment shows that this equation is true, we have good reason to think that all the work of the battery between the points B and C is transformed into heat.

If an electrolytic cell, a storage battery, or an electric motor be inserted into the circuit between *B* and *C*, Ohm's Law no longer holds for this part of the circuit where the work of the current is partly spent in overcoming the counter E.M.F. of the cell, the battery, or the motor.

Joule's equation thus shows us that with any given current the heat will be distributed throughout the circuit in exactly the same ratio as the resistance. It is for this reason that in incandescent electric lamps the resistance of the circuit is concentrated in the carbon filament of the lamp. An ordinary 16-candle-power 110-volt lamp has about 300 ohms resistance. For this same reason fuse wires for protecting electric circuits are made of a material which has high resistivity as well as a low melting point.

THERMOELECTRIC CURRENTS

412. In addition to the voltaic cell there is a second method of producing current which deserves notice, partly on account of its wide application in science and partly because of its simplicity and directness, namely, the method in which the energy of the electric current is obtained directly from heat. This mode of generating electric currents was discovered by Seebeck and reported to the Berlin Academy of Sciences in 1822. We have already learned that the essential feature of a Voltaic cell is that it will maintain at a different potential the two conductors of the first class which have their respective ends immersed in a conductor of the second class. It has also been noted (§ 367) that, when two conductors are joined together so as to form a circuit at uniform temperature, whatever tendency there is at one junction to produce a current is exactly counterbalanced by an opposite tendency at the other junction. See Fig. 286.

But Seebeck discovered that when one of these junctions is at a different temperature from the other, the equilibrium is destroyed and a direct continuous current flows in the circuit so long as this difference is maintained. From this phenomenon, known as the *Seebeck effect*, it appears that when a metal is heated, its electrical properties are changed just as definitely as if its chemical composition were altered. Not only so, but different metals are differently affected by the same change of temperature. These thermoelectric currents, as they are called, are in general very minute; but the modern galvanometer is so extraordinarily sensitive that even these small effects are easily observed and measured. As noted above (§ 273) the thermocouple is already a standard instrument for measuring temperatures. The reason for this is that the strength of the thermoelectric current depends, other things remaining the same, upon the difference of temperature between the heated and the cooled junction.

But the E.M.F. in a circuit of this kind is also a function of the mean value of the two temperatures at the junction, giving rise to the curious phenomenon of *inversion*, discovered by Cumming in 1823.

The important fact appears to be that at any single junction a change of temperature produces a change of potential difference. And the important constant connected with any particular couple is its **thermoelectric power, which is defined as the rate at which this potential difference changes with temperature.**

Since the differences of potential produced at thermoelectric junctions are as a rule very small, they are generally expressed in terms of microvolts, *i.e.* millionths of a volt.

In 1834, Peltier, a French watchmaker, discovered the converse of the Seebeck effect, namely, when an electric current, say from a voltaic cell, is passed through a junction of two dissimilar metals the junction is either heated or cooled, depending upon the direction of the current. This is known as the *Peltier effect.*

Problems

387. How long will it take a current of one ampere to deposit a gram of pure silver? *Ans.* 14 m. 55 sec.

388. An ampere is 10^{-1} C.G.S. units; a volt is 10^8 C.G.S. units. Find the heat in ergs developed by a current of one ampere flowing for one second between two points whose potential difference is one volt.

389. How much power in watts will be required to keep up a current of 40 amperes charging a storage battery at 55 volts?

390. The resistance of a 50-volt lamp is 50 ohms. How much heat in joules will be developed in the filament in 10 seconds? What power in watts is required to feed such a lamp? If the lamp is 16-candle power, how many watts per candle power are required?

391. How many amperes will be required to deposit 0.75 gram of copper in 2 hr.?

392. Suppose the current in the preceding problem to be furnished by a battery of gravity cells. How much zinc will be used up in the battery while the 0.75 gram of copper is being deposited?

393. A chain is made of alternate links of platinum and silver. If the links are identical in all other respects, find the ratio of the heat developed in two adjacent links.

394. What weight of water will be decomposed by a current of 10 amperes flowing for 1 hr.?

395. Suppose the earth is used as the return wire on a direct current lighting circuit. If the ground connection is made by use of two iron plates, and if 100 lights each using ½ ampere are employed for 4 hours each day, find loss of weight at the iron anode in the course of one year.

396. A dynamo supplies a circuit of 600 incandescent lamps. Each lamp requires 100 volts and ½ ampere. What must be the resistance of the leads and the armature in order that the energy loss in them shall amount to 5% of the total electrical output of the machine?

397. A waterfall is situated 10 mi. from a certain town which is lighted by electrical energy derived from this fall. The power required to light the town is 150 kilowatts. Will it be more economical to transmit this power, using 1500 amperes and 100 volts, or by employing a voltage of 10,000 to transmit 15 amperes? Suppose the resistance of 10 miles of line wire to be 40 ohms; compute the loss on the line in these two cases.

398. Taking the atomic weight of silver as 108 and the electrochemical equivalent of hydrogen as 0.0000104, find the electrochemical equivalent of silver.

399. A copper wire of given length is replaced by one of aluminium. What cross section must the aluminium wire have in order that a given current will produce no more heat in it than in the copper?

400. Two cells are joined in series with a given resistance and produce a current of 1.5 amperes. One of the cells is now reversed, its poles being interchanged with their previous position. The current is now 2 amperes. Find the ratio of the E.M.F.'s of the two cells.

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> "For after all, the noblest definition of Science is that breadth and impartiality of view which liberates the mind from specialties."
>
> LOWELL, *Harvard Anniversary Address*, 1886.

CHAPTER XI

ELECTROMAGNETISM

413. First Law of Electromagnetism. The reader has already used a galvanometer for the detection of electric currents and has learned that it was H. C. Oersted who discovered the first connection between magnetism and electric currents.

In order to briefly review the magnetic field which Oersted found to exist about every electric current, it is well to try the experiment illustrated in Fig. 326.

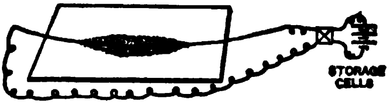

FIG. 326. — Iron filings arrange themselves in rings about an electric current.

A current is passed through a bare copper wire of small diameter. The copper wire should be just large enough not to melt. On dipping this wire into a box of fine iron filings, we observe that the filings, which, by induction, become temporary magnets, arrange themselves in circles about the wire, and as the wire is lifted out these rings of filings remain clinging about it, as indicated in Fig. 326. But when the current is interrupted they fall off, showing that these circular lines of force disappear with the current.

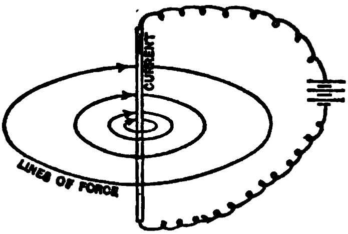

FIG. 327. — Showing right-handed screw relation between current and lines of force due to that current.

The magnetic field about a straight wire conveying a current is then represented by a series of circles drawn about the wire as center and in planes perpendicular to the wire, as illustrated in Fig. 327. As an experimental fact it is observed that the direction of these lines of force is related to the direction of the current, as indicated in Fig. 327, that is, in the right-handed screw relation. But this is exactly the state of affairs described by Biot and Savart's Law, which takes the following form when the C.G.S. unit of current is employed: —

$$H = \frac{2i}{r}. \qquad \text{Eq. 158}$$

If the wire be bent into a loop, as in Fig. 328, lines of force will pass through the plane of the loop in a direction perpendicular to that plane. If the loop be a circular one, then the intensity of the magnetic field at the center of the loop is $2\pi i/r$, where r is the radius of the loop.

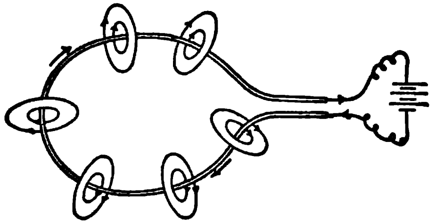

FIG. 328. — Lines of force about a single loop conveying a current.

414. Suppose now that one takes a positive unit magnet pole and carries it once around (*i.e.* along) one of the circular lines of force which surround a long straight wire and which are depicted in Fig. 327. The force acting upon the unit pole will be numerically equal to the magnetic intensity H. If the work done in taking the unit pole once around the circuit be indicated by W, then,

$$W = 2\pi r \cdot \frac{2i}{r} = 4\pi i. \qquad \text{Eq. 159}$$

This is the general form (although here demonstrated only for a special case) of the great principle discovered by Oersted, namely, **the electromagnetic measure of any current is the work done by carrying a unit magnet pole once about the entire current; and the amount of work done is 4 π times the strength of the current inclosed by the path of the pole.**

This summary of fact is known as the First Law of Electromagnetism. No exception, indeed, has ever been discovered to the general principle that about every electric current, whatever its path, there are linked lines of magnetic force in the right-handed screw relation.

For the illustrations given make it clear that the field about an electric current is a magnetic circuit; and since electric circuits are always closed, these two circuits (the magnetic and electric) are always linked. One might define the E.M. unit of current as one which produces in a magnetic circuit *of unit length* a field intensity of 4 π.

This definition is strictly the same as that given above (§ 381).

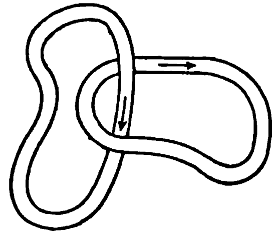

FIG. 329.—Illustrating the right-handed screw relation between lines of force and currents.

415. Second Law of Electromagnetism. The reader has already seen that the region about a magnet is endowed with properties totally different from other portions of space. This magnetic field, as it is called, does not appear to the naked eye to be different from any other portion of space; but we know that it is different, for a magnet brought into such a field behaves in a manner peculiar to this magnetic field. As has been pointed out, Faraday described such a magnetic field by conceiving it to be filled with lines of force — purely imaginary lines — drawn, at each point in the field, in the same direction in which the axis of a magnet would set itself if placed at that point. The earth itself being a great magnet, the whole of the space inhabited by the human race is a magnetic field. **In these fields the lines are distributed just far enough apart to make the number which pass through unit area, drawn perpendicular to the direction of the field, numerically equal to the intensity of the field.**

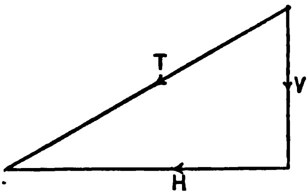

FIG. 330.—Resolution of the earth's magnetic field into two components.

The lines of force which describe a magnetic field being directed, *i.e.* vector, quantities, they can be resolved into components. We accordingly speak of the vertical component and the horizontal component of the earth's field, as represented in Fig. 330, where T represents in direction and in amount the total magnetic force of the earth on any magnet pole; V the vertical, and H the horizontal component.

The total number of lines of magnetic force passing through any definite surface is spoken of as the *magnetic flux* through that surface. Thus if a single magnet pole of strength m be surrounded by a sphere of any radius, say 1 cm., the magnetic flux through this surface is $4 \pi m$.

If the preceding is clear, we are now in a position to consider the beautifully simple method by which Faraday, in the autumn of 1831, first succeeded in producing currents without the use of the voltaic cell and without the use of heat.

Let us consider any closed circuit (say a piece of copper wire bent into the form of a circle or square) placed in a magnetic field; we may think of it as containing or inclosing a certain number of lines of force, just as a fence might inclose a group of trees.

In Fig. 331 six lines of force are represented as being inclosed by the wire circle. If by any means the magnetic field were made twice as strong, we should represent this change by drawing the lines of force twice as close together, so that now the circle would inclose twelve instead of six lines. Or, if the copper circle were carried parallel to itself to another region where the magnetic field is not so strong, then we should describe this change by drawing *fewer* lines of force through the circle.

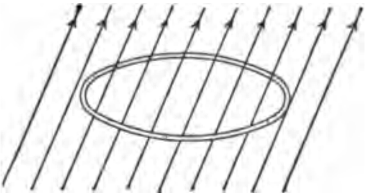

FIG. 331.— Magnetic lines of force inclosed by an electric circuit.

Faraday's discovery may then be described in a quantitative manner as follows: **When, by any means whatever, the total number of lines of force passing through any circuit is changed, an electric current is produced in that circuit. Such a current is called an induced current.**

This statement is at once a description of the third method of producing electric currents and a qualitative expression of the Second Law of Electromagnetism. As will be seen later, nearly all our electric lighting and telephone currents are produced in this way.

Some Illustrations of Induced Currents

416. *Experiment* 1. — Connect the terminals of a D'Arsonval galvanometer with a copper wire, so that with the suspended coil a closed circuit is formed. Take a single loop in this circuit, and thrust through it one pole of a permanent magnet. Such a magnet carries its magnetic field (*i.e.* its lines of force) about with it. And when these invisible lines are thrust through the loop, we observe that for the instant a current passes in the galvanometer. See Fig. 332. Now remove these lines by removing the magnet; a current passes, but now in the opposite direction. **Observe that in either case the current lasts only so long as the motion of the magnet continues.**

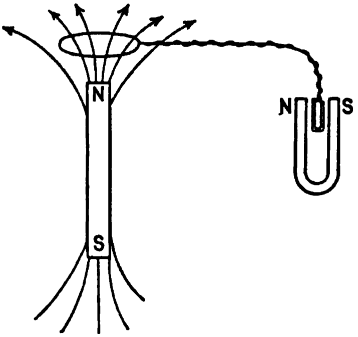

FIG. 332. — An electric current produced by thrusting lines of force through a circuit.

417. *Experiment* 2. — Now support the magnet on some convenient stand, and slip the loop of wire over one end of the magnet. See Fig. 333. A current is again observed. If we slip the coil off, the *direction* of the current is reversed.

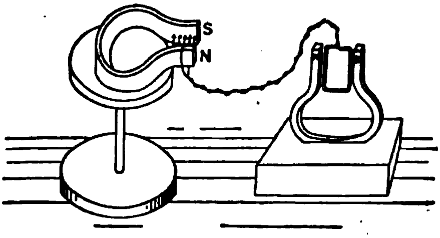

FIG. 333. — An electric current produced by moving a coil in a magnetic field.

If the loop of wire be slipped on and off quickly, a current will be produced which flows first in one direction, then in the other.

(i) A current which changes its direction in a regular and continuous manner is called an " alternating " current.

(ii) But if the current, during certain intervals of time, ceases entirely, it is said to be "intermittent."

(iii) Currents which flow continuously in one direction are said to be " direct."

From these two experiments, it is evident that it makes no difference whether it is the loop of wire or the lines of force that are moved. **The relative motion of the wire and the magnetic field changes the number of lines of force inclosed, and hence produces the current.**

418. *Experiment* 3.—Since we live on the surface of a large magnet, it becomes interesting to see how we must move a closed circuit in the earth's magnetic field in order to produce an electric current.

If a large loop of wire be made in one part of a closed circuit containing a galvanometer, we have only to hold this loop alternately in an east-and-west and in a north-and-south vertical plane. When the plane of the loop is vertical and north-and-south, no magnetic lines from the earth pass through it; they pass along each side of it. But if the coil be suddenly rotated about a vertical axis so as to lie in an east-and-west plane, the earth's lines of force will thread through it, and we observe in the circuit an induced current. If we rotate the coil of wire back into a north-and-south plane, the current is in the opposite direction. **If in either of the preceding experiments we leave the total length of wire unchanged and take two turns of wire instead of one, the deflection of the galvanometer is observed to be twice as great as before.**

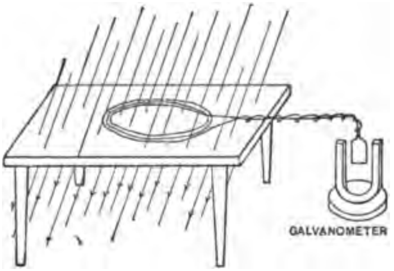

Fig. 334.—An electric current produced by capsizing a coil of wire in the earth's magnetic field.

If a coil of wire which lies flat on the table (Fig. 334) be quickly turned over, the lines of force passing through it will be changed by twice the original number. For if in the first case there were N lines passing through in, say, a *positive* direction, when the coil is turned over there will be N lines passing in a *negative* direction, *i.e.* the change will be

$$N - (-N) = 2N.$$

419. *Experiment* 4.—But there are still other ways in which we may alter the number of lines of force passing through a circuit. We have seen that a wire conveying a current is surrounded by a magnetic field. If we bring a loop of our galvanometer circuit near such a battery current, we shall find that the magnetic lines from the battery current will in general thread through the galvanometer circuit, and will thus produce an induced current. In Fig. 335 lines of force marked 3, 4, 5, and 6 each pass through the galvanometer circuit as well as the battery circuit. When the motion ceases the induced current ceases.

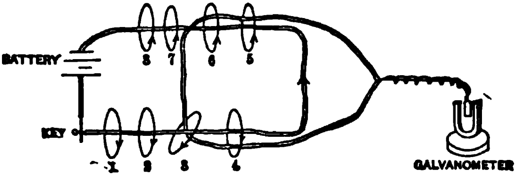

FIG. 335. — An electric current produced in a loop of wire by making or breaking a battery circuit in the neighborhood of the loop.

If instead of bringing the loop and the battery current near together, we simply place the circuits side by side and suddenly open or close the battery circuit, we observe in each case an induced current in the galvanometer circuit. To interrupt or break the battery current is equivalent to carrying it off to a very great distance. To close the battery circuit is electrically equivalent to bringing it suddenly into the neighborhood of the galvanometer circuit. Indeed, anything which increases the battery current — as, for instance, shortening the wire in the circuit — is equivalent to moving it nearer to the galvanometer circuit; while anything which diminishes the battery current has the effect of motion away from the other circuit.

The Idea of Electromotive Force

420. The essential feature of the voltaic cell is that it will maintain a difference of potential either between its terminals if the circuit be open, or between any two points on a circuit which is closed.

But Faraday showed, by experiments such as the four preceding, that, without touching the closed battery circuit, it is possible to exactly counteract this potential difference and make the current disappear by simply thrusting lines of magnetic force through the circuit in the proper sense and at the proper rate. The introduction of the lines of force is therefore equivalent to the introduction of an E.M.F.; the effect is in each case to make the electric current flow; and in each case the effect is said to be produced by an "electromotive force," just as the motion of a stream of water might be said to be due to a "hydromotive force." And just as in the case of a basin of water one can make the water flow around in the pan by dragging a towel in a circle over the bottom, without any difference of level, so also in the case of an electric circuit one can make the current flow, without any difference of electrostatic potential, by merely introducing lines of force. In other words, while a difference of electrostatic potential always acts as an electromotive force, it does *not* follow conversely that all electromotive forces are differences of electrostatic potential. Electromotive force is the more general term.

Concerning the E.M.F. which is produced by magnetic induction, the three following experimental facts should be verified by each student for himself in the laboratory: —

(i) Other things being the same, the amount of E.M.F. varies directly as the number of lines, say N, introduced into the circuit.

(ii) Other things remaining the same, the E.M.F. varies inversely as the time occupied by their introduction.

(iii) To produce a current in any given sense, lines of force must be thrust through the circuit in a sense which is just opposite to that of the lines produced by this current. Thus in Fig. 336 the lines of force due to the current i would be directed downward; and hence any lines of force which are thrust downward through the circuit will produce an induced current which is opposite in sense to i and may be made to just annul the battery current i.

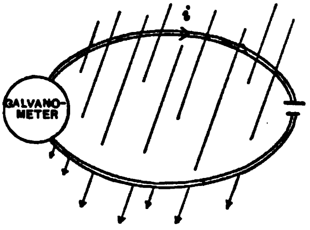

FIG. 336. — Showing how potential difference of a battery may be counterbalanced by introducing lines of force.

In 1845, F. E. Neumann formulated these facts in the following simple manner: Let N_1 and N_2 denote the total number of lines of force threading any given circuit at the instants t_1 and

t_2 respectively, and k a constant of proportionality. Then for this circuit

$$\text{E.M.F.} = -k\left[\frac{N_2 - N_1}{t_2 - t_1}\right]_{t_2 \doteq t_1} \qquad \text{Neumann's equation.} \qquad \text{Eq. 160}$$

This brief expression, which says that the E.M.F. of any induced current is to be measured by the rate at which lines of force are cut by that circuit, is the Second Law of Electromagnetism.

In a modified form it is also the engineer's formula for computing the E.M.F. of a dynamo which he is designing. The negative sign means that the sense of the induced E.M.F. and the sense of the lines thrust through the circuit are related as the rotation of a left-handed nut to its direction of travel on the screw.

The advanced student will find that these two principles, the first and second laws of electromagnetism, can be described in a general and quantitative way only by the two fundamental equations of Maxwell. Here, however, we are not seeking after the utmost generality, but rather after a clear grasp of a few special and concrete but exceedingly important cases. For these particular cases lead naturally to a clear understanding of the two fundamental principles.

Unit of Electromotive Force

421. The Volt. About one half a century ago it was agreed by the British Association for the Advancement of Science and assented to by the remainder of the scientific world that we should take as unit E.M.F. that which is produced in any closed circuit when it cuts one line of force per second. This definition has the effect of making $k = 1$ in Eq. 160, so that the defining equation for E.M.F. is

$$\text{E.M.F.} = -\left(\frac{N}{t}\right), \qquad \text{Defining equation for E.M.F.} \qquad \text{Eq. 161}$$

where N is the number of lines of force added during the interval t. We have already learned how to measure the number of lines passing through unit area, *i.e.* the intensity H of a magnetic field; and N is, of course, simply the product of H by an area.

This unit of E.M.F. is called the electromagnetic C.G.S. unit; more briefly the E.M. unit; but it is entirely too small for practical purposes. Hence the British Association agreed to adopt the following "practical unit," called the "volt."

The "volt" is defined as that electromotive force which is induced in any closed circuit cutting lines of magnetic force at the rate of one hundred million per second.

Or in terms of arithmetic, —

$$1 \text{ volt} = 10^8 \text{ E.M. units of E.M.F.}$$

This "practical" unit of E.M.F. has, however, been superseded by the "international volt," adopted by the London Congress of 1909 and defined as follows: *The international volt is that electrical pressure which when applied steadily to a conductor whose resistance is one international ohm will produce a current of one international ampere.*

Standard of Electromotive Force

422. In order to measure electromotive forces in pratice, it remains only to establish a standard E.M.F. with which others may be compared. This was much facilitated by the invention and perfection of the Weston cell, which when prepared according to the international specifications furnishes $\frac{10183}{10000}$ volts at 20° C. See §§ 376, 377, and 389.

Electrical Measuring Instruments

423. Electrical measurement is almost a complete and independent science by itself, so well established are its standards and methods. Yet the entire structure is built essentially upon two fundamental facts of electromagnetism, one of which connects the E.M.F. in any circuit with the rate at which the magnetic field in this circuit is varying, while the other connects the current strength with the strength of the magnetic field produced by the current.

Most instruments for measuring voltage and current strength are based directly upon the Oersted phenomenon. But this first law of electromagnetism can be formulated in several ways.

Ampère described the fact by saying that any closed loop of wire conveying a current — say a voltaic circuit — is equivalent to a thin disk magnetized normally to the plane of the disk, and having edges which coincide with the electric circuit. A disk of this kind whose lines of force are precisely the same as those of the circuit is called an equivalent *magnetic shell.* The strength of such a shell will, of course, vary directly as the current in the circuit.

From this point of view a portion of an electric circuit which is movable will set itself in a magnetic field just as the equivalent magnetic shell would do. Thus if a coil of wire be suspended between the two poles of a magnet as shown in Fig. 337, it will behave as a magnet (with its north pole on the face of the coil turned toward the reader) as soon as a current is passed through it in the direction indicated by the arrow. Such a magnetic shell will at once twist the suspension wire and turn its N pole so as to face, as nearly as possible, the S pole of the permanent magnet. The amount of rotation indicates the amount of the current, even when, as in large deflections, the rotation is not directly proportional to the current.

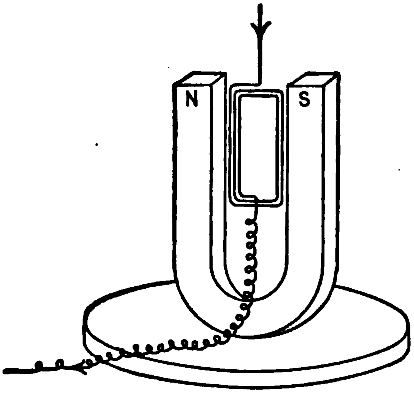

FIG. 337. — A galvanoscope in which a part of the wire conveying the current is movable, while the magnet is fixed.

Maxwell's point of view is more helpful, and the student is urged to master it and adopt it at once. The behavior of an electric circuit as a whole or of any portion of an electric circuit placed in a magnetic field is described by Maxwell as follows: **Every portion of the circuit is acted upon by a mechanical force urging it in such a direction as to make it inclose the largest possible number of lines of force in the right-handed screw direction.**

The classical method of illustrating this fact is to float a small voltaic cell in a large dish of water. A small coil of copper wire, consisting of half a dozen turns, with zinc and copper plates soldered to its respective terminals, can be fitted, through a cork, into a small and rather deep glass beaker, as shown in Fig. 338.

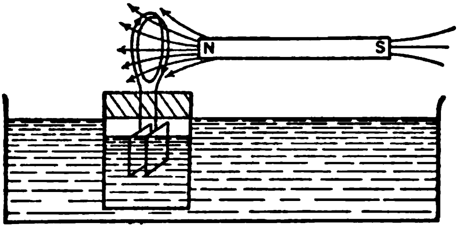

FIG. 338. — Movable coil *rotates* so as to include lines passing through in the right-handed direction.

As soon as the glass is half filled with dilute sulphuric acid and floated on water we have at our disposal a current in a circuit which is free to move. If the floating cell is inclined to be unstable, a small piece of sheet lead laid in the bottom will remedy the difficulty.

Two phenomena are observed when a strong bar magnet is brought near this floating coil, viz.: —

(1) If the N pole of the magnet be extended toward the coil, the circuit will always *rotate* so as to present that face in which the current is flowing in a clockwise sense, *i.e.* the south face of the coil is attracted by the north pole of the magnet.

(2) But this motion of rotation is not the whole of the phenomenon. For if the magnet be held in a fixed position, end on *in front of the coil*, it is observed that the coil is also *translated* as a whole, moving so as to include more lines of force.

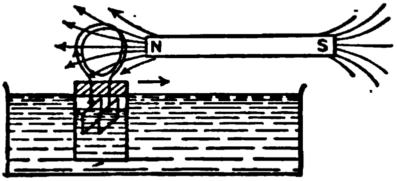

FIG. 339. — Movable circuit is *translated* so as to include as many lines of force as possible.

And, in general, it has been proved that an electric current always sets itself so as to include as many lines of force as possible in the right-handed screw direction.

A slack wire of gold tinsel carrying a current shows the same tendency to wrap itself about a straight magnet, that lines of force have to wrap themselves about a straight wire conveying a current.

The advanced student will discover that this general electrodynamical principle is merely a special case of the general dynamical principle that any body tends to move, from rest, into such a position as to make its potential energy a minimum.

424. The Moving Coil Galvanometer. This instrument, shown in Fig. 340, is the one which Lord Kelvin devised in 1867 for receiving messages over the Atlantic cable, and is based upon the principle which we have just been considering. It was improved for laboratory use by D'Arsonval; hence this type of instrument is frequently called a *D'Arsonval* galvanometer.

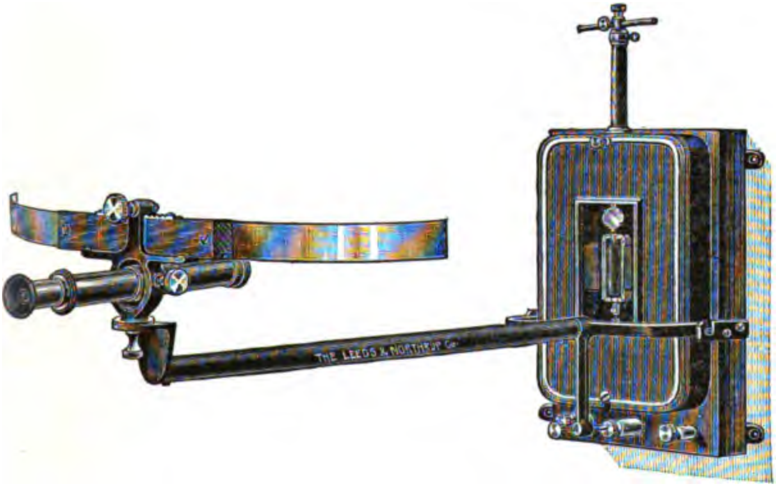

FIG. 340. — A sensitive moving coil galvanometer.

The sensibility of this device almost surpasses belief. American makers are now able to furnish moving coil galvanometers that measure currents less than one thousandth of a millionth of an ampere.

425. The Weston Voltmeter. This justly celebrated and widely used instrument also consists essentially of the magnet and movable coil. Here, however, an iron core is placed midway between the poles of the magnet and the light movable coil is held in position by pivots which work in jeweled bearings. The flat spiral springs which hold the coil in its zero-position are used also to introduce and lead off the current. See Fig. 341.

In series with this movable coil there is placed, inside the case of the voltmeter, a resistance — called a resistor — the value of which is high compared with the resistance existing between the two points whose potential difference is to be measured. Let us denote by R the constant and large resistance of the voltmeter itself, and by r the resistance of the source of the E.M.F. which is to be measured. If we denôte this E.M.F. by E, then the current i through the movable coil will be

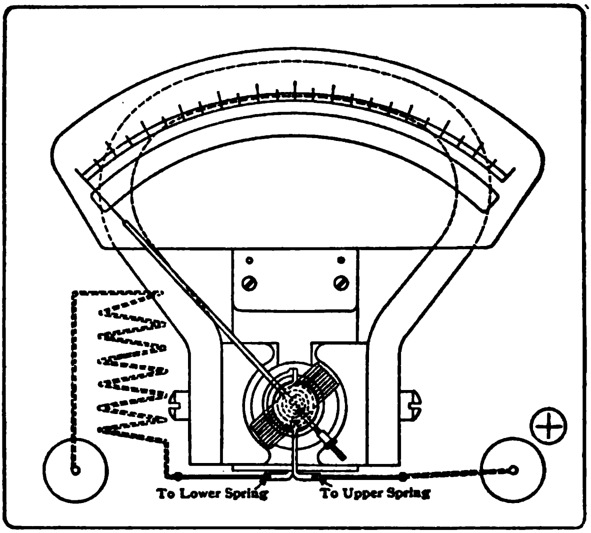

FIG. 341. — A typical Weston direct-current movable-coil system.

$$i = \frac{E}{R + r} = \frac{E}{R}, \text{ very approximately,}$$

since R has been made so large that r may be neglected in comparison with it; that is, the indications of the instrument are directly proportional to E. The portable type is shown in Fig. 342; the type used in power stations is essentially the same, only here they are permanently fastened to a switchboard.

A *millivoltmeter* is merely a voltmeter of higher sensibility, such that the divisions on the scale indicate hundredths or thousandths or ten-thousandths of a volt.

426. The Weston Ammeter. It is evident at the outset that any instrument which is to be inserted in a circuit for the measurement of current must have a resistance which is so small as

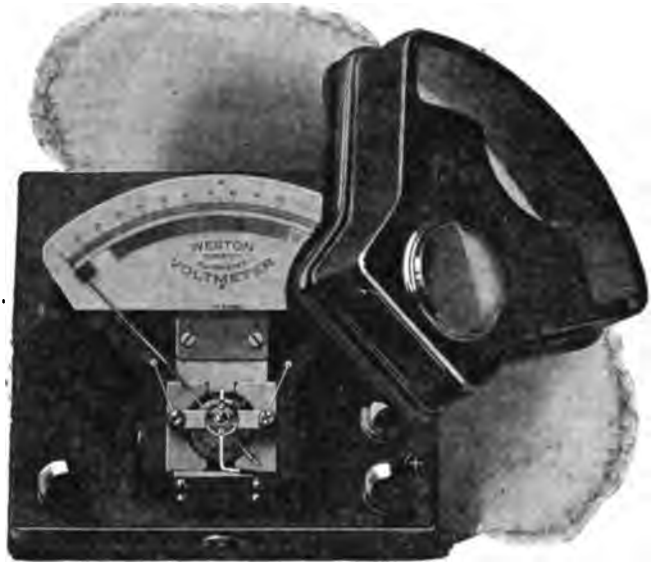

FIG. 342. — A Weston voltmeter with its cover removed.

not to appreciably change the current to be measured. For this reason an ammeter is made up of a short thick piece of wire which has a millivoltmeter shunted about. This short thick conductor is called the *shunt*. Sometimes it is placed inside the case of the instrument — as in portable ammeters; sometimes it is placed a number of feet away from the moving coil, as in the case of power station ammeters; in either case the whole constitutes an ammeter.

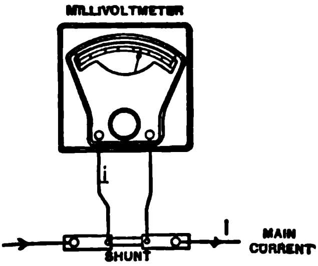

FIG. 342 *bis*. — The ammeter.

Let us denote by S the very small resistance of the shunt and R the resistance of the millivoltmeter. Then if I is the current to be measured, IS will be the drop in potential between the ends of the shunt and will also be the E.M.F. acting upon the millivoltmeter. See Fig. 342 *bis*. The current through the moving coil of the ammeter is then i, where, by Ohm's Law,

$$i = \frac{SI}{R}.$$

But S and R are here each constants — parts of the ammeter. Hence the indications on the dial, which are determined by i, are proportional to I, the main current. Such a device is called an indicating ammeter. No student should leave this subject until it is perfectly clear to him that current-measuring devices of this type are voltmeters, and that all voltmeters of the type described in the preceding section are current-measuring devices.

427. The Wattmeter. We have already (§ 407) seen that in order to measure the amount of power absorbed in any part of a circuit — say in an arc-lamp — we must know the *amperes* passing through that part of the circuit and also the *volts* between the terminals of that part. Any instrument which indicates the product of *amperes* and *volts* indicates *power* and is called a *wattmeter*. One obtains this end by replacing the permanent magnet of the Weston voltmeter by a pair of coils of heavy wire, through which the main, or "load," current is passed. In the magnetic field produced by these coils is mounted the movable coil of a voltmeter. If now the coil of the voltmeter, including its high resistance as usual, be shunted about the terminals of that part of the circuit in which the power is to be measured; and if the heavy coils be placed *in series* with this part of the circuit, then the indications of the instrument will be proportional to the currents in each of these coils — and hence to their product. In other words it is a power-measuring device, and is called an *indicating wattmeter*.

The so-called wattmeter which is installed in private residences by electric lighting companies is really an energy meter, since it records the product of *amperes* × *volts* × *hours*. It is a watt-hour meter; often called a *recording* or *integrating wattmeter*.

There is an endless variety of ammeters, voltmeters, etc., invented for particular purposes, of which the four preceding are merely typical.

Some Applications of Electromagnetism

The Electromagnet

428. Promptly after Oersted's great discovery in 1820, it was found by Davy in England and Arago in France that an iron core is magnetized when placed in a helix of wire conveying a current. This device, which enables one to control the magnetism of the iron core by controlling the current in the helix, was called by William Sturgeon an *electromagnet.* Professor Joseph Henry of Princeton College first showed how to wind the helix in order to produce a strong magnetic effect at the end of a long line of wire, *i.e.* at a distant point, namely, provide the helix with a large number of turns of fine wire.

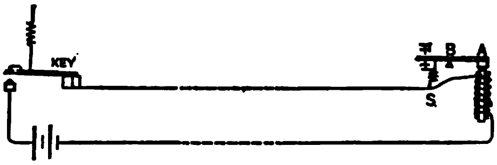

Fig. 343. — Representing the key at the sender's end and the sounder at the receiver's end of the telegraph line.

The behavior of the electromagnet is well illustrated in the ordinary telegraph sounder. Here the operator at a distant station closes the circuit; the electromagnet furnishes a strong magnetic field, attracting to its poles a small piece of iron *A*, called an "armature." This armature is pivoted at *B*, and hence can rotate slightly about an axis through *B*. When the circuit is closed the armature hits a stop which gives the sharp metallic click familiar to every one who has ever been in a telegraph office. When the circuit is broken, the armature is pulled back against another stop by means of a spiral spring *S*. The details of the "sounder" are shown in Fig. 344.

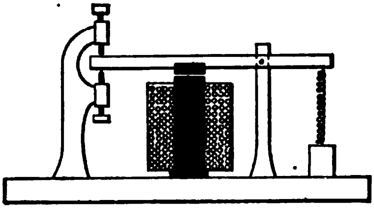

Fig. 344. — The telegraph sounder.

The electric door bell, which makes and breaks the circuit automatically, is very much like the telegraph sounder, only here when the circuit is first closed, by pushing the button at *K* (Fig. 345), the current runs through the armature; but the iron core of the magnet immediately becomes magnetized; the armature is pulled away from the metallic stop on which it rested, and the current is thus interrupted; but as soon as the current is broken the electromagnet ceases to attract, the armature falls back by means of a spring into its initial position, and the circuit is again closed.

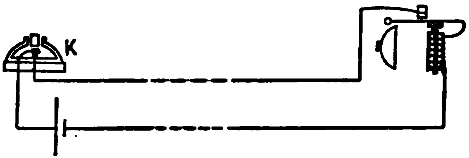

Fig. 345. — The electric door bell.

In this manner the armature is kept in continual vibration, and may therefore be employed as a bell clapper, the bell continuing to ring so long as the push button is held down at *K*.

The Electric Generator

429. By an electric generator is meant a machine which can be employed to transform mechanical energy into electrical energy. The fundamental principle of all such machines is, of course, the Second Law of Electromagnetism, and the earliest form of generator was invented by the man who discovered this law.

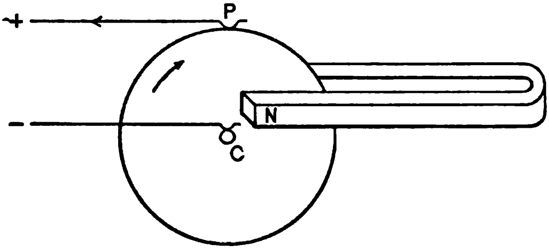

Fig. 346. — Faraday's disk machine, the earliest type of electromagnetic generator.

Faraday's machine consisted merely of a copper disk mounted on an axle so as to rotate between the two poles of a horseshoe magnet as shown in Fig. 346. The brushes make contact, one at the periphery, the other at the axle, of the disk. The radius *CP* is continually cutting lines of force in such a direction as to produce a current in the direction indicated by the arrow on the lead wire.

But the magnetic field given by a permanent magnet of this kind is comparatively weak, and the E.M.F. generated is therefore feeble.

Some thirty years after the invention of Faraday's disk it was shown, by Wilde, Siemens, Wheatstone, and others, how an electric generator might be made to furnish a powerful magnetic field by use of its own current. The method is as follows.

430. The Dynamo. Consider the iron core of the electromagnet shown in Fig. 347. It will always exhibit a little residual magnetism. Consequently if a single turn of wire be mounted on an axis perpendicular to the lines of force which run from the north to the south pole of this magnet, this turn of wire will, on rotation, alternately include more and fewer lines of force. Hence this loop of wire will be the seat of an E.M.F. first in one sense, then in another.

The rotating coil *A*, whether it consists of one turn or many turns, is called the *armature*.

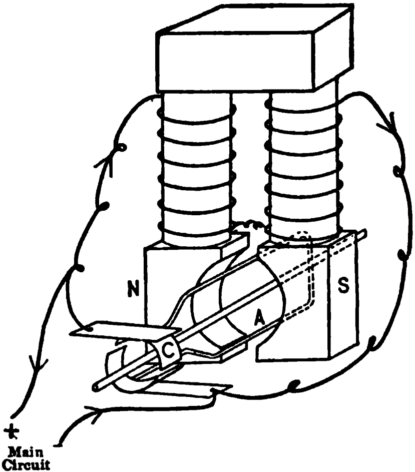

Fig. 347. — The dynamo.

In order to obtain a current from the machine which shall be constant in direction, the shaft of the armature is provided with a split ring called a commutator. If there is but one turn on the armature the commutator consists of two parts only, as shown in Fig. 347. They are insulated from each other and from the shaft. The brushes are placed in such a position that they shift from one part of the commutator to the other just at the instant when the current in the armature changes direction. In this manner the E.M.F. in the external circuit is always kept in one sense; in other words, one brush is always at a higher potential than the other. During any one rotation of an armature carrying a single turn, the E.M.F. would twice vary from zero to a maximum. In order to avoid this pulsation the armature is wound with a large number of turns distributed in all azimuths about the iron core of the armature. The effect of this is to give a steady E.M.F. between the brushes. The effect of the iron core is to make the magnetic field stronger; but the magnetic field would still be very feeble were it not for the fact that a part of the current from the brushes is shunted off and sent around the iron cores of the field magnet. It is this ability of exciting its own magnetism that gives to the machine the name *dynamo*.

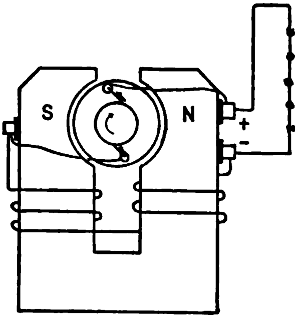

Fig. 348. — A series-wound dynamo.

This magnetizing current may be employed in any one of three different ways: (i) The *whole* of the current may be led around the field magnet and then off to the main circuit, as indicated in Fig. 348. Such machines are said to be *series wound;* (ii) Sometimes only a *part* of the current is diverted from the armature to excite the field. Such a design is shown in Fig. 347. These machines are said to be *shunt wound;* (iii) Both the shunt and series winding may be combined. These are known as *compound wound* machines.

Any machine of this type is known as a direct current generator; or more briefly as a *D.C. generator.*

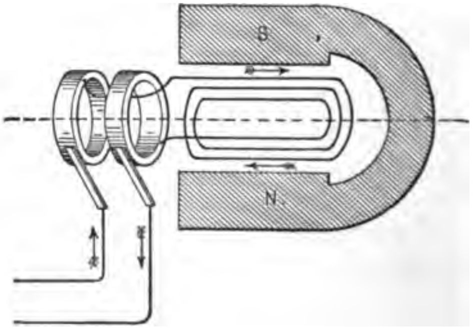

Fig. 349. — Collector rings of alternating current generator.

431. The Alternator. Sometimes the armature of a generator is not provided with a commutator; but has instead a pair of *collector rings* as shown in Fig. 349. Each brush is, in this manner, permanently connected to one end of the armature coil; and since the current in the armature is constantly changing direction so also is the current in the main circuit. Such a machine is called an *alternating current generator*, or, more briefly an *A.C. generator*. For sake of simplicity, the field in Fig. 349 is furnished by a permanent magnet. In practice, the field is furnished by an electromagnet excited by a D.C. generator. Engineers frequently find it more convenient to rotate this field magnet and allow the armature to remain stationary; but the relative motion of the two remains the same.

432. Theory of the Generator. When it comes to a quantitative consideration of the dynamo, the engineering principles upon which the machine is designed and operated are nothing less than the two fundamental equations (159 and 160) upon which are based nearly the whole of electrical science. The first of these describes Oersted's great discovery; the second, Faraday's.

Consider any single straight copper wire on the armature of a dynamo. Suppose it to lie parallel to the axis of rotation so that its motion as well as its direction is nearly everywhere perpendicular to the lines of force — magnetic flux, as it is called — which run from one pole piece to the other through the armature. Let us call this single straight wire an "inductor"; then if

L = length of the inductor in centimeters;
V = linear velocity of inductor in centimeters per second, and
H = intensity of magnetic field — lines per square centimeter — in the air gap, between pole piece and armature,

the rate at which the inductor cuts lines of force will be HLV, and the electromotive force E generated in this inductor will be

$$E = HLV \times 10^{-8} \text{ volts.} \qquad \text{Eq. 162}$$

The various inductors are laid on the armature in such a way that all their electromotive forces are added together; and it is the sum of these E.M.F.'s which makes up the voltage of any particular machine.

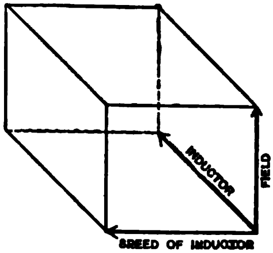

FIG. 350. — Three factors which determine E.M.F. of dynamo.

The above equation gives us the E.M.F. of the dynamo in terms of its speed and its magnetic field.

We next proceed to derive a second equation, which will give us, in terms of the current, the force required to drive the inductor through the field.

From Joule's Law and from the definition of E.M.F. we have already learned that the rate at which a current does work between any two points is the current strength i, multiplied by the E.M.F. overcome between these two points. Neglecting friction, the rate of work — the power — which must be exerted upon the inductor is therefore iE. But we also know from mechanics that the power used in moving any body is the product of the force F exerted upon the body, and the velocity V with which the body is moved.

Hence
$$FV = iE,$$

or, substituting for E its value from Eq. 162, we have

$$F = iHL \text{ dynes}$$

as the mechanical force required to drive the inductor through the field. If i be measured in amperes instead of C.G.S. units, then

$$F = HLi \times 10^{-1} \text{ dynes,} \qquad \text{Eq. 163}$$

which is the second fundamental equation of the dynamo.

The student of electric engineering will find that in practice these equations call for certain corrections and modifications depending upon whether the inductors are bound upon a ring or upon a drum; but he will also find that they, together with Ohm's Law, cover essentially the entire electrical theory of the dynamo and the motor, for the motor is merely a reversed dynamo. The magnetic behavior of iron is also a vital factor in dynamo design.

The Electric Motor

433. The electric motor is a device for transforming electrical into mechanical energy. Its construction rests upon the same principles which are fundamental to the generator. We have already seen (§ 423) that every portion of a circuit conveying a current sets itself so as to include the maximum number of lines of force; and we have just found (Eq. 163) that the force with which a straight conductor of length L, carrying a current of i amperes, tends to move at right angles across a field of strength H is $\frac{HLi}{10}$ dynes. Accordingly if a current be sent through an armature coil placed in the field of an electromagnet, one may expect rotation until this coil includes a maximum number of lines of force, in the right-handed screw direction; and if, when this has been accomplished, a commutator reverses the current through this armature coil then the rotation will be continuous. When the armature current I, and the field strength H are given it is easy to compute the torque on a single turn of the armature winding at any instant.

One interesting feature of any motor is the fact that the rotation of its own armature produces an E.M.F. in the circuit tending to stop the flow of the current driving the motor. Let us denote this *counter-E.M.F.*, as it is called, by E; the resistance of the armature by R and the E.M.F. impressed at the brushes by E_I. Then, at any instant,

$$E_I = E + RI. \qquad \text{Eq. 164}$$

In practice, R is generally very small; so that after the motor has once started and gotten up to speed, E is not much less than E_I. When the armature is not rotating there is no counter-E.M.F. Hence in order to save the armature from being "burned" out by a large rush of current at starting, it is customary to put in series a resistance which is gradually removed as the motor comes up to speed.

The general principle that an efficient dynamo will also work as an efficient motor, if supplied with current — the principle of reversibility — was first enunciated, in 1864, by Pacinotti, then Professor of Physics in the University of Pisa.

The counter-E.M.F. of the motor is a fine illustration of Lenz's law that the induced current is always in such a direction as to oppose the action which produces it. The work spent in overcoming this opposition maintains the induced current.

The Telephone

434. An exceedingly valuable illustration of electromagnetic induction is found in the telephone, which was invented by Graham Bell and Elisha Gray in 1876.

The instrument consists merely of a thin iron diaphragm, underneath which is supported a permanent magnet, carrying on the end, near the diaphragm, a helix of fine wire. See Fig. 351. The lines of force which spread out from the upper end of the magnet tend to pass along the iron disk and keep within it as much as possible. Connect the two ends of the helix to a moderately sensitive D'Arsonval galvanometer. It is observed that when one pushes the disk, with his finger, a little nearer to the end of the magnet, a current shows itself on the galvanometer; and when the diaphragm is released and springs back into its position of equilibrium, a current in the opposite direction is seen. Evidently the distribution of the lines of force which pass through the helix is, in each case, changed in such a way that the total flux through the helix is altered, thus producing an E.M.F. in the helix. When for the push of a finger one substitutes the vibrations of the air caused by the human voice he has the "Bell transmitter." When the two ends of the helix are connected to a pair of line wires having a similar telephone at the distant end, the currents induced in the transmitter will pass through the helix at the distant end and will alternately magnetize and demagnetize — to some extent — the magnet at the receiving end. The effect of these variations in the magnetism of the receiving telephone is to set into vibration the diaphragm of the receiver and reproduce the voice which is talking into the transmitter. Such was the early telephone, the transmitter being practically an alternating current dynamo, the receiver being a motor.

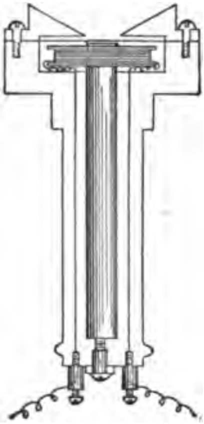
FIG. 351. — The Bell telephone.

In 1877, it was discovered by Berliner that the efficiency of the transmitter could be vastly improved by allowing the vibrations of the diaphragm to merely vary the resistance of a battery circuit. The battery thus furnishes the energy for the induced current and the voice merely controls its distribution. A form of this instrument, which is due to Blake and which has been widely used in the United States, is shown in Fig. 352. The diaphragm *D* presses a small platinum point against a carbon disk *C*; this diminishes the resistance and hence increases the current in a local battery circuit; the variations of the local current, acting through a small transformer coil, send out over the line the induced currents, which reproduce at the receiver each vibration of the transmitter disk. This transmitter is essentially the microphone which was invented by the English electrician, Hughes.

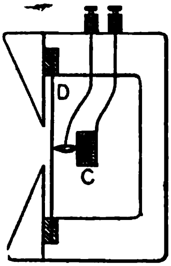

Fig. 352. — The Blake transmitter.

The essential principle which the world owes to Graham Bell and to Elisha Gray is that the successful reproduction of human speech requires a continuously varying, instead of an intermittent, current.

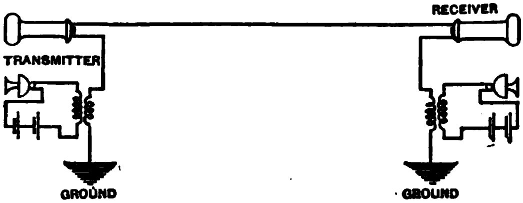

Fig. 353. — Illustrating the connections of a telephone circuit during conversation.

Long distance telephony is a recent achievement. Conversation between New York City and San Francisco dates from January, 1915. The use of such long lines in telephony has been made possible largely through the inventions of Professor M. I. Pupin of Columbia University.

The Transformer

435. Take an iron ring such as that shown in Fig. 354, wrap it with two coils of wire; pass an alternating current through one of them. It will then be observed — perhaps by means of an incandescent lamp — that a current is passing through the other coil. **A pair of electric circuits, each insulated from the other and interlinked with a common magnetic circuit, is called a static transformer.** Whenever the current in the circuit P is made to vary, an induced current appears in the circuit S. Conversely, if a current be passed through the circuit S and made to vary, a current will be induced in the circuit P. An instrument of this kind is in wide use for lighting houses by means of alternating currents. The coil P consists, in this case, of many turns of fine wire, while the coil S consists of a few turns of heavy wire. The result is that a small alternating current of high pressure passed through P induces a large current of low pressure in S.

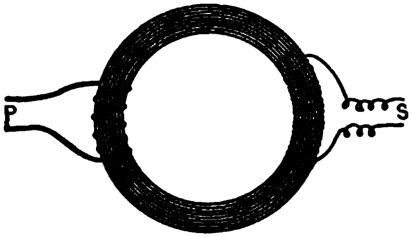

FIG. 354. — A static transformer.

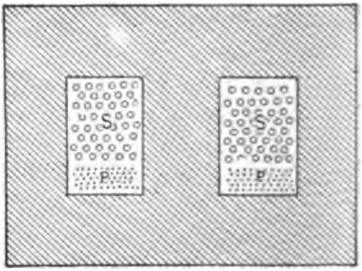

FIG. 355. — Transformer cores are made of a pile of iron plates having this shape.

Barring a slight loss of energy in heating the iron core, the power delivered to the transformer by one circuit is equal to that taken up by the other circuit. Hence if E_P and E_S denote the voltages in the windings and i_P and i_S the corresponding currents we may write approximately,

$$E_P i_P = E_S i_S, \qquad \text{Eq. 165}$$

which means that the currents in the two windings are inversely proportional to the voltages in these windings. The two windings are generally called "high-tension" and "low-tension." When the high tension side is connected to the source of power the instrument is said to be a *step-down* transformer; and when the low-tension side is connected to the source of power the device is employed as a *step-up* transformer. For lighting houses by alternating currents, it is the step-down transformer which is ordinarily used, the power being supplied at, say, 2200 volts and distributed at 110 volts. A transformer of this kind would have 20 times as many turns on the high-tension side as on the low; for since the magnetic flux is practically the same through both coils it is evident that, on open circuit, **the ratio of the voltages is the same as the ratio of the turns**, or

$$\frac{E_S}{E_P} = \frac{N_S}{N_P}, \qquad \text{Eq. 166}$$

where N_S and N_P represent the number of turns in the respective windings. The rotary converter must be left for advanced study.

The Induction Coil

436. The Primary Induction Coil. We have just learned how an alternating E.M.F. may be transformed into a higher or lower voltage. The problem of transforming a low *direct* E.M.F., such as that of a voltaic cell, into a high alternating and intermittent E.M.F. is solved by means of the induction coil. This device, in its simplest form, consists of a coil of wire wound upon a compact bundle of iron wires. When this coil is placed in series with one or more voltaic cells and with a make-and-break contact, such, for example, as that used in the ordinary doorbell circuit, we have a *primary* induction coil — often called a *spark-coil*. When the circuit is closed, the current in this electromagnet — for that is what it really is — rises rather slowly, occupying perhaps $\frac{1}{100}$ of a second, because the lines of force which are being established in the iron core are in such a sense as to create a counter-E.M.F. in the battery circuit. But when the circuit is suddenly broken, the lines of force disappear with great rapidity, and by so doing produce a high E.M.F. in the same sense as that of the battery. The curve OA in Fig. 356 represents the rise, and the curve AB the fall, of the current. At the instant of

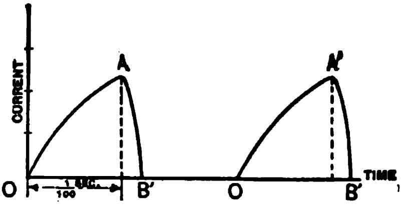

FIG. 356. — Rise and fall of current in a spark coil.

break, the wires separate and leave an enormous resistance just at the spark-gap. The result is that a large portion of this high induced E.M.F. is located in the spark-gap. A large portion of the energy of the current is therefore delivered to this gap.

Such coils are used for purposes of ignition in gas and oil engines, where of course the make-and-break contact must be placed in the end of the cylinder. Experience shows that a reliable spark requires the expenditure of energy amounting to $\frac{2}{100}$ of a joule in the spark-gap at each break.

437. The Secondary Induction Coil. The secondary induction coil differs from the primary only in having *two* separate coils wound on an iron core. One of these contains but a few turns of thick wire and is placed in series with the battery and interrupter. It is called "the primary." The other coil, which is usually very long and made of comparatively fine wire, is known as "the secondary." This secondary is so long that it is wrapped many times about the lines of force which pass through the primary. The result is that when the primary is quickly broken, an enormous number of lines of force N are suddenly removed from the secondary. And since the average E.M.F. induced in the secondary is given by

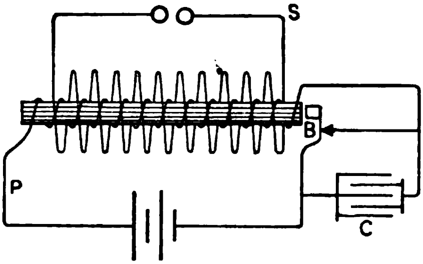

FIG. 357. — Diagram of secondary induction coil.

$$\text{E. M. F.} = -\frac{N}{t}, \qquad \text{Eq. 161}$$

it is clear that the voltage between the terminals of the secondary may become enormous, so large, indeed, as to discharge through even a five-foot gap of air or other insulator.

It will be evident from Neumann's equation (161) that it is quite as important to make t, the duration of break, small as it is to make N large.

Accordingly, for the purpose of reducing *t*, it is customary to join one terminal of a condenser to each side of the break

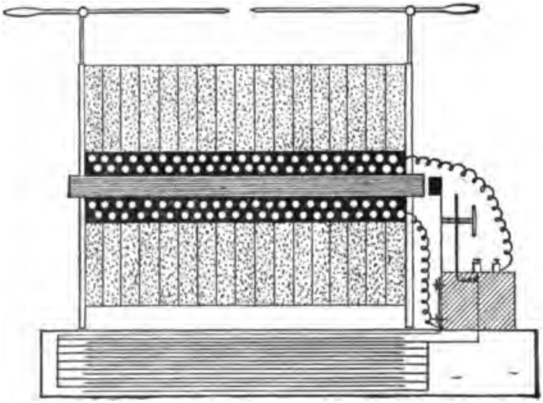

FIG. 358. — Longitudinal section of induction coil.

in the primary as indicated in Fig. 357. The effect of this condenser is to hasten the "break" and slow up the "make," by receiving the charge which would otherwise pile up at each terminal of the primary gap and thus prolong the primary current.

The simultaneous variations of the primary and secondary currents are shown in Fig. 359. Why does the sign of the secondary current change, while that of the primary is always in one direction?

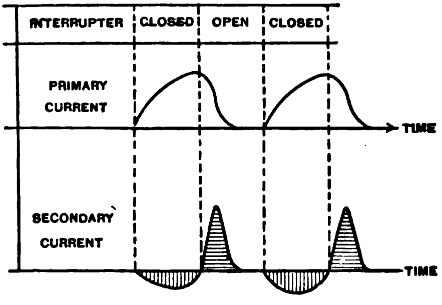

FIG. 359. — Showing the rise and fall of the primary and secondary currents in an induction coil.

Electromagnetic Waves

438. The Oscillatory Discharge. It was first observed by Savary in 1826, that, when a Leyden jar is discharged through a coil of wire so as to magnetize a sewing needle placed inside the coil, sometimes the needle is magnetized in one way, sometimes in the opposite way, even though the inner coating of the jar be always charged with electricity of the same sign and the coil connected in the same way. See Fig. 360.

This fact, known as "anomalous magnetization," was first explained by Joseph Henry, in 1842, by supposing that the charge oscillated from one coating to the other, but became a little smaller at each oscillation. Accordingly the last discharge which was strong enough to reverse the magnetization of the needle was the one which determined which end of the needle would be the north pole and which the south.

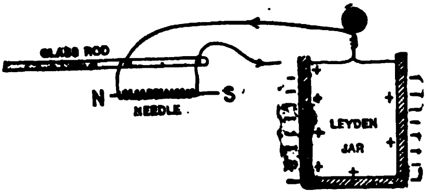

Fig. 360.—Illustrating Henry's experiment.

Later (1857) Feddersen examined the spark of a Leyden jar discharge by means of a rotating mirror, such as that which is used for viewing a manometric flame (§ 227), only turning much more rapidly, and found that in this mirror the image of the spark looked something like a string of beads, thus furnishing irresistible evidence that the spark was, under his conditions, a periodic phenomenon. The oscillatory discharge has recently been photographed many times.

The advanced student will be interested in following Lord Kelvin's beautiful prediction of this periodic phenomenon, made in 1855 before Feddersen had proved it by experiment. For this see *Ency. Brit.*, 9th Ed., art. "Electricity," p. 81. Here Kelvin proves, what has since been verified by experiment, that the period of the discharge is

$$T = 2\pi\sqrt{\frac{L}{1/C}} = 2\pi\sqrt{LC}, \qquad \text{Eq. 167}$$

where C is the capacity of the jar and L a quantity which represents the *electrical* inertia of the circuit and which is known as *self-inductance.* The *reciprocal of capacity* represents therefore, by analogy with the pendulum, spiral spring, and other vibrating systems, the *electrical elasticity* of the circuit. But this is a matter which can be cleared up only by more advanced study.

439. Electrical Resonance. If two Leyden jars which are alike in size and shape so as to have practically the same period of oscillation be provided each with a wire frame such as that shown in Fig. 361, then one of them can be set into electrical vibration by means of a discharge at the other. The wire frame serves to make the period definite and the same for each jar.

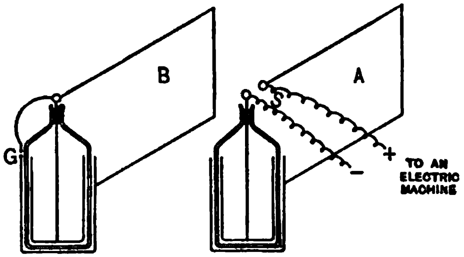

Fig. 361. — Lodge's resonance experiment.

The spark-gap *G* in the circuit *B* is very small, say ½ millimeter; while that in *A* is larger, say ½ inch. When the spark of the electric machine passes across *S*, a periodic disturbance is produced in *A* which sets up surgings of electricity in *B*, on the same principle that one pendulum will set into motion another of equal period, if there be any elastic connection between the two.

When these surgings become sufficiently large, the charge " slops over," so to speak, and a small spark is seen at *G*. The effect of the discharge in the first jar is to send through the medium which separates the jars a series of properly timed electrical impulses, first in one direction and then in the other; these impulses separate the two electricities in the circuit *B* and produce the spark.

440. Electromagnetic Waves. Whenever an electric charge surges to and fro, as it does from the inner to the outer coating through the spark-gap *S*, in Fig. 361, we have a varying electric current. The current flows first to the right, then to the left, through the gap *S*; so that at one moment the inner coating is charged positively; at the next moment, negatively. But when one changes the *charges* on a body in this manner, he inevitably changes the *electrical forces* in the surrounding medium — in this case the air. Not only so but when the *current* from one part of the conductor to the other alternates rapidly, the magnetic field in the surrounding medium also alternates rapidly. Experiment shows that these changes in the electric and magnetic fields are propagated away from the electric oscillator with the speed of light.

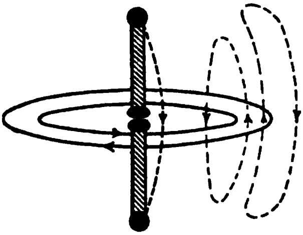

Fig. 362. — Electric and magnetic waves generated by an oscillatory circuit.

These variations of electric and magnetic force are the electromagnetic waves whose properties Clerk Maxwell predicted in 1864 from a mathematical discussion of the two laws of electromagnetism; but waves of this kind were first produced in the laboratory by H. Hertz in the autumn of 1888.

In Fig. 362, the shaded portion represents an electric oscillator which one may think of as connected with the terminals of an induction coil. The lines of magnetic force are shown in solid lines; those of electric force in broken lines. These two together constitute an electromagnetic wave. The direction of propagation at any point is perpendicular to the plane containing both electric and magnetic forces. The two waves shown in Fig. 362 are traveling to the right; the electrical forces are vertical, the magnetic, horizontal. The form of oscillator here represented is one employed by Hertz in his early work.

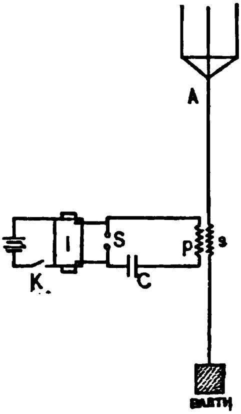

Fig. 363. — Transmitting apparatus. *A*, antenna; *C*, condenser; *S*, spark; *p*, primary; *s*, secondary of oscillatory circuit.

441. Wireless Telegraphy. The manner in which electromagnetic waves have been utilized for sending messages through space will be clear from Fig. 363. Here the oscillator is a long vertical wire called the *antenna*, in which electric oscillations are started by induction from a neighboring oscillatory circuit. The spark at *S* is produced by depressing the key at *K*. With each spark, a train of waves leaves the aerial, *A*, traveling in all directions, except vertically, with the speed of light.

442. The Receiving Apparatus. When electromagnetic waves impinge upon a conductor, such as the aerial represented in Fig. 364, they set up oscillations in that conductor which may be detected in a large number of ways. One of the simplest of these detectors is a metal point bearing on a silicon crystal. The simplicity, however, is in the construction rather than the theory. Suffice it to say that such a crystal contact permits the current (which oscillates up and down in the aerial) to pass in one direction only.

In other words, it blocks one part of the alternating current and sends it around through the telephone *T*. Each train of incoming waves thus gives an impulse to the diaphragm of the telephone; each spark at the sending end is represented by a click at the receiving end.

Wireless telephony is a fascinating chapter into which we cannot here enter; the principles are precisely those which we have just been considering; the receiving apparatus is very similar to that used in telegraphy. But in the case of wireless telephony, persistent, instead of intermittent, trains of waves are used at the sending end.

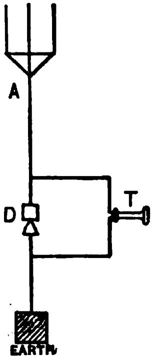

FIG. 364. — Receiving apparatus. *D*, crystal detector; *T*, telephone.

The Conduction of Electricity through Gases

443. When a current from an induction coil or other source of high E.M.F. is made to pass between two metal points a *spark* ensues. The phenomena which attend this spark are, except at its very starting, almost the same as those of the arc which we have already considered.

If, however, the two metal electrodes have been fused into the ends of a glass tube from which the air can be gradually removed by a pump, a series of remarkable phenomena presents itself. The investigation of these phenomena has led to the most far-reaching and radical discoveries made in physics within the last generation.

As a type of the discharge which occurs in a vacuum tube of this kind we may consider Fig. 365, which represents the appearance of a tube when the pressure is in the neighborhood of one millimeter of mercury. About the cathode is a non-luminous region called the *Crookes Dark Space.* In front of this a second dark region called the *Faraday Dark Space.* The positive column often fills nearly the whole tube with a diffuse pinkish light — many times stratified. As the exhaustion of the tube proceeds the dark space of Crookes expands and fills practically the entire tube, and brings with it an apple-green fluorescence. For lighting these tubes there is nothing superior to a small transformer wound so as to give a small fraction of an ampere at 5000 volts.

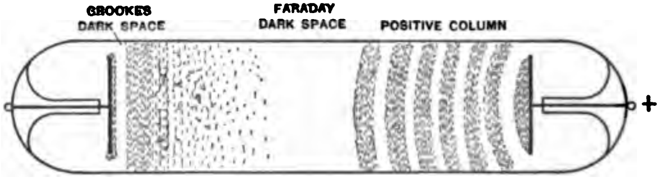

FIG. 365. — Vacuum tube discharge.

Among the dozens of important phenomena which are exhibited in a tube of this kind, the most fundamental are perhaps the cathode rays and the X-rays, which we proceed to consider.

444. Cathode Rays. Sir William Crookes, the eminent English chemist, showed that the cathode emits some sort of radiation. For on placing any body (transparent or opaque) between the cathode and the walls of the tube one finds a geometrical shadow of this obstacle on the walls of the tube. Crookes imagined this radiation to consist of small particles of matter, negatively charged. He thought that they were negatively charged because these rays are deflected by a magnet in the same direction as would be a current of positive electricity flowing through the tube *toward* the cathode. Perrin, in France, in 1895 demonstrated conclusively that the radiation from the cathode is negatively charged. This he did by sealing in a vacuum tube a small hollow conductor *F.* About this vessel is placed a hollow metal cylinder *A, B, D, E,* which is connected to earth and is used as an anode. There is a small opening in each end of this cylinder. The cathode *C* is shown at the right-hand end of the figure. Perrin found that, when by means of a magnet they deflected the rays so that they struck on the face *ED* of the anode, the electrometer was not affected; but just as soon as the rays were allowed to enter the opening in *ED* and strike the vessel *F*, the electrometer indicated a negative charge. The name of *Cathode rays* (*Kathodenstrahlen*) was given to this radiation by Goldstein of Berlin.

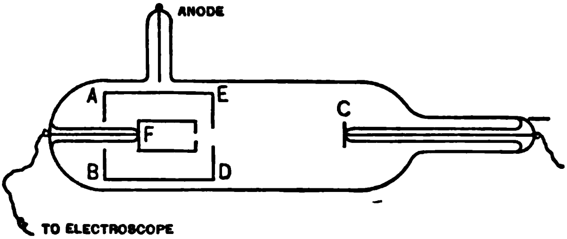

FIG. 366. — Perrin's experiment.

445. X-rays. In the autumn of 1895, in the city of Würzburg, Bavaria, Professor W. K. Röntgen made this remarkable discovery, namely, that whenever cathode rays strike matter they produce a new and different kind of radiation. The properties of this new radiation — called *X-rays* because their nature was not understood at the time — are very different from those of cathode rays. Most striking of all is their extraordinary ability to penetrate opaque bodies of very considerable thickness. Hardly less striking is their ability, on passing through an ordinary nonconducting gas, to break up this gas into ions and render the gas a conductor. X-rays are not deflected by a magnet as are cathode rays.

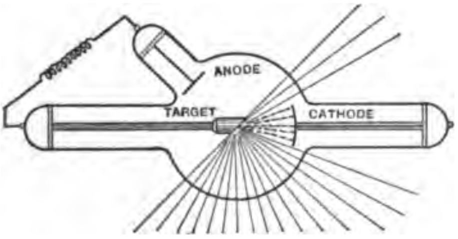

FIG. 367. — The X-ray bulb.

For the actual production of these rays in the laboratory and the hospital, tubes of many designs have been invented. The one shown in Fig. 367 is typical. The cathode is a small portion of a spherical bowl. This shape has the effect of concentrating the cathode rays on the target. This target — called the *anticathode* — when struck by cathode rays becomes the source of the X-rays. Since X-rays not only ionize the gas through which they pass, but also affect the photographic plate, it is an easy matter to record on a plate the shadow of any obstacle placed between the anticathode and the plate. Such a record is called a *radiograph*. Recent experiments on the passage of X-rays through crystals, by Laue, the Braggs, and others have proved that these rays consist in a wave motion in the ether; their wave length is about $\frac{1}{1000}$ that of ordinary light.

The Charge of the Electron

446. The reader of this and the three preceding chapters can hardly have failed to observe what an important rôle the electron plays in the explanation of all electrical and magnetic phenomena. The idea of the electron is, indeed, the most unifying conception ever introduced into electrical science. The value of this elementary charge appears to be one of the fundamental constants of nature. Experiment shows that ions, produced in the greatest variety of ways, each carry either the charge of a single electron or some simple integral multiple of this charge.

In 1909, Professor R. A. Millikan of the University of Chicago devised an exceedingly clever method for measuring the charge carried by an ion. In *Science*, 30 Sept., 1910, he describes his method as follows: —

" A cloud of fine droplets of oil, or of mercury, or of some other non-volatile substance is blown by means of an atomizer over a horizontal air condenser and a few of the droplets in this cloud are allowed to fall through a pinhole in the middle of the upper plate of this condenser into the space between the plates. The pinhole is then closed for the sake of shutting out air currents. The condenser used consists, in most of the experiments, of two heavy, circular, and accurately planed brass plates, 20 cm. in diameter, held exactly 16 mm. apart by means of three small ebonite posts. The plates are inclosed and the temperature controlled so that the air within the condenser is altogether stagnant.

The droplet, once inside the condenser, is illuminated through a small window by a beam from an arc light, so that it appears in the field of view of the observing cathetometer telescope like a bright star on a black background. This star, of course, falls under the action of gravity toward the lower plate, but before it reaches it, an electrical field of strength between 3000 volts and 8000 volts per centimeter is thrown on between the plates, and, if the droplet had received a charge of the proper sign and strength as it was blown out through the atomizer, it is pulled up by this field against gravity, toward the upper plate. Before it strikes this plate the field is thrown off, the plates short circuited, and the time required by the drop to fall under gravity the distance corresponding to the space between the cross hairs of the observing telescope is accurately determined. Then the rate at which the droplet moves up under the influence of the field is measured by timing it through the same distance when the field is on. This operation is repeated and the speeds checked an indefinite number of times, or until the droplet catches an ion from among those which exist normally in air, or which have been produced in the space between the plates by any of the usual ionizing agents like radium or X-rays. The fact that an ion has been caught, and the exact instant at which the event happened are signaled to the observer by the change in the speed of the droplet under the influence of the field. From the sign and magnitude of this change in speed, taken in connection with the constant speed under gravity, the sign and the exact value of the charge carried by the captured ion are determined. The error in a single observation need not exceed one third of one per cent."

Millikan's next step is to derive the laboratory equation.

Let m denote the mass of one of these droplets, and let e denote its electric charge. If we call V_1 the limiting speed which such a drop acquires when falling freely in the gravitational field, g, alone, then we may write

$$V_1 = kmg,$$

where k is a constant of proportionality; for experiment shows that the limiting speed of a falling drop in any given medium is proportional to the force acting upon it. When the electrical field of intensity, F, is applied the drop again acquires a limiting speed in "falling up," so to speak; let us denote this speed by V_2. Then one may write

$$-V_2 = k(mg - Fe).$$

Between these two equations one may eliminate k and solve for e, thus obtaining,

$$e = \frac{mg\,(V_2 + V_1)}{V_1F}. \qquad \text{Eq. 168}$$

All of the quantities on the right-hand side of this equation are easily measured except the mass, m, of the droplet. In order to obtain the value of m, or rather to eliminate it, one more equation is necessary. This equation is furnished by a theorem established by Stokes in 1850. This theorem expresses the limiting speed, V_1, of a sphere falling under gravity, through air in terms of its *mass* and *density*, or what is the same thing, in terms of its *radius* and *density*. By use of Stokes' equation, each quantity in the right-hand member of Eq. 168 is either directly observed or easily computed.

The charge of the electron obtained in this way and checked by a number of other methods turns out to be

$$e = 4.77 \times 10^{-10} \text{ E.S. units.}$$

Knowing e, and also the ratio e/m_0, where m_0 is the mass of the electron, it is easy to compute m_0. This turns out to be

$$m_0 = 9.01 \times 10^{-28} \text{ gram,}$$

which is approximately $\frac{1}{2000}$ of the mass of a hydrogen atom. The value of e/m_0 is easily obtained from the deflection of cathode rays. Other considerations lead us to think that the radius of an electron is 1.87×10^{-13} cm., while the radius of a hydrogen atom is 1.21×10^{-8} cm. Some one has suggested that the size of the electron is therefore to the size of the hydrogen atom as the size of a fly is to the size of the dome on St. Paul's in London.

It should be clearly understood concerning the electron that, apart from its charge of electricity, it has no existence and no mass. It consists of the smallest known charge of electricity — quite dissociated from ordinary matter.

Summary

447. No student should leave the subject of electricity without a clear grasp of the three laws of nature and the two definitions which are expressed symbolically in the following five equations: —

$$W = 4\pi i. \qquad \text{Eq. 159}$$

$$E = -\frac{N}{t}. \qquad \text{Eq. 161}$$

W

$$i = \frac{E}{R}. \qquad \text{Eq. 146}$$

$$Q = it. \qquad \text{Eq. 150}$$

$$C = \frac{Q}{E}. \qquad \text{Eq. 138}$$

In these equations appear five, and only five, unknown quantities, namely, the electromotive force E, the current i, the resistance R, the quantity Q, and the capacity C. These unknown quantities are expressed in terms of the three known quantities, namely, the time t, the number of magnetic lines N, and the work W. By elimination, then, one is enabled to express any of the five electrical quantities in terms of known or directly measurable quantities. This list does not include all the fundamental equations of electrical science; but it does represent the minimum upon which one can construct a system of electrical measurement.

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"**Provando e riprovando.**"
Motto of Florentine Academy.

CHAPTER XII

LIGHT

448. A very large proportion of all that we know concerning the external world is obtained from an examination and comparison of our sensations of sight.

As under the head of Sound we considered those phenomena which produce the sensation of hearing, so now, under the head of Light, we proceed to consider those phenomena which are recognized by our visual sense. And just as in the case of sound we did not consider either the physiological or psychological side of the question, so here **the discussion is limited to an external physical process which the human eye interprets as light.** There may be in a room objects to be seen, also an eye to see them; but until radiant energy from some source is supplied there is no seeing.

449. Following the method used in previous chapters of this volume, we shall begin and end with a study of some of the fundamental phenomena of light, accompanying the discussion with certain definitions and applications, hoping thus to learn something of modern views concerning the "nature of light."

Of these fundamental phenomena the following seven are selected as being perhaps the more important for our purpose: —

I. Rectilinear Propagation.
II. Finite Speed.
III. Evidence that light is a wave motion.
IV. Polarization.
V. Reflection.
VI. Refraction and Dispersion.
VII. Interference and Diffraction.

Neither in thought nor in practice are these phenomena separated by water-tight bulkheads. On the contrary, they are, indeed, intimately connected, from which it follows that any grouping of facts such as that indicated above is purely a matter of convenience. For the sake of orienting ourselves and starting together, we shall now take a hasty and rather qualitative glance at the first four of these fundamental facts, and shall then proceed to take up the other three in a more quantitative way.

Some Preliminary Definitions

450. When we are looking at the various objects in a room, it is not difficult, in general, to select those which we might still be able to see in case no light were admitted to the room from the outside. If the room were completely shut off from any outside light, the chairs and tables in it would become quite invisible, while a burning match, a red-hot coal in the fireplace, a lighted gas jet, or an incandescent lamp would each illuminate the room, and at the same time be seen by its own light. **Bodies of this latter class which shine by their own light are said to be "self-luminous"; while bodies of the former class, being seen only by reflected or diffused light, are described as "non-luminous."** Luminous bodies are generally, but not always, hot. A phosphorescent clock face is easily seen at night, but can hardly be called a hot body. The student who recalls his daily duties will find that most of his work is performed by the aid of borrowed, or diffused, light. For instance, you are now reading this page by light which is reflected from the white paper to the eye.

Some substances, such as water, air, thin layers of gelatine, many kinds of glass, and some crystals, permit us to see with distinctness other objects through them. But there are other substances, such as iron, wood, ink, through which bodies cannot ordinarily be seen. **Bodies through which one can see other objects distinctly are said to be "transparent"; bodies through which vision is impossible are said to be "opaque."** But every one will recall certain other substances, such as milk in thin layers, thin tissue paper, lightly ground glass, even ink in thin layers, thin shavings of wood, through each of which objects are faintly or indistinctly seen. These bodies form a class intermediate between transparent and opaque bodies. **Substances which transmit some light, but not enough for distinct vision, are said to be "translucent."**

If bodies are taken in layers of proper thickness, it may be easily shown that there is no sharp line between the most opaque and the most transparent substances. For metals, such as gold and silver, which are very dense and ordinarily very opaque, become translucent, and even transparent, when taken in sufficiently thin layers, as may be easily shown by depositing a thin film of silver on glass; while, on the contrary, what we call "perfectly clear" water, when taken in sufficiently thick layers, permits almost no light to pass through it. Thus geographers who have studied the bottom of the sea have found that those parts of the ocean which lie at a depth of several kilometers are in almost complete darkness. No substance is either perfectly transparent or perfectly opaque.

Experiment shows that, *in an inclosure,* where all bodies are at the same temperature they are invisible, regardless of whether any particular body is opaque or transparent. The sum of the radiated, reflected, and transmitted light is the same for each object. No outlines will therefore show.

I. LIGHT MOVES IN STRAIGHT LINES

451. An object behind an opaque screen is invisible to a person in front of the screen. As every one knows from experience, the reason of this is that the light proceeding from any part of, say, a man behind the tree cannot pass through the opaque tree, cannot take a curved path around the tree, and, therefore, cannot reach the eye of the observer. Evidence of this kind, such as we have all been collecting since early childhood, shows us that if light does not travel in straight lines, it travels in lines which are very nearly straight.

One of the simplest illustrations of the rectilinear propagation of light is obtained by placing a naked arc light in front of a white screen. Intercept the light which falls on the screen by means of a sheet of lead foil, placed rather close in front of the arc. If now this sheet of foil be punctured with a pin, an inverted image of the arc will appear upon the screen. Each new puncture will create a new image. The image thus produced is, as regards both size and position, what we should expect if it had been traced by a straight line, always passing through the pinhole, one end tracing out the arc while the other end traces out the image on the screen.

452. But, as will be seen from the following experiment, the statement that light travels in straight lines is not quite true without modification.

In Fig. 368, let *S* represent a tin lamp chimney with a small hole in one side. If a flame, Nernst glower, or incandescent filament be placed inside this chimney, the hole in the side will act approximately as a point source. By use of a lens, an image of this hole may be focused upon a point *P* on a white screen.

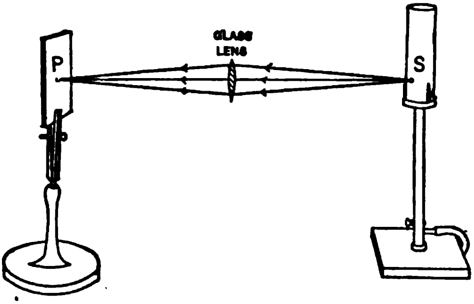

FIG. 368. — If a medium is not homogeneous, light does not travel through it in straight lines.

To do this we must place the spectacle lens so that the straight line *SP* passes approximately through the center of the lens. Let us now interpose a coin somewhat smaller than the lens immediately in front of the lens so that the straight line passes through the center of the coin. We observe that this coin does not prevent all the light from the luminous point reaching the point *P*. The image still remains at the point *P*, the only difference being that it is now not quite so bright as before. The path of the light from the source to the point *P* is no longer a straight line. All the light which forms the image now passes around the edge of the coin, as shown in the adjoining figure. But the reason of this is very evident, for we have now interposed a piece of glass in the path of the light, and, indeed, it has been found that, **in general, when light has to pass through materials of different kinds, it does not travel in the same straight line through them all.**

If now we remove the spectacle lens and allow the penny to remain, in Fig. 369, we shall find that the coin prevents any light from the source reaching the hole in the cardboard. The entire path of the light is here through air alone.

Accordingly we may say that when the medium is not homogeneous light does not travel through it in straight lines.

453. But there are certain circumstances in which light does not move in straight lines, even when the medium is perfectly homogeneous. If a luminous object be viewed through a very narrow aperture, it will in general be found that the image is distorted. This experiment is most easily realized perhaps by viewing a bright linear source of light through a single cut made with a penknife in a visiting card. As a linear source one can use either a single slit in the tin chimney of an Argand burner or an ordinary incandescent lamp filament, or better still a vertical Nernst glower. The slit in the opaque screen should be held very close in front of the eye. The linear source **appears now very much diminished in brightness, but very much wider than to the naked eye.** This phenomenon is still better seen if we substitute for the visiting card a single straight cut made on the opaque film of an ordinary photographic plate. A small piece of a lantern-slide or silvered mirror is best; for it is difficult to cut a slit in the film of a "fast" plate without making the edges ragged. The effect of a small aperture on a bundle of rays passing through it is then to spread the rays out, so that the slit *S* (Fig. 370) appears to have a width *CD*. For the source *S* would not appear to have the width *CD* unless the rays were spread out. The ray *PH*, prolonged backward, gives us one edge of the apparent slit; the ray *QK*, prolonged backward, gives us the other. For it is a general fact that we appear to see any object, not in

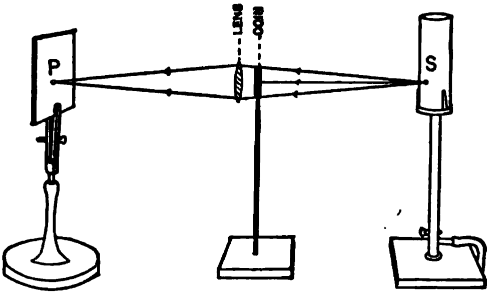

FIG. 369.

the direction in which the rays leave the object, but in the direction in which the rays enter the eye. The light from the linear source does not, therefore, pass in straight lines through the small opening *HK*.

From the preceding experiments, it is evident that **light travels in straight lines**: —

(i) **When the medium is homogeneous**; and

(ii) **When the rays are not compelled to pass through any very small openings.**

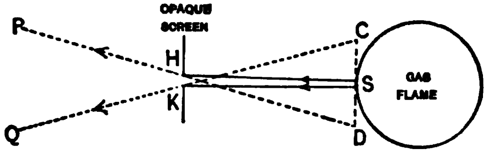

FIG. 370. — A luminous line appears widened when seen through a small aperture.

The most accurate evidence for this general conclusion is, however, derived from the computation of lenses, mirrors, and other optical instruments. Predictions which rest upon the assumption that light travels in straight lines through homogeneous media and large openings have always been verified by the experimental result.

Photometry

454. Photometry is the science and art of measuring the relative amounts of light given out by two different sources. The purpose is to compare the illuminating powers of two or more sources. This purpose is accomplished in a great variety of ways all of which are based upon the rectilinear propagation of light.

Imagine any point-source of light, or any source small enough to be considered as a point-source, and around this draw a spherical surface of one meter radius. A perfectly definite amount of light will fall on this spherical surface. This is called the *total luminous flux* of the source and may be indicated by F. The amount of luminous flux falling upon unit area of any body at any point is defined as the *illumination* at that point, and may be indicated by E. In the case of a sphere of radius r, with the source at the center of the sphere, we have therefore

$$E = \frac{F}{4\pi r^2}. \qquad \text{Eq. 169}$$

It is customary to write I for the quantity $F/4\pi$, so that one obtains

$$E = \frac{I}{r^2} \qquad \text{Eq. 170}$$

and

$$F = 4\pi I. \qquad \text{Eq. 171}$$

The quantity I, defined by Eq. 170, is known as the *intensity of the source.* Since any sphere subtends a solid angle of 4π at its center, it follows from Eq. 171 that the intensity of any source which radiates uniformly in all directions is measured by the amount of light (*i.e.* luminous flux) which passes out through unit solid angle.

It is clear from Eq. 170, that the illumination produced at any point by any given source varies inversely as the square of the distance of that point from the source. This is known as the *inverse square distance law.* It is, of course, a mere consequence of the rectilinear propagation of light, and holds for all kinds of radiant energy whether luminous or not.

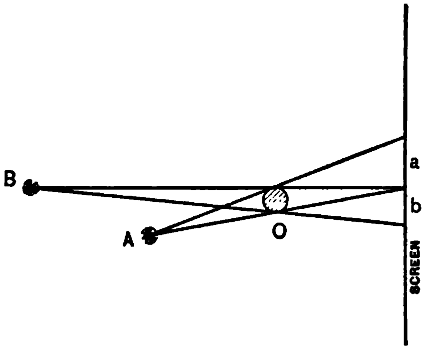

Fig. 371. — The Rumford Photometer.

It must not be forgotten that only a small portion of the radiant energy from any artificial source is perceived by us as light. Thus only ½ of 1 per cent of the energy radiated by an incandescent carbon lamp reaches us as light; most of its radiation is invisible.

455. Rumford's Photometer. One of the earliest, easiest, and most practical methods of comparing the intensity of one source, A, with another, B, is that indicated in Fig. 371. The shadow of an obstacle, O, say a vertical rod, is thrown on a screen simultaneously by both sources. The positions of the sources are adjusted so that these two shadows, a and b, lie in immediate juxtaposition. Observe that the shadow a is illuminated by the source at B alone, while the shadow b receives light from the source

A only. Now it happens that the human eye is very keen in judging when two illuminations are equal, provided each illumination has the same color. Accordingly the distances of the two sources are adjusted until each gives the same illumination on the screen.

Let I_A and I_B denote the intensities of the two sources, and r_A and r_B their distances from the screen; then by Eq. 170, we have in the case of equal illuminations

$$\frac{I_A}{I_B} = \frac{r^2_A}{r^2_B}.$$

Since both quantities on the right are easily measured, we have a means of comparing the intensity of one source with another which is taken as unity.

Many other and more accurate methods than this of Rumford's have been devised by Bunsen, Swan, Lummer and Brodhun, Joly, Weber, and others; but they all employ the common principle of equating illuminations. Their consideration, together with Lambert's Cosine Law, must be left for the advanced student.

456. Photometric Units. It has not yet been found practical to measure luminous intensity in terms of C.G.S. units. Accordingly France, England, and America have by international agreement (1909) adopted an arbitrary standard of intensity which is known as the *international candle*. The international candle is perhaps most easily realized in the laboratory by use of a Hefner lamp which burns pure amyl acetate, has a wick of definite dimensions and a flame of definite height. This standard Hefner lamp gives 0.90 international candles. Most convenient in the laboratory is an incandescent lamp whose candle-power has been previously determined by comparison with a standard flame. Such lamps must be used, of course, with the prescribed voltage between their terminals.

The *unit of luminous flux* is that which passes out from one international candle in a cone of unit solid angle. This quantity of light is called a *lumen*.

The metric *unit of illumination* is that produced by one international candle at a distance of one meter. It is sometimes called a *meter-candle*, sometimes a *lux*.

The student should distinguish very carefully between the candle-power of a lamp and the electrical power required to operate it; the former measures the ability of the lamp to stimulate the retina and is measured in terms of an arbitrary unit, the international candle, while the latter is ordinary mechanical power and is measured in joules per second, *i.e.* in watts. They are totally different kinds of quantities.

II. LIGHT TRAVELS WITH FINITE SPEED

457. Passing now to the second fundamental fact of optics, it is not surprising that the ancients should have considered the velocity of light as infinite. For every part of a landscape illuminated by a flash of light is seen apparently at the same instant as the flash. It is evident that if light did not travel at an enormous speed, this would not be the case. If, for instance, light traveled with the same speed as sound, it would be a full second after the flash of lightning before we could see points as far away as 1200 feet; and four seconds later we could see points a mile away. It was such evidence as this, perhaps, that led the ancients to think that light traveled with an infinite speed.

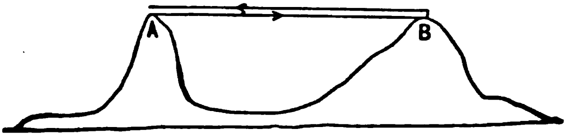

FIG. 372. — Galileo's plan for measuring the speed of light.

458. But Galileo suspected that the velocity of light was not infinite, and proposed to *measure* it as follows: Two observers, each provided with dark lanterns, were to be stationed on two distant hilltops at night. When the first observer, at A (Fig. 372), sent a flash of light from his lantern, the second observer, at B, was to make a similar signal as soon as the first flash was perceived. The interval between sending the first signal and receiving the answer might be supposed to be the time taken by the light in making one round trip between the hills. This experiment was tried by the Florentine Academy, in 1663, between Florence and Pistoia. But the measures were not satisfactory to the Academy and were therefore never published in their proceedings. We now know, however, that the time lost by the second observer in perceiving the first flash and deciding to send a return signal would occupy more time than is required for light to make a trip around the world.

459. But the following most ingenious method was discovered and tried by the Danish astronomer, Ole Roemer, during the years 1675–1676 at the Paris Observatory.

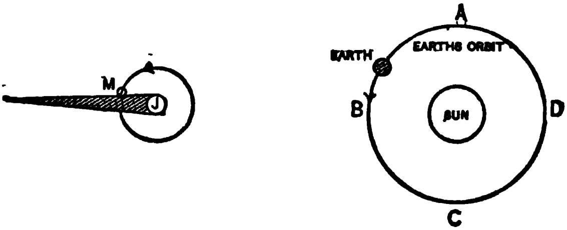

FIG. 373. — Roemer's method for measuring the speed of light.

Jupiter has nine moons. One of these, *Io*, astronomers call the "first satellite." Every time this first satellite, which we shall denote by *M*, revolves about the planet which we shall call *J*, it is eclipsed to an observer on the earth, *i.e.* it passes into the shadow of Jupiter, as indicated in Fig. 373. Roemer measured the period of this moon very carefully, *i.e.* determined very exactly the *average* interval of time between two of its successive eclipses. Knowing this, he could of course predict the time of future eclipses. This he did; but on comparing the predicted times with the observed times, he found that whenever the earth was in that half of its orbit next to Jupiter *ABC* (Fig. 373), these eclipses occurred each a little earlier than the predicted time. But when the earth was in that half of its orbit away from Jupiter, *i.e.* in the half marked *ADC*, the eclipses were each a little tardy in their appearance. And in particular when the earth was at the point *B*, nearest to Jupiter, the eclipse occurred about eight minutes earlier than the predicted time; while at *D*, a point most remote from Jupiter, the eclipse took place about eight minutes late. From this Roemer concluded that eight minutes is the time required for light to travel from the point *B*, or the point *D*, to the center of the earth's orbit, *i.e.* from the sun to the earth. This distance is 154 million kilometers. Hence, for the speed of light *S* he obtained approximately —

$$S = \frac{154 \times 10^9 \text{ meters}}{498 \text{ seconds}} = 309 \text{ million meters per second.}$$

This, of course, is the rate at which light travels in a vacuum; for the region between the earth and sun is probably a very perfect vacuum. But experience shows that the speed in air is only about one part in four thousand less than that in a vacuum. Such a speed as this, which would carry a point seven times around the earth's equator in one second, is practically beyond the grasp of the imagination.

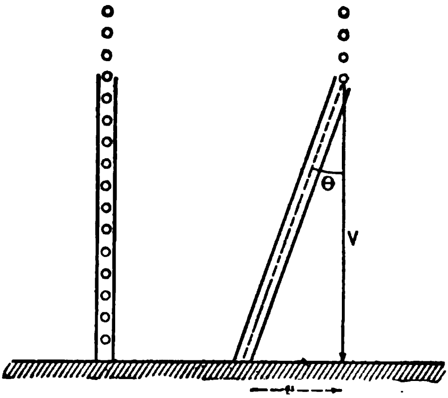

FIG. 374. — Aberration of a raindrop.

460. Roemer's measurements remained unconfirmed for half a century, until, in 1729, Bradley, then Professor of Astronomy at Oxford, discovered that each of the " fixed " stars during the course of a year appears to describe a small ellipse in the sky. This he explained by assuming that the apparent position of a star depends not only upon its actual direction from the earth, but also upon the velocity of light and upon the velocity of the observer. The matter will be clear to one who considers how he would have to hold a straight tube in order that raindrops might fall axially through it without touching the sides.

Let us assume that the rain is falling vertically; then if the observer is at rest, the tube must be held vertical. But if the observer is traveling with a speed v in any direction, the bottom of the tube must be held back toward the observer by an amount just equal to the distance which the observer would travel while the raindrop is passing through the tube.

If V be the speed of the raindrop and v that of the observer, it will be clear from Fig. 374 that the angle θ at which the tube must be inclined is given by

$$\tan \theta = \frac{v}{V}. \qquad \text{Eq. 172}$$

The velocity of the earth in its orbit about the sun is approximately nineteen miles a second. This stupendous speed, which would carry one across the American continent in less than three minutes, is the speed of every one who observes a star from the earth. Accordingly, if we imagine the path of the raindrop to be replaced by a ray of light from a fixed star, and the tube to be replaced by a telescope, provided with cross hairs in the eyepiece, we shall have all the conditions for observing the aberration of light. Bradley found that in looking at a star whose rays fall perpendicularly upon the plane of the earth's orbit, as indicated in Fig. 374, he had to incline the telescope at an angle of 20″ with the mean direction of the star.

Knowing θ and v, it became a simple matter to substitute these values in Eq. 172, and thus determine the value of V, that is, the speed with which light travels down the tube of the telescope. The numbers which he obtained were 308,300,000 meters per second. The velocity of light thus measured by Bradley coincided so nearly with the value obtained by Roemer as to leave little doubt that each of these men had correctly explained the phenomena which they observed.

461. More than a century elapsed before any one succeeded in measuring the speed of light along a path confined to the surface of the earth. This was first done by the French physicist Fizeau, in 1848. He employed essentially the method suggested by Galileo, only, instead of placing a *second* observer on a distant hill, he placed there a mirror so arranged as to send back to the *first* observer any flashes of light which he directed upon it.

These flashes of light were obtained by rapidly rotating a toothed wheel at a known rate, and in such a way that each tooth in succession intercepted a beam of light directed steadily towards the mirror. Since the wheel was rotated at a measured rate, the time required for any one tooth to pass from its present position to the position occupied by its predecessor was a known quantity. Call this T. When the wheel was rotated at a certain critical speed, it was observed that while the beam was traveling from the source to the mirror and back again the open space between two teeth (through which the beam had passed out to the mirror) had been replaced by the succeeding open space, so that an observer looking along the path of the beam would see the reflected beam. At other critical speeds the reflected beam was eclipsed by an opaque tooth, so that the observer looking in the direction of the reflected beam saw nothing.

If we call the distance from the wheel to the mirror L, and denote by T the interval between two successive flashes, so timed that the light which passes out *just in front of* any particular tooth is reflected back *just after* that tooth has passed, then we may write, for the speed of light: —

$$v = \frac{2L}{T}.$$

In this manner Fizeau obtained a value of 315,000,000 meters per second. But his method has been greatly improved, especially by Cornu and by Michelson. These improvements, together with the advantages and disadvantages of various methods, are clearly set forth in Edser's *Light for Students*.

The best of these modern measurements make it practically certain that the velocity of light does not differ from the round number, 300 million meters per second, by as much as one part in three thousand.

III. LIGHT IS A WAVE MOTION

462. We have already seen (§ 208) that energy is, in general, transmitted in one of two ways, viz. either by currents or by waves. From the time of Aristotle until the time of Huygens, and even later, men believed that light consisted in the motion of small particles — corpuscles — emitted either from the eye or from the luminous body, with great speed. In other words, they believed light to be propagated by currents rather than waves.

This hypothesis is, indeed, recommended by the ease with which it explains the rectilinear propagation of light through a vacuum; but it involves other difficulties which, as we shall presently see, are insuperable. Newton made a masterly attempt to avoid these difficulties; but the hypotheses which he was compelled to introduce led to consequences which contradicted the experimental facts.

463. The first decisive evidence which led men to consider light a wave motion was given by Dr. Thomas Young (1773–1829), a London physician. He argued that **if light is a wave motion, we ought to be able to add two trains of light waves together and produce darkness,** just as (§ 202) we add together two trains of waves in strings and thereby produce rest. If, however, light consists of currents of matter emitted by the luminous body, as Newton had suggested, one cannot hope to obtain darkness by adding two of these currents together. On the contrary, the two currents would produce *more* disturbance than either one alone, *i.e.* more light than either one alone.

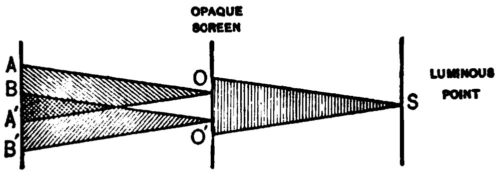

FIG. 375. — Showing how diffraction allows two rays to interfere.

The manner in which Dr. Young added two rays of light is very simple and clear to any one who has examined the manner in which a beam of light is flared out — diffracted — on passing through a narrow aperture. This phenomenon has already been described in § 453. We there found that a beam of light on passing through a small slit was spread out into a wedge of light. **This spreading apart which a bundle of rays experiences on passing through a narrow aperture is called diffraction.**

Imagine now *two* slits, instead of a single slit, to be made side by side in the opaque screen, each will furnish a cone of light, as represented in Fig. 375. These two cones, AOA' and $BO'B'$, will overlap in the region between B and A'. An eye placed anywhere in this region will receive upon its retina two different series of disturbances, one from O, and another from O'.

Under these circumstances one sees in the region between B and A' an image of the opening at S; and the image is filled with an alternation of bright and dark bands. These bands are symmetrically placed on each side of a bright central band.

This is precisely what is to be expected (Dr. Young argued), if light consists in a wave motion. If from any point in the region BA' **the respective distances of the two openings O and O' differ by one half a wave length, these two trains of waves annul one another and produce darkness**; but at any other point where the distances differ by one whole wave length, or any whole number of wave lengths, there the brightness will be greater than that due to either train alone. And this is exactly what he found to be the fact. **Light from a bright point shining through two small openings in a card behaves like water waves from the sea rushing through two small openings in a breakwater.** Two trains of waves which combine with each other to produce alternately bright and dark bands in this manner are said to **interfere**; and the phenomenon which is called interference of light appears to be strictly analogous to the interference of waves in strings and of waves in air which we have already observed and studied. These and other similar experiments seem to furnish irrefutable evidence for thinking light to be a wave motion.

IV. LIGHT WAVES ARE TRANSVERSE

464. We have already learned (§ 200) that there are in general two fundamental types of wave motion, namely, *transverse* and *longitudinal*. If we accept the evidence which has just been adduced for thinking light a wave motion, the next pertinent inquiry is, are these light waves transverse or longitudinal? Are they similar to the waves which run along a stretched clothesline when struck with a stick, or do they resemble sound waves?

To answer this question it is necessary to first establish a criterion which will distinguish between these two wave types. Such a test is found in the symmetry which accompanies longitudinal waves and the asymmetry which is characteristic of transverse waves. When a spherical raindrop is viewed *along the direction of its path,* its appearance is precisely the same however the eye or the raindrop may be turned about its path as an axis; so also, in hearing a sound the same sensation is produced however the ear or the sounding body be rotated about the direction of the sound as an axis. In general, a longitudinal wave is symmetrical about its direction of propagation.

With transverse waves such as those in a stretched string, the matter is quite otherwise. Let two boys stand holding a rope stretched between them, each boy holding one end. If one boy shakes his end of the rope transversely in a single direction it is an easy matter for the other boy to tell what this direction is merely from the wave pulses which strike his hand. In other words, the waves in the string are not symmetrical with respect to the direction of their propagation, *i.e.* with respect to the direction of the string. If a third boy were to stand between the other two and hold a slotted board so as to allow the string to pass through the slot, the vibrations may or may not be stopped. If the direction of the slot is parallel to the motion of the particles of the rope, the vibrations will pass through the slot undisturbed; but if the slot is held perpendicular to the motion of the particles of the rope, the vibrations will at once be stopped in that part of the rope behind the slot; no waves can pass through the slot.

465. In 1669 it was discovered by Bartholinus, a Danish philosopher, that a single ray of light on passing through a crystal of calcium carbonate (calcite) is split up into two separate rays traveling in different directions. So that if one lays a piece of transparent calcite over an ink spot on a piece of white paper, he will in general see two images of the spot. This phenomenon, which is called double refraction, now concerns us only because a few years later Huygens (1629–1695) discovered that each of these two rays refracted by the calcite exhibits precisely the phenomenon which we have seen in the stretched string. In other words he found that, on looking through one piece of calcite at a beam of light which had just passed through another piece of calcite, the intensity of the light which reaches the eye depends very much upon the angular position of the crystal about the ray of light as an axis. The ray of light is not therefore symmetrical about its direction of propagation.

466. An experiment due to the French physicist Biot illustrates this fact in an even simpler way. Biot discovered that tourmaline, unlike most other crystals, gives only a single refracted ray, the second ray being absorbed by the substance of the crystal. When light is passed through one tourmaline and then examined through a second tourmaline, it is found that the light may be completely extinguished by rotating either tourmaline, with reference to the other, about the ray as an axis (see Fig. 376). Here again light waves exhibit asymmetry with reference to their direction of propagation.

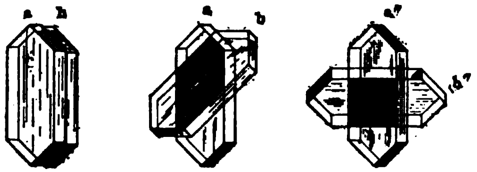

FIG. 376. — Biot's experiment with the crossed tourmalines.

467. Another illustration of this asymmetry was discovered in 1808 by Malus, a French army officer, who found that light which has been reflected from a surface of water or from glass or from any polished surface, when viewed through a tourmaline, has its intensity changed in a marked manner as the tourmaline is rotated in front of the eye. He showed also that for each substance there is a certain angle at which, if the light be reflected, it may be completely extinguished by the tourmaline. In other words he discovered that *reflected* light presents this same lack of symmetry which had already been observed by Huygens and Biot in the case of transmitted light.

Malus in 1811 introduced the word "polarized" to describe light which exhibits different properties in different directions at right angles to the line of propagation. This peculiar asymmetry he called polarization. Common light, such for instance as that by which you are reading this page, does not possess this peculiarity. The explanation of this must be taken up later.

As might have been predicted from Malus's discovery, polarization can be detected, as well as produced, by reflection. This is best shown perhaps by a pair of glass mirrors, such as those drawn in Fig. 377. The incident ray is so directed as to make an angle of 57° with the normal to the mirror. In the *left-hand* figure the mirrors are parallel, the vertical beam is polarized by reflection at the lower mirror; the upper mirror reflects it along the direction *BC*. In the right-hand figure the mirrors are crossed, *i.e.* at right angles to each other; the vertical ray is polarized precisely as before, but the upper mirror in this position is unable to reflect any ray.

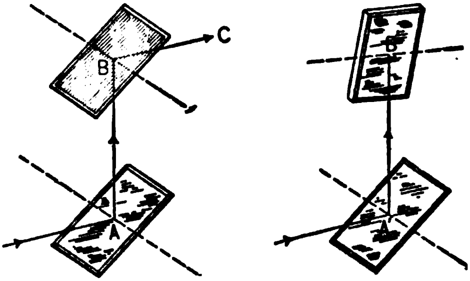

FIG. 377. — Polarisation produced and detected by reflection.

468. From evidence such as the foregoing a brilliant young French engineer, Fresnel, was led to the conclusion that light waves are transverse. For more than a century following the publication of Newton's *Optics*, in 1704, his corpuscular theory had held undisputed sway. But between the years 1812 and 1826 Fresnel produced and coördinated such a mass of cogent evidence for thinking that light consists of transverse waves that there has since been almost as little doubt concerning this principle as concerning any of the principles of ordinary mechanics.

Summarizing, then, we may consider it a fact, well established by such evidence as the four fundamental phenomena just studied, that light is a transverse wave motion propagated through what we sometimes call a "vacuum" — sometimes the "free ether" — with a velocity closely approximating 300 million meters per second.

Upon this assumption, if you prefer, and with this general view of the entire subject, we proceed to examine some of the other fundamental facts of optics. As will be seen later, many of the following facts might have been deduced as mere inferences from the four which precede.

V. The Reflection of Light

469. We have just been considering the behavior of light when it has a free path in which to travel. Suppose, however, a beam of light strikes upon some body lying in its path.

If the body has a very smooth surface, it will generally happen that a large part of the light which falls upon it from any direction is sent off in some one other direction. This is called **reflection.** If the body is a rough one, such as ground glass or a piece of newspaper, the light which falls upon it is scattered in all directions. Such light is said to be **diffused**; and the process is called **diffuse reflection.**

But some of the light which is incident upon a body will penetrate the body; this we know because objects can be seen through all substances if only the body is made thin enough. The rays which pass through a body are said to be **transmitted.** This process of transmission is, however, always accompanied by some loss of light. Light which is lost in this way is said to be **absorbed**; and the process is called **absorption.** When the body is so thick that all the rays are absorbed the body is said to be opaque.

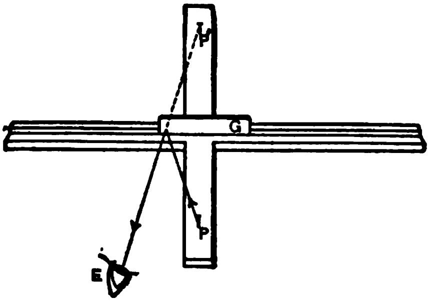

Fig. 378. — A convenient device for studying the laws of reflection.

Reflection

470. The simplest and most elegant manner of studying the reflection of light is perhaps the following. A cross is made of two pieces of pine wood mortised together, as indicated in Fig. 378. One piece is slotted so as to receive a strip of plate glass *G*.

If a pin or an incandescent lamp be placed at *P*, this strip of glass will act as a mirror. To an eye placed somewhere in front of the glass, say at *E*, the image of the pin or lamp will appear at *P'*. We can now easily place a *second* pin or lamp at the point *P'*, so that it coincides with the *image of the first* pin. The second pin at P' should be viewed over the top of the glass strip. The glass strip may well be ground or greased on the rear surface so that the reflection from the first surface only is seen.

The first pin now locates the position of an object; while the second pin locates the position of its image, produced by a plane mirror. Concerning this image it is important to note the four following facts: —

1. On observing this image from various points of view, it is seen that the image does not change position as the eye changes position. The image has a fixed position in space quite independent of the observer.

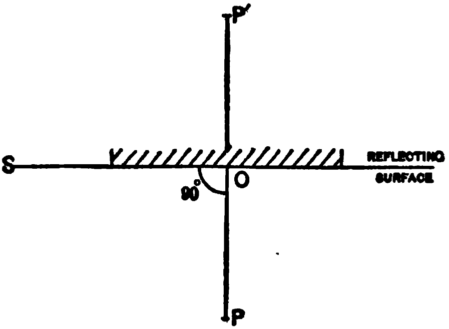

Fig. 379. — $P'O = PO$.

2. If, however, either the object be moved or if the reflecting surface be moved out of its own plane, the image also moves.

3. If now we draw a straight line from P to P', we find by use of a steel square that the reflecting surface is at right angles to this line.

4. Not only so, but on measuring the perpendicular distance $P'O$ from the image to the reflecting surface, and also the perpendicular distance PO from the object to the reflecting surface, it is found that these distances are equal.

These facts, which are familiar to nearly every one, may be summarized by saying that *the image of a point in a plane mirror lies on the perpendicular let fall from the point to the mirror, and lies as far behind the mirror as the point lies in front of the mirror.*

THE LAW OF PLANE REFLECTION

471. The general fact may be more simply described as follows: —

The point P is sending out rays of light in all directions. Let O' (Fig. 380) be a point at which one of these rays strikes the mirror. Then the two right-angled triangles $P'O'O$ and $PO'O$

are equal for they have one common side OO', and the sides OP' and OP are equal. Hence the angles $O'PO$ and $O'P'O$ are equal. Draw the line NO' perpendicular to mirror at O'. Then the angles $NO'E$ and $NO'P$ are equal, since they are respectively equal to $O'P'O$ and $O'PO$.

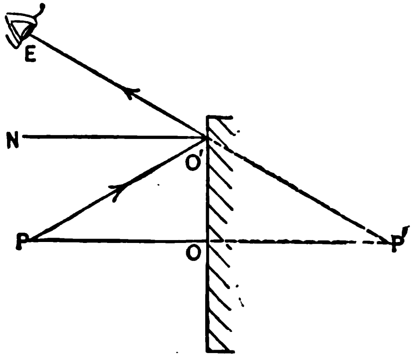

Fig. 380. — Illustrating the law of reflection at plane surfaces.

Definitions

The angle between the normal to the reflecting surface and the incident ray $NO'P$ is called the **angle of incidence.** The angle between the normal and the reflected ray $NO'E$ is called the **angle of reflection.**

But these are the angles which we have proved to be equal, in the case of the ray PO'; the same evidently is true of any other ray. Not only so, but this experiment shows that the reflected ray lies in the plane of the normal NO' and the incident ray PO'.

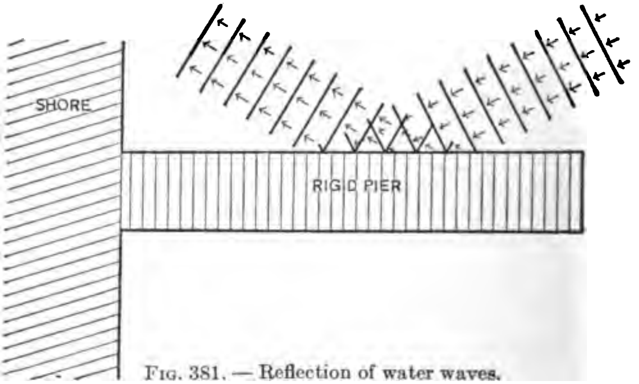

Fig. 381. — Reflection of water waves.

Accordingly, if we denote the angle of incidence by i and the angle of reflection by r, we may state the laws of reflection as follows: —

(i) **At each point of the reflecting surface the angle of reflection is equal to the angle of incidence.**

(ii) **The reflected ray lies in the plane defined by the normal and the incident ray.**

Or, in terms of algebra

$$i = r. \qquad \text{Eq. 173}$$

But this is precisely the manner in which we have found (§ 192) water waves behaving. When a wave coming from any direction strikes a solid pier at a definite angle of incidence, the reflected wave leaves the pier at the same angle on the other side of the normal. If we employ Professor Hastings' excellent definition of a mirror as " a barrier to the further progress of the waves which does not destroy their motion," it is clear that the rigid pier is a mirror for water waves quite as truly as the plate glass for light waves.

Huygens' Explanation of the Laws of Reflection

472. The first man who was able to reduce the reflection of light to anything simpler and more general was Huygens, who, in his *Treatise on Light*, published in 1690, enunciated a general principle which has proved to be of the utmost fertility in almost every branch of optics.

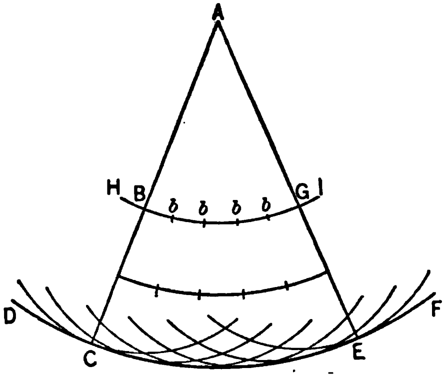

FIG. 382. — Illustrating Huygens' Principle.

The principle is derived from a consideration of what must happen according to the principles of ordinary dynamics if a wave motion started by any luminous particle is propagated through any material medium, including the free ether. Let A denote a luminous particle and $DCEF$ the position of the front of a spherical wave at any instant t. Let b, b, b, b denote particles of the medium through which this wave has passed on its way from A to its present position. Then *at some instant earlier than t*, each of these particles b must be considered as a source of disturbance sending out spherical wavelets of its own. This being true, we must consider the disturbance at any point on the wave front CE as the resultant of a large number of disturbances arising from all the particles of the medium over which the wave has passed.

Huygens' Principle consists, then, in the statement that the wave front in a train of light waves is a surface of disturbance which results from and envelops (*i.e.* is tangent to) the secondary waves sent out by each particle lying in the wave front at an earlier instant.

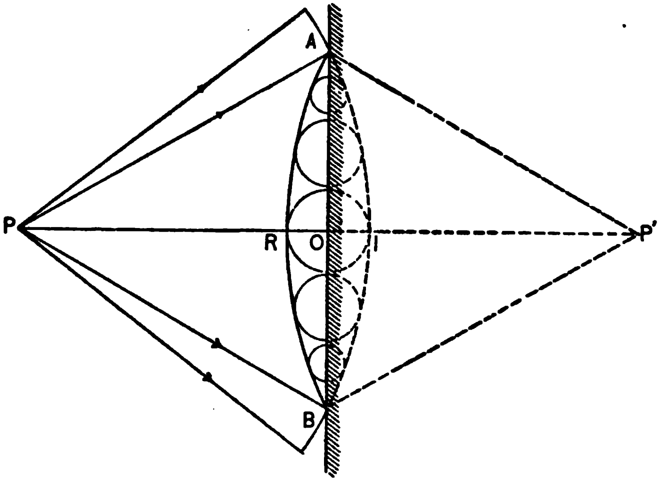

Fig. 383. — Reflection of a spherical wave in terms of Huygens' Principle.

473. Let us now apply this principle to determine the image of a luminous point in a plane mirror, the case which we have already solved in § 470. If P be the luminous point, and AOB a plane mirror, a series of spherical waves will be emitted by the point P and will proceed toward the mirror with the speed of light. The first point on the mirror which will be struck by this advancing wave is the point which is nearest P. Call this point O. Then O becomes, according to Huygens' Principle, a source of light at the instant when the wave first touches the mirror. At immediately succeeding instants, the successive points between O and A on the mirror will be reached by the incident wave front and they will in turn become sources of secondary waves. If there had been no mirror at AB, the incident wave would have gone on and occupied the position AIB. As it is, the spherical wavelets emitted by each successive point on the mirror have for their envelope the spherical surface whose trace is ARB. This envelope is therefore the reflected wave surface. By symmetry the center of curvature P' of ARB is at the same distance behind the mirror that the source P is in front of the mirror. To an observer in front of the glass the reflected light appears to come from the point P'; this point is therefore the image of P.

In this manner Huygens derives from his principle the laws of reflections at plane surfaces. The student will find it profitable to draw a diagram, corresponding to Fig. 383, but showing the particular case in which the luminous point P is at an infinite distance and in a direction which is not normal to the mirror, thus giving rise to a plane wave which is reflected at a plane surface.

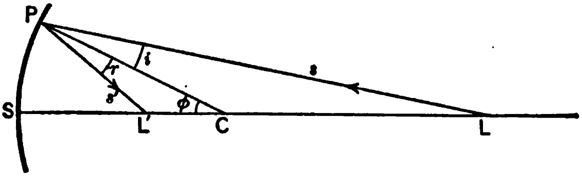

FIG. 384. — Reflection at a spherical mirror.

The Reflection of Spherical Waves at Spherical Surfaces

474. This is the most general case of reflection met with in ordinary optics, inasmuch as spheres are the only surfaces which can be accurately ground with ordinary machinery. The position of the image can here be easily determined by means of Huygens' Principle; but the method which depends upon the use of rays is more generally useful and practical, for which reason we here adopt it.

In Fig. 384 let SP indicate a spherical mirror whose center of curvature is at C. Let L denote a source of light — a luminous point — and L' its image in the mirror. In general we shall denote points by capital letters and distances by small letters. The line joining the apex of the surface S and the center of curvature C is called the *axis of the mirror*. We shall consider only that case in which the luminous point is situated somewhere on the axis of the mirror.

The center (or apex) of the mirror S is chosen as the origin; and, as in ordinary geometry, distances measured to the right will be called positive, and those measured to the left, negative. Let us now consider any ray of light LP leaving the source L and incident upon the mirror at P. The mirror surface at P is normal to the radius CP. The reflected ray will therefore cross the axis of the mirror at some point L', such that the angle of reflection $L'PC$ is equal to the angle of incidence LPC

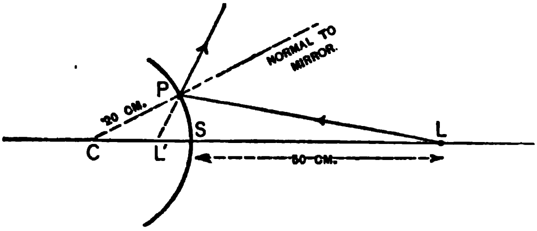

FIG. 385. — Reflection at a convex mirror.

Let $LP = s$, $L'P = s'$, $CP = r$, and the angle $PCS = \phi$; then if ϕ be small, $CL = s - r$ and $CL' = r - s'$, approximately.

By geometry, $\dfrac{s}{s - r} = \dfrac{\sin \phi}{\sin i}$ and $\dfrac{s'}{r - s'} = \dfrac{\sin \phi}{\sin r}$. Since, however, $\sin i = \sin r$, Eq. 173, it follows that

$$\frac{s}{s'} = \frac{s - r}{r - s'}, \qquad \text{Eq. 174}$$

an equation which gives us s', the distance of the image from the mirror, as soon as we know the radius of the mirror and the position of the source. Clear of fractions and then divide each term in the equation by $ss'r$; you obtain

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{r}, \qquad \text{Eq. 175}$$

a more easily workable form of the same equation.

475. Thus if one be asked to find the image of a luminous point placed 50 cm. in front of a convex mirror having a 20 cm.

radius of curvature, he has merely to substitute the following values in Eq. 175: —

$$s = +\ 50 \text{ cm.},$$
$$r = -\ 20 \text{ cm.}$$

From which it follows that $s' = -\ 8\frac{1}{3}$ cm.; that is, the image lies $8\frac{1}{3}$ cm. to the left of the mirror; in other words, the reflected rays appear to come from an image $8\frac{1}{3}$ cm. behind the mirror.

Real and Virtual Images

An image of this kind from which rays merely appear to proceed, but do not actually proceed, is called a **virtual image**. But when a mirror so changes the direction of a group of rays coming from one point so as to make them intersect at another point and actually proceed from that point, this second point is said to be a **real image** of the first.

476. The case of reflection at a plane mirror is also included in Eq. 175; for a plane mirror is described by writing $r = \infty$, whence

$$s = -\ s'. \qquad \text{Eq. 176}$$

In other words, the image is just as far behind the mirror as the object is in front of it, a conclusion which we have already (§ 470) derived from experiment. The two positions L and L', defined by s and s' in the general equation, are sometimes spoken of as "conjugate points."

Focal Length of Mirror

477. If the source L be at an infinite distance, *i.e.* if the incident rays be all parallel to the axis of the mirror, this state of affairs may be described by writing

$$s = \infty,$$

from which it follows that

$$s' = \frac{r}{2} = +f. \qquad \text{Eq. 177}$$

The length f, defined by this equation, the distance from the mirror to the image when the incident rays are parallel, is called

the focal length of the mirror. Introducing this definition into the general equation (175), we may write,

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}, \qquad \text{Eq. 178}$$

from which we may always obtain the position of the image in terms of the focal length of the mirror. From the symmetry with which the two variables s and s' enter this general equation, it is evident that, when the object is shifted so as to occupy the position of its former image, the image will shift to the former position of the object. Values of s and s' being interchangeable, the positions of object and image are interchangeable.

Size of Image produced by Mirror

478. So far we have considered only the position of the image of a luminous point situated on the axis. We now pass to a body of finite size situated on or near the axis. The position of the image of each point of this object will be found exactly as before. But in order to determine the *size* of the image as compared with that of the object, it will be necessary to first observe two very convenient facts, namely: —

(i) **An incident ray which passes through the center of curvature of the mirror has its direction unchanged by reflection.**

(ii) **An incident ray which is parallel to the axis of the mirror passes, on reflection, through the principal focus of the mirror; and *vice versa,* an incident ray which passes through the principal focus will be reflected in a direction parallel to the axis of the mirror.**

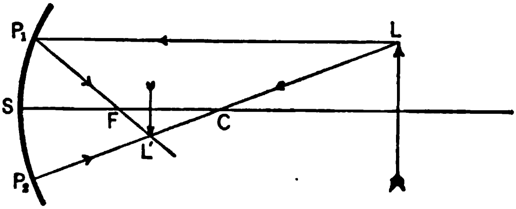

FIG. 386. — Showing how to determine size of image.

With these two rules in mind, we can see at once, from Fig. 386, that the object and the image each subtend the same angle at the center of curvature of the mirror, from which it follows that their sizes are proportional to their respective distances from the center. For let LP_1 be an incident ray from the point of the arrow parallel to the axis; the reflected ray must pass through the principal focus F; and hence the image of the arrow point must lie somewhere on the line P_1F.

Let LP_2 represent an incident ray passing through the center; then the image of the point L must lie also on this line P_2C. And since it lies on both P_1F and P_2C, it must lie at their intersection, L'. If the angle SCP_2 is not very large, we may equate LC to $s - r$, and $L'C$ to $r - s'$, which enables us to describe the size of the image, as follows: —

$$\frac{I}{O} = \frac{\text{Size of Image}}{\text{Size of Object}} = \frac{r - s'}{s - r}. \qquad \text{Eq. 179}$$

Or, in virtue of Eq. 174,

$$\frac{I}{O} = \frac{s'}{s}.$$

The student will find it an interesting problem to show that this construction holds as well for convex mirrors as for concave; only in the case of a convex mirror it is the *prolongations backward* of the two incident rays which pass through the center and focus respectively.

Problems

401. An incandescent lamp is placed 40 ft. in front of a concave mirror whose radius of curvature is 8 ft. Find the position of the image.

402. A candle placed 7 cm. in front of a concave mirror gives an image at 21 cm. behind the mirror. Prove that the radius of curvature of the mirror is 21 cm.

403. Find the focal length of a concave mirror required to produce at a distance of 20 cm. from the mirror an image of an object which is located 40 cm. in front of the mirror.

404. Where must an object be placed in front of a concave mirror in order to give an image which shall be erect and of the same size as the object?

405. Where must an object be placed in front of a concave mirror in order that the image may be inverted and of the same size as the object?

406. An object 6 in. high is placed 12 ft. in front of a concave mirror whose focal length is 2 ft. Find the size of the image.

407. Prove that when an object lies between a concave mirror and its principal focus, the image is always erect and virtual.

408. An object lying 16 cm. in front of a convex mirror gives an image at a distance of 3 cm. behind the mirror. Find the radius of the mirror and then the size of the image.

409. Why do convex surfaces such as polished door knobs and the rounded wooden knobs on furniture give such "high lights" when one is photographing "interiors"?

410. If you were required to project an arc light upon a screen and to magnify it three times, what kind of a mirror would you use and where would you hold it?

411. An international candle and an incandescent lamp are placed 8 ft. apart. Where must a screen be placed, on the line joining these two sources of light, in order that it may be equally illuminated by each? Assume the lamp to be 20-candle power.

412. In comparing a gas flame with a 16-candle power electric lamp, by means of a Rumford photometer, I find the shadows are equally deep when the gas flame is 2 ft. from the screen and the lamp 2 ft. and 6 in. Find the candle power of the flame.

413. Assume that the moon produces the same illumination as does one international candle at a distance of 5 ft., and compute the candle power of the moon.

414. Two arc lights of 500 and 300 candle power respectively are placed at a distance of 40 meters apart. Show that there are two points on the straight line passing through the arcs where the illumination produced by each of the two lamps is the same. Find these two points.

415. What candle power lamp must be placed in the center of a cubical room, measuring 6 meters on the side, in order to produce an illumination of 20 meter-candles at a point directly beneath the lamp?

416. Show that, in the case of a plane mirror, the image will move away from the object twice as fast as the mirror moves away from the object. Show also that when the mirror is rotated the reflected beam will rotate twice as fast as the mirror.

417. Two plane mirrors are hinged together and placed so that their polished surfaces include an angle of 90°. Prove that a candle placed anywhere within this right angle will give three images in the two mirrors.

418. Interpret Eq. 174 to prove that, while the size of the image bears to the size of the object the same ratio as their distances from the *center*, the ratio of their sizes is also the same as the ratio of their distances from the *mirror*.

419. Prove that when an object is placed midway between a mirror and its principal focus the image is twice as large as the object.

420. Assume the sun to be 1 million mi. in diameter and 100 million mi. distant. Find the size of the sun's image produced by a concave mirror having a focal length of 52.8 ft.

421. Explain how it is that a man standing in front of a plane mirror can see his complete image by use of a portion of the mirror which is only half as high as he is.

422. Two plane mirrors are placed with their polished surfaces parallel and facing each other. An incandescent lamp placed between them is 6 in. from one mirror and 8 in. from the other. Locate the positions of the first two images in each of the mirrors.

423. An incandescent lamp is moved from a position which is 120 cm. to one which is 80 cm. in front of a convex mirror of radius 10 cm. Find through what distance the image is shifted by this motion of the object.

424. Show that when an arc light is shining obliquely on a mirror an object in front of the mirror may cast two shadows instead of one.

425. Two concave mirrors, each of 12 cm. radius, stand facing one another. The distance from the apex of one mirror to the apex of the other is 36 cm. A luminous object is placed 12 cm. in front of the first mirror. Prove that its image in the second mirror after one reflection occupies the same position as the image in the second mirror after two reflections, *i.e.* after being reflected once in the first mirror and again in the second.

VI. THE REFRACTION OF LIGHT

479. The first man who succeeded in measuring the speed of light in any medium other than air or " vacuum " was the French physicist Foucalt, who in 1850 proved beyond all doubt that light travels at a much slower rate in water than in air or vacuum. Since then Michelson has measured the ratio of the velocity in air to that in water and finds it 1.33. The ratio of the velocity in air to that in carbon bisulphide is 1.77. Each of these results is for white light.

All experiments show that light travels more slowly in any kind of ordinary matter than it does in a vacuum. Kundt, Drude, and others have proved that there are apparent exceptions to this statement under certain highly special conditions. We are now prepared to trace some of the consequences of this general result.

Indians in spearing fish show themselves very familiar with the fact that the fish is not exactly where he appears to be in the water, but is always a little lower down. Boys in wading a creek, generally learn that they have to roll their trousers higher than at first they thought necessary; for the creek is always deeper than it appears to be. A lead pencil placed in a tumbler of water and looked down upon from one side appears to be sharply bent at the point where the pencil enters the surface of the water.

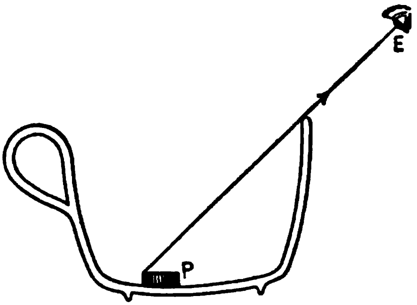

Fig. 387.

480. Every student should repeat for himself the following experiment which is described by Lord Bacon, but which probably belongs to a century still earlier than his. He places a small piece of metal, say a coin, in the bottom of an opaque bowl; and then gives his eye such a position *E* that he just *cannot* see any part of the coin *P* (Fig. 387). If now water be carefully poured into the bowl while neither the position of the eye nor that of the coin is changed, it will be seen that the coin is lifted into view, as indicated in Fig. 388. The change in direction of the ray *PS*, when it leaves the water and passes into air, is called **refraction.**

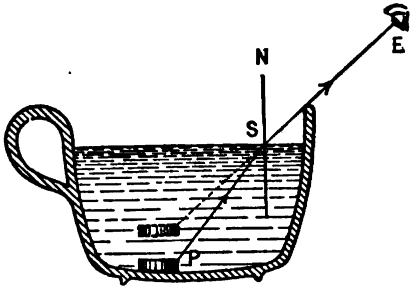

Fig. 388. — Coin is lifted into view by the water.

In general, it is found that **the ray emerging from water to air is bent away from the normal *NS*, while a ray which enters the water from the air is always bent toward the normal.**

Another Illustration of Refraction

481. The laws of refraction may be illustrated by means of the "optical disk," one form of which, devised by Professor R. R.

Tatnall, is shown in Fig. 389. Half of a circular disk of glass *A* is mounted on a wooden base *B*. A horizontal beam of light from the sun or from a projection lantern falls on the mirror *M* and is reflected thence to a second mirror *N*. After leaving *N*, the beam is incident on the upper plane face of the glass disk *A*.

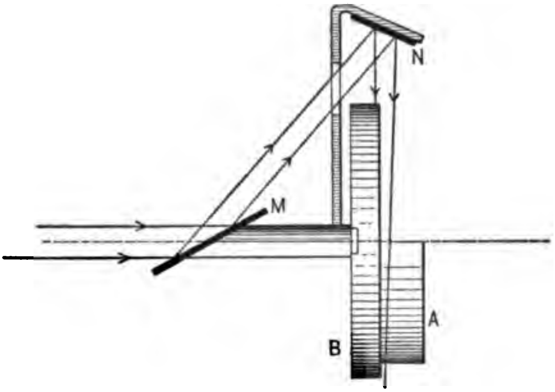

FIG. 389. — Tatnall's form of optical disk.

The surface of the mirror *N*, with the exception of a narrow strip, is covered with black paper, as shown in Fig. 390. By means of this diaphragm, the light incident on the refracting surface is restricted to a narrow beam. This beam falls upon the whitened face of the plate *B* at a small angle, so that the course of the incident ray in air and the refracted ray in glass may be plainly seen. The frame which carries the two mirrors may be revolved around the pivot by which it is attached to the base *B*, and in this way the angle of incidence may be varied at will.

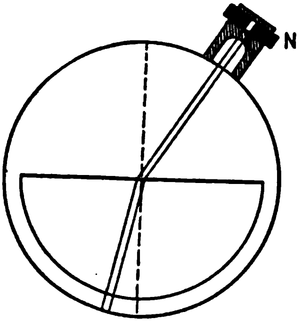

FIG. 390.

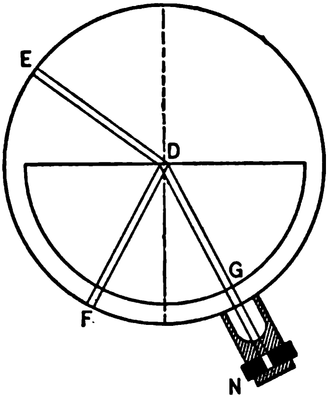

FIG. 391.

If the mirror N be placed in the position shown in Fig. 391, the refraction will take place on passing from glass into air. The incident beam GD, the refracted beam DE, and the reflected beam DF are now all seen at the same time. The refracted beam is bent away from the normal, while the incident and reflected beams make equal angles with the normal.

By moving the mirror N toward the right, in Fig. 391, the angle of incidence is increased; but the angle of refraction increases more rapidly than the angle of incidence. When the critical angle of incidence is reached, the angle of refraction has become a right angle, and the beam DE just grazes the refracting surface. On increasing the angle of incidence beyond this critical value, it will be found that there is no refracted beam, the light being now totally reflected.

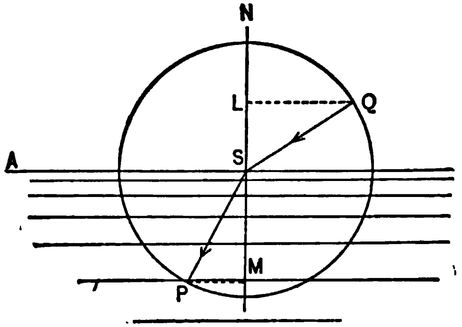

FIG. 392.—Snell's description of the refracted ray.

482. The man who first described all these phenomena in a simple manner was Willebrord Snell (1591–1626), a Dutch mathematician. His description is as follows: —

Let AS (Fig. 392) be any bounding surface separating two different media, say water and air. Take any point S on this surface and describe a circle about it as a center. Let QS be any ray incident at S; the problem is to find the direction of the refracted ray SP. Snell discovered that this could always be done as follows; viz. from Q let fall a perpendicular QL on the line NS, which is drawn normal to the surface at S. Then, if we define PM as the perpendicular distance of P from this same normal, the ratio of QL to PM is constant for any two media. This constant is usually indicated by n, and is called, when the upper medium is air, the **index of refraction**.

$$QL/PM = \text{constant} = n = \text{index of refraction.}$$

For water and air this constant is about 1.33; for ordinary glass and air its value is about 1.52. Experiment shows also that the refracted ray SP lies in the same plane with the normal NS and the incident ray QS.

Definitions. The angle QSL, between the incident ray and the normal, we have called the **angle of incidence**; it is generally denoted by i. The angle PSM, between the refracted ray and the normal, is called the **angle of refraction**; it is generally denoted by r.

The Two Laws of Refraction

Accordingly we may summarize these two results as follows: —

(i) **The angle of refraction r bears to the angle of incidence i the relation**

$$\frac{QL}{PM} = \frac{\sin i}{\sin r} = n = \text{constant.} \qquad \text{Eq. 180}$$

(ii) **The refracted ray lies in the same plane with the incident ray and the normal to the refracting surface.**

Explanation of Refraction

483. Assuming a fair familiarity with the phenomena of refraction, we proceed to inquire whether it is like any other phenomenon which we have already studied; whether there is any simpler class of facts to which we can refer it as a special case; whether, in short, we can "explain" the phenomenon of the bending of the rays.

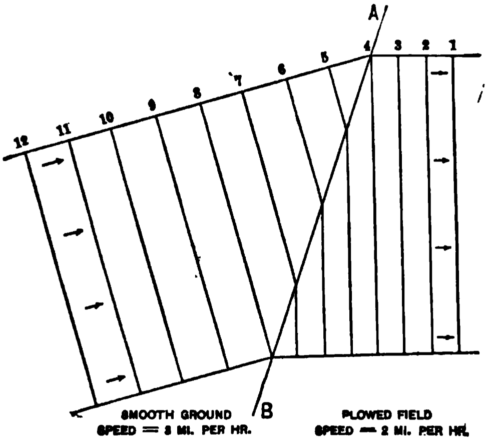

Fig. 393. — Refraction due to change of speed.

Let us suppose that a regiment of soldiers is marching over a country interspersed with plowed fields; and suppose also that the regiment moves with a speed of 3 miles an hour over the smooth ground, and 2 miles an hour over the plowed ground. Let AB (Fig. 393) represent the line which separates the smooth ground from the plowed. The arrows indicate the line of march, *i.e.* the arrows are at every point perpendicular to the front of the ranks.

Consider any rank, say No. 8, which is just entering upon the rough ground. Evidently the right-hand end will be slowed up, for the left end is now traveling 1 mile an hour faster than the right — the front will now form a slightly bent line, until the whole rank has crossed into the plowed region; and then the rank will again be straight, **but it will be headed in a direction slightly different from that which it previously had.** It will, in fact, be "refracted." Not only so, but the line of march will be refracted toward the normal to the line BA.

We have already seen (§ 479) that light travels more slowly in water than in air. The refraction of rays of light is therefore strictly analogous to the refraction of the line of march, each depending upon a change of speed in passing from one medium to another.

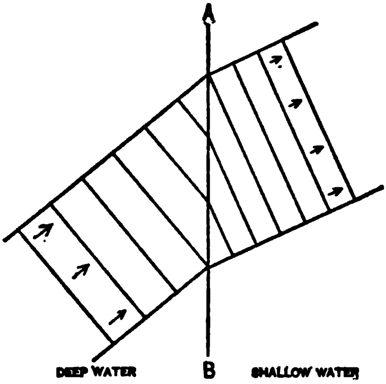

FIG. 394. — Refraction of water waves due to change of speed.

Another Type of Refraction

484. We have already found (§ 191) by experiment that the speed of water waves depends upon the depth of the water in which they travel — the deeper the water the faster the wave travels. If now we consider a series of waves, whose crests are represented by the straight lines in Fig. 394, it is evident that when these waves cross the boundary line AB into more shallow water, they will be deflected just as the soldiers are, and just as the rays of light are. This analogy between water waves and light is essentially due to Huygens.

From what precedes it will be evident that the amount of bending (refraction) which a ray undergoes in passing from one medium to another depends upon the ratio of the speeds of light in these two media. If the speed is the same in each, there is no change in the direction of the ray.

From Fig. 395 it is evident that, if CA represents the distance which light travels *in air*, during the time required for it to travel from B to D *in glass*, we have

$$\frac{\text{Speed of incident light}}{\text{Speed of refracted light}} = \frac{CA}{BD} = \frac{\sin i}{\sin r} = n, \qquad \text{Eq. 180}$$

as defined in § 482. For all transparent liquids and solids n is greater than unity.

485. Since n for any particular medium varies inversely as the speed of light in that medium, we may generalize the above equation by writing the refractive index for a ray passing from, say, water to glass, thus,

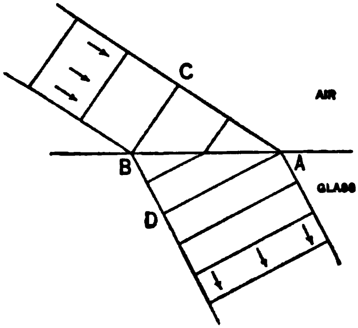

FIG. 395. — The physical meaning of refractive index.

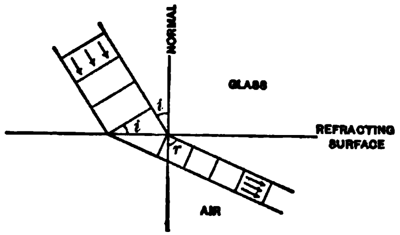

FIG. 396. — Refraction away from the normal.

$$\frac{\text{Speed in water}}{\text{Speed in glass}} = \frac{\sin i}{\sin i'} = \frac{\frac{1}{n}}{\frac{1}{n'}} = \frac{n'}{n},$$

where n is the refractive index for water and n' the index for glass; i is the angle of incidence and i' the angle of refraction.

A more significant way of writing this equation is the following: —

$$n \sin i = n' \sin i' = \textit{optical invariant}. \qquad \text{Eq. 181}$$

Total Reflection

486. Up to this point we have been considering the passage of a ray of light from a medium where it travels with a certain speed to one where it travels less rapidly. Let us now reverse the sense in which the light moves so that the incident ray travels along the path of what was formerly the refracted ray. The angle of refraction will in this case, by Huygens' construction (Fig. 396), always be greater than the angle of incidence,

$$r > i.$$

But the maximum value of r is $\frac{\pi}{2}$. Hence the maximum value of i, corresponding to any refracted ray, is given by

$$\frac{\sin i}{\sin \frac{\pi}{2}} = \sin i = \frac{1}{n}, \tag*{Eq. 182}$$

where n is the refractive index of the first medium with respect to air or vacuum.

We now proceed to consider what happens when the angle of incidence is greater than the maximum value indicated above. The phenomena which occur under these circumstances are familiar to every one who has examined minutely a freshly fallen snowflake and found it to be made up of small but transparent crystals of ice. How does transparent ice acquire the brilliant whiteness of snow? The same phenomenon has been observed by every one who has ever broken up a piece of clear ice with a

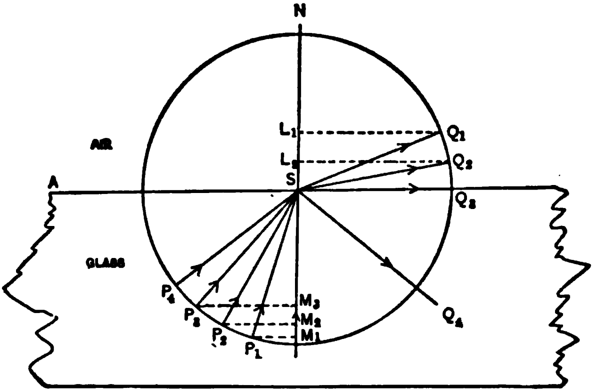

Fig. 397. — The phenomenon of total reflection.

hammer. The moment the ice is broken into small pieces it appears white. The same is true of finely broken glass. To explain this phenomenon, let us consider any point in the interior of a solid piece of glass. Suppose a ray of light to leave the point P_1 (Fig. 397) and pass on through the surface AS. The angle of incidence is P_1SM_1. The incident ray has the direction P_1S, but is refracted in the direction SQ_1. Imagine another ray to start from the point P_2; the angle of incidence now becomes P_2SM_1, the angle of refraction increases to, say, Q_2SL_1. As the angle of incidence increases still farther, the refracted ray finally takes the direction SQ_3, that is, the refracted ray just grazes the refracting surface. The perpendicular from Q_1 let fall upon the vertical line NM_1 has now its largest possible value. What will happen, then, if we increase the angle of incidence to P_4SM_1? If refraction now occurs, the ratio $\frac{QL}{PM}$ can no longer remain constant. It is observed that, **under these circumstances, refraction does not occur.** But if the angle of incidence ever exceeds the limit P_3SM, the ray is wholly reflected back at S into the glass. This phenomenon is called **total reflection. There is then a certain limiting angle of incidence — called the critical angle — for any ray in glass, defined by the fact that if this angle of incidence be exceeded, the ray will not escape from the glass but will be sent back into it.** This is the angle defined by Eq. 182.

The following table gives the critical angles for a few of the more important substances: —

TABLE OF CRITICAL ANGLES

Diamond	24°
Ruby	36°
Rock Salt	41°
Crown Glass	42°
Turpentine	43°
Water	48°

487. Every student should examine for himself the reflection which occurs on the lower side of the free surface of water in a tumbler placed a little above the level of one's eyes, as indicated in Fig. 398. He will then understand why snow appears so white. Any stray ray of light which enters a body of snow will, in general, penetrate only a few of the transparent crystals before it will strike some refracting surface at an angle greater than the critical angle. It will then be totally reflected, and will finally find its way out into the air again. A large proportion (not all) of the rays of light which fall on snow will therefore be sent back. But this is all that we mean by a white body, viz. one which reflects a large portion of all the light which falls upon it. From the preceding table it is clear also why rubies and diamonds are more brilliant than glass bodies of the same size, shape, and polish.

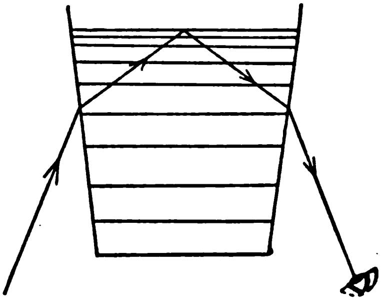

FIG. 398. — Total reflection in a tumbler of water.

488. An important application of this phenomenon is seen in the prismatic pavement lights which are now widely used in large cities. These prisms receive the light from that portion of the sky which is directly overhead, and send it by total reflection back into an otherwise dark basement. It will be evident from the figure that the slope of the prism must be such as to make the angle of incidence greater than the critical angle for glass.

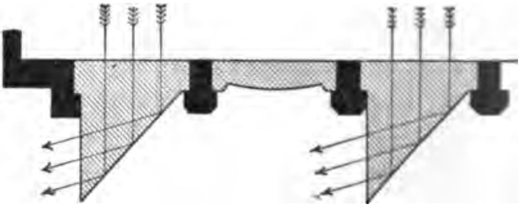

FIG. 399. — Showing the path of rays through a prismatic pavement light.

CASE I. *Refraction at a Single Spherical Surface. Axial Rays.*

489. Up to the present we have been considering the general law of refraction, which we now proceed to apply to some particular cases which are highly important in the arts.

In Fig. 400 let *PS* denote a spherical surface separating two media whose refractive indices are n and n' respectively. We proceed to determine the position of the image produced by refraction at this surface, for which purpose we adopt the following nomenclature, which is essentially that used in the Zeiss optical works at Jena.

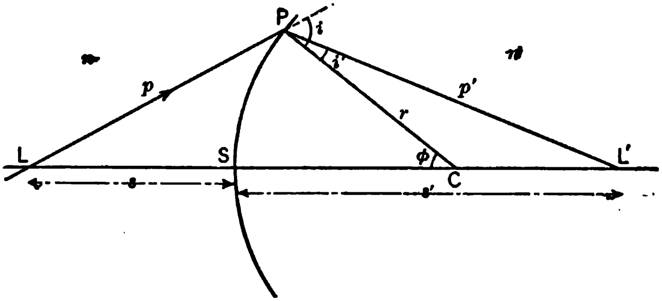

FIG. 400. — Case of refraction at a single spherical surface.

Let r = radius of curvature of refracting surface — positive when convex side of surface is toward incident ray,
C = center of curvature,
L = luminous point, source on axis,
LC = axis of refracting surface, an unlimited straight line,
L' = point at which refracted ray crosses the axis, image of L,
S = apex of refracting surface,
s = distance from apex of surface to luminous object; negative to left, positive to right,
s' = distance from apex to image L'; negative when measured in a sense opposite that in which the light is traveling, positive when measured in the same sense,
$c = s - r$,
$c' = s' - r$,
ϕ = angle subtended at C by the arc SP,
i = angle of incidence,
i' = angle of refraction.

Starting now with the optical invariant, Eq. 181

$$n \sin i = n' \sin i',$$

divide each side by $\sin \phi$, whence

$$\frac{n \sin i}{\sin \phi} = \frac{n' \sin i'}{\sin \phi}.$$

But for axial rays, that is, for those which make a small angle with the axis, we have, by geometry,

$$\left.\begin{array}{l} \dfrac{\sin i}{\sin \phi} = \dfrac{s - r}{p} = \dfrac{s - r}{s} \\ \text{and} \\ \dfrac{\sin i'}{\sin \phi} = \dfrac{s' - r}{p'} = \dfrac{s' - r}{s'} \end{array}\right\} \text{very approximately,}$$

where the meanings of p and p' are evident from Fig. 400.

Hence
$$n\left(\frac{s - r}{s}\right) = n'\left(\frac{s' - r}{s'}\right).$$

Dividing through by r, we have

$$n\left(\frac{1}{r} - \frac{1}{s}\right) = n'\left(\frac{1}{r} - \frac{1}{s'}\right) \quad \text{Laboratory equation for spherical surface.} \qquad \text{Eq. 183}$$

As soon as s, the position of the source, is given, this equation enables one to determine s', the position of the image.

This equation was first obtained, in a general form, in 1693 by the English astronomer, Halley.

SPECIAL CASE II. *First Medium, Air; Second Medium, Glass, Water, etc.*

490. Here $n = 1$ and $n' = \mu$, whence Eq. 183 becomes

$$\frac{\mu}{s'} - \frac{1}{s} = \frac{\mu - 1}{r}. \qquad \text{Eq. 184}$$

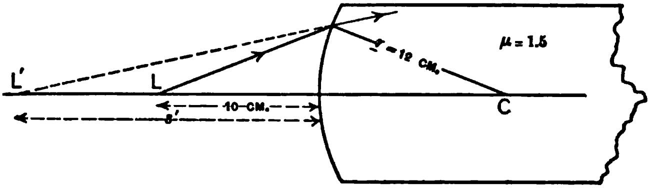

FIG. 401.

Illustration. Let it be required to find the image of a luminous point, placed 10 cm. to the left of a block of glass ($\mu = 1.5$)

bounded by a convex surface of 12 cm. radius as shown in the figure.

Here $\mu = 1.5$

$r = + 12$ cm.

$s = - 10$ cm. Find s'.

Hence
$$\frac{1.5}{s'} + \frac{1}{10} = \frac{0.5}{12}.$$

Or
$$s' = - 25.7 \text{ cm.}$$

Accordingly the image is virtual, and lies 25.7 cm. to the left of the surface of the glass.

SPECIAL CASE III. *Incident or Refracted Rays Parallel. Focal Lengths*

491. Let $s = \infty$, then the incident rays are parallel to each other, and our general equation (184) becomes

$$s' = \frac{\mu r}{\mu - 1} = f' = \text{second focal length.}$$

Or if, instead of making $s = \infty$, we make $s' = \infty$,

$$s = - \frac{r}{\mu - 1} = f = \text{first focal length.}$$

Observe that the second focal length, *i.e.* the one in the glass, is μ times greater than the first focal length. Note also that these focal lengths have opposite signs, and that the sign of each depends upon the sign of r.

Employing the first focal length f, we may write, Eq. 184,

$$\frac{\mu}{s'} - \frac{1}{s} = - \frac{1}{f}. \qquad \text{Eq. 185}$$

Employing the *second* focal length one obtains

$$\frac{\mu}{s'} - \frac{1}{s} = \frac{\mu}{f'}. \qquad \text{Eq. 186}$$

SPECIAL CASE IV. *Refraction at a Plane Surface*

492. To describe a plane refracting surface in algebraic terms we have merely to write $r = \infty$. On introducing this condition into our general equation (184) we obtain,

$$s' = \mu s. \qquad \text{Eq. 187}$$

From this it follows that the image in the glass or in the water is μ times as far from the surface as the object is. A fish swimming in the water and looking at a fishing rod above the surface sees it 1.3 times as far away as it really is.

Conversely, since $s = \frac{s'}{\mu}$, an Indian spearing a fish sees it only $\frac{1}{\mu}$, or $\frac{3}{4}$ as far below the surface as it really is.

SPECIAL CASE V. *Reflection*

493. By placing $\mu = -1$, and *again measuring r from the origin according to the rules of ordinary geometry,* we may obtain the entire theory of *reflection* at plane and spherical mirrors; for then our general equation (184) becomes,

$$\frac{1}{s'} + \frac{1}{s} = \frac{2}{r}. \qquad \text{Equation of spherical mirror.} \qquad \text{Eq. 188}$$

If, in addition, we put $r = \infty$, we have the description of a plane mirror,

$$s = -s'. \qquad \text{Eq. 189}$$

The student will observe that the one equation which we have been using throughout this discussion (Eq. 184) is perfectly general and holds not only for concave, convex, and plane refracting, but also for concave, convex, and plane *reflecting* surfaces.

Problems

426. Find the refractive index when the angles of incidence and refraction at a plane surface are 45° and 30°, respectively.

427. The critical angle for a certain medium is 45°. Find its refractive index.

428. A pebble lies at the bottom of a pool of water which is four feet deep. How far below the surface will this pebble appear to a man looking vertically down through the water? *Ans.* 3 ft.

429. An air bubble imprisoned in a piece of glass appears to lie $\frac{4}{19}$ cm. below the surface of the glass, which has a convex surface of 20 cm. radius. If the refractive index of the glass is 1.5, what is the real distance of the air bubble below the surface? *Ans.* 0.3 cm.

430. In what direction must an incident ray strike a spherical glass surface in order that this direction may not be changed by refraction?

431. A luminous point is placed $6\frac{2}{3}$ cm. to the left of a surface of glass of which the refractive index is 1.6. The image which is virtual lies 16 cm. to the left of the surface. Find the radius of curvature of the glass surface. *Ans.* + 12 cm.

432. Show from the general equation (184) that there are two positions on the axis at which a luminous point may be placed such that, after refraction at a single spherical surface, the image will coincide with the object.

433. What refractive index will be required in a glass ground with a convex surface of 40 cm. radius in order that a point situated in air 25 cm. to the left of the convex surface may be brought to focus at a distance of 90 cm. to the left of the surface? *Ans.* $\mu = 1.8$.

434. A microscope is focused upon a scratch on the upper surface of the flat bottom of a glass beaker. Liquid is now poured into the beaker to the depth of y cm., when it is found that the microscope must be lifted through a distance h in order to remain focused upon the same scratch. Prove that the refractive index of the liquid μ is given by the following equation,

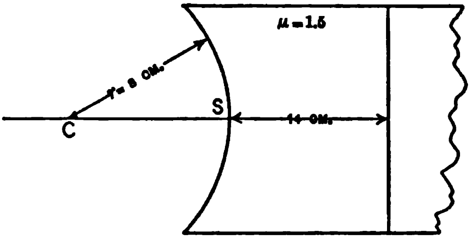

FIG. 402.

$$\mu = \frac{y}{y - h}.$$

435. From the general equation (184) find the two principal foci for a refracting surface S, such as that indicated in Fig. 402.

436. Referring to Fig. 402, the block of glass, instead of being left unfinished at the right hand, is ground to a plane perpendicular to the axis and is silvered. A small incandescent lamp is placed 48 cm. to the left of the surface S. Find the position of its image after refraction at S and reflection at the silvered surface.

Ans. 46 cm. to right of S.

CASE VI. *Refraction of a Spherical Wave Surface at Two Refracting Surfaces in Succession. Lenses*

494. *Definition.* **A portion of a refracting medium which is limited by two curved surfaces is called a lens.**

The only surfaces which can be ground in lathes with accuracy and ease are spherical surfaces, hence all lenses, practically, are made with spherical surfaces, *i.e.* the shape of a lens may be considered as the shape of a figure bounded by two spheres.

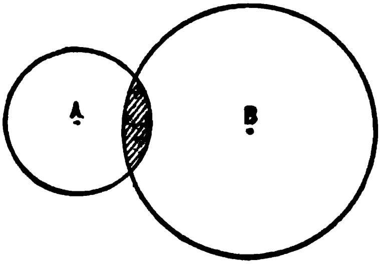

FIG. 403. — A converging lens.

Definition. **The line joining the centers of curvature of the two refracting surfaces is called the axis of the lens.**

If one face of the lens is a plane, we may consider this plane as a portion of a sphere of infinite radius. The axis of such a lens is the line through B (Fig. 404) normal to the plane surface.

It will be observed that the axis of a lens is normal to each surface of the lens at the point where it passes through the surface.

We proceed now to trace the history of a wave surface through a lens defined as above. And for the purpose of obtaining a general solution, we shall choose a lens limited by two spheres of radii r_1 and r_2 which are each positive, as indicated in Fig. 407. For, as before, we shall consider the radii positive, when the convex surface is turned toward the incident light. Here also we shall use the same nomenclature as before, following the conventions of ordinary geometry and allowing the subscript "1" to refer to the first surface, and the subscript "2" to the second surface.

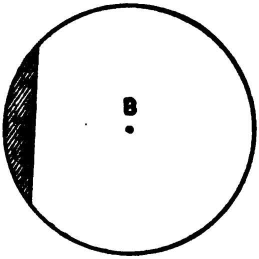

FIG. 404. — A converging lens with one plane surface.

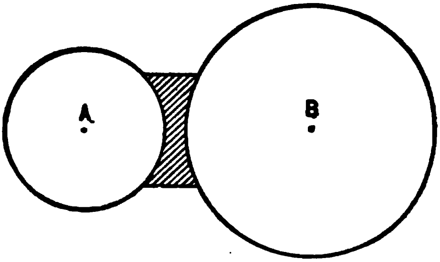

FIG. 405. — A diverging lens.

The first problem is to obtain the position of the image L'

due to a luminous source L; in other words to find s' in terms of s, r_1, r_2, and μ.

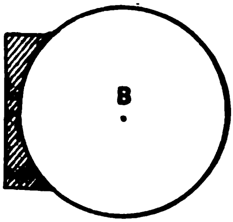

Fig. 406. — A diverging lens with one plane surface.

This might be done, as in the case of a single refracting surface, by following through the lens the paths of a pair of rays; but it is more simply accomplished by following through the lens a single wave surface. Since L is the point source whose image is sought, it will be the center of the incident wave front. Let AIP be the position which this incident wave front would have assumed at the instant t, *if the lens had not been there.* Owing to the presence of the glass, however, the speed of this wave has been very much slowed up. The portion which is *most* retarded is that part which has had the greatest thickness of glass to travel through, *i.e.* the center of the wave which was incident at S_1. Now, on the wave theory, the focus which is conjugate to L, that is, the image of L, is simply that point where the disturbances due to each element of the emergent wave front meet in the same phase. In other words the image, L', of L is a point such that the light occupies the same time in traveling from L to L', whatever be the path. Evidently this point is identical with the center of curvature of the emergent wave surface.

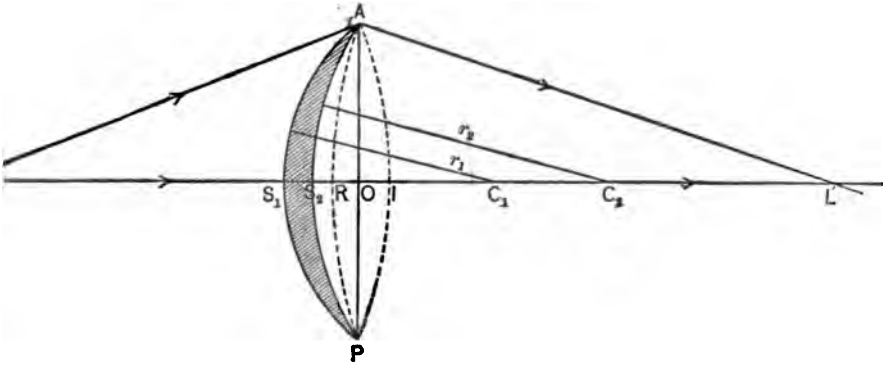

Fig. 407. — Illustrating refraction through a medium bounded by *two* spherical surfaces.

Let ARP represent this emergent wave front in Fig. 407. Draw the chord *AOP* perpendicular to the axis *LC*. If *a* denote half of this chord, we may write, very approximately, for *RO*, which is called the *sagitta* of the emergent wave front,

$$RO = \frac{a^2}{2s'}, \text{ for the emergent wave front.}$$

And in like manner,

$$S_1O = \frac{a^2}{2r_1}, \text{ for the refracting surface } AS_1P.$$

$$S_2O = \frac{a^2}{2r_2}, \text{ for the refracting surface } AS_2P.$$

$$IO = \frac{a^2}{2s}, \text{ for the incident wave front, } AIP.$$

Now the time occupied by each part of the wave front in traveling from *L* to *L′* is the same as that of any other part. Let us therefore equate the time occupied by the outer edge of the wave traveling over the path *LAL′* to the time consumed by the central portion of the wave in going over the route LS_1S_2L'.

If we call the speed of light in air unity, we have

$$\begin{aligned} LA + AL' &= LI - S_1I + \mu S_1S_2 + S_2L' \qquad \text{Eq. 190} \\ &= LI - (S_1O + OI) + \mu(S_1O - S_2O) \\ &\quad + S_2O - RO + RL'. \end{aligned}$$

But since $LA = LI = -s$
and $AL' = RL' = +s'$ } approximately for small pencils,
we have $0 = -S_1O - OI + \mu(S_1O - S_2O) + S_2O - RO.$

Introducing now the value of the sagitta, as just defined, and remembering that when the source is to the left, as in this case, *s* is negative, we have

$$0 = -\frac{a^2}{2r_1} + \frac{a^2}{2s} + \mu\left(\frac{a^2}{2r_1} - \frac{a^2}{2r_2}\right) + \frac{a^2}{2r_2} - \frac{a^2}{2s'}.$$

Dividing by $\frac{a^2}{2}$ and transposing, we obtain

$$\frac{1}{s'} - \frac{1}{s} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \qquad \text{Eq. 191}$$

which is a perfectly general equation *for pencils of small aperture coming from points near the axis and incident upon thin lenses.* If r_1 is positive and r_2 negative, we have a **double convex lens**; but if r_1 is negative while r_2 is positive, we have a **double concave lens.** Eq. 191 describes completely all the following special cases, which need therefore be merely indicated.

SPECIAL CASE VII. *Incident Wave Front Plane. Focal Length*

495. Let $s = \infty$.

Hence $s' = \frac{1}{\mu - 1}\left(\frac{r_1 r_2}{r_2 - r_1}\right) = f$, say = second focal length.

Let $s' = \infty$.

Then $s = \frac{-1}{\mu - 1}\left(\frac{r_1 r_2}{r_2 - r_1}\right) = -f$ = first focal length.

In terms of f, the *second* focal length, the general equation (191) becomes

$$\frac{1}{s'} - \frac{1}{s} = \frac{1}{f}. \qquad \text{Eq. 192}$$

These expressions for the focal length were first obtained by Cavalieri, the Italian, in 1647.

The point at which the emergent wave front converges when the incident wave front is plane is called the **principal focus.** Note that a plane incident wave is equivalent to a bundle of incident rays each parallel to the axis. The **power** of a lens, which we may denote by P, is then defined as **the reciprocal of the focal length.** For, the shorter the focal length of a lens, the greater the change it produces on the direction of rays passing through it; hence its power is said to vary inversely as its focal length.

$$P = \frac{1}{f}. \qquad \text{Eq. 193}$$

Defining equation for power of a lens.

A lens whose focal length is 1 meter is said to have a power of 1 **dioptric.** The dioptric is the **unit of power** for lenses. This unit is universally used by oculists in prescribing spectacles. A lens whose focal length is ½ meter has a power of 2 dioptrics; and so on. The power of a lens is simply its ability to change the divergence or convergence of rays incident upon it.

SPECIAL CASE VIII. *One Refracting Surface Plane*

496. Let $r_1 = \infty$, and r_2 be negative, then we have a **plano-convex** lens, for which

$$\frac{1}{s'} - \frac{1}{s} = \frac{1-\mu}{r_2}. \qquad \text{Eq. 194}$$

When, however, r_1 is positive and $r_2 = \infty$, we have a **convexo-plane** lens, for which

$$\frac{1}{s'} - \frac{1}{s} = \frac{\mu-1}{r_1}. \qquad \text{Eq. 195}$$

Graphical Construction of Image

497. So far we have concerned ourselves only with the *position* of the image produced by a lens. Before we attempt to determine its size, it will be convenient to add one more term to our optical vocabulary.

Definition. A ray of light falling upon any point of a lens may have such an angle of incidence that after refraction in the lens it will emerge in a direction parallel to the direction of incidence. If we draw a tangent plane to the front surface of the lens at the point of incidence, and then draw another plane, tangent to the back surface and parallel to the first tangent plane, we may at once construct the ray in the glass; for its path will be the line joining these two points of tangency. Indeed, the lens will behave as a plate of plane parallel glass for a ray which is incident at A and emergent at B in Fig. 408.

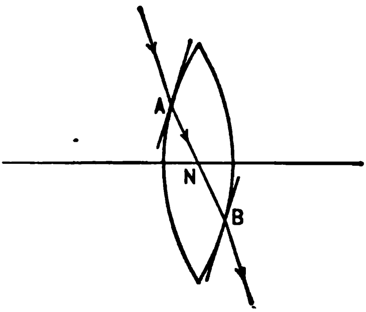

FIG. 408. — Illustrating the optical center of a lens.

That point where an undeviated ray in passing through the substance of a lens crosses the axis is called the optical center of the lens.

In any actual lens, that is, in any lens of finite thickness, the emerged portion of an undeviated ray is more or less shifted at right angles to the direction of the incident ray while it remains parallel to the incident ray. But in thin lenses, such as we are now discussing, this shift is negligible. The advanced student will find this part of the subject beautifully simplified by the consideration of "principal points" and "nodal points," whose interesting properties would here lead us too far afield.

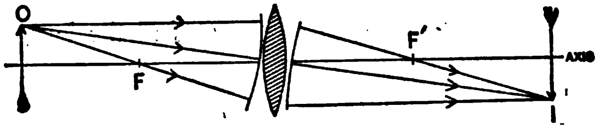

Fig. 409. — Illustrating the construction of an image in a double convex lens.

To construct the optical image of a point (or of any aggregation of points) we have therefore only to employ any two of the three following general principles: —

(i) An incident ray parallel to the axis passes, on emergence, through the principal focus.

(ii) Or, conversely, an incident ray which passes through the principal focus will emerge in a direction parallel to the axis.

(iii) An incident ray which passes through the optical center emerges without change of direction.

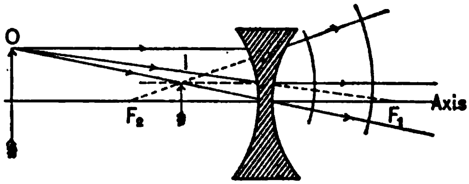

Fig. 410. — Illustrating the construction of an image in a double concave lens.

Just how these principles are applied will be clear from a careful consideration of Figs. 409 and 410. From Fig. 409 will be seen how the three typical incident rays are drawn when the lens is double convex and when the object is placed at a distance from the lens which is greater than the focal length.

From Fig. 410 will be seen the behavior of a double concave lens. In each case the top ray, being parallel to the axis, emerges in a direction which passes through the focus. Another ray, being directed toward the optical center, emerges without change of direction. The remaining incident ray passes through the focus and hence emerges parallel to the axis. In the particular cases here cited, it will be observed that the double convex lens gives an image which is real but inverted; while the double concave gives an image which is virtual but erect.

Magnification

498. *Definition.* The ratio between the linear magnitude of an image and the corresponding linear magnitude of the object, both measured at right angles to the axis of the lens, is called the *linear magnification* of the lens. It is evident from the typical Fig. 411, that this ratio $A'L'/AL$ is identical with the ratio s'/s, as will be seen by comparing the two similar triangles ANL and $A'NL'$.

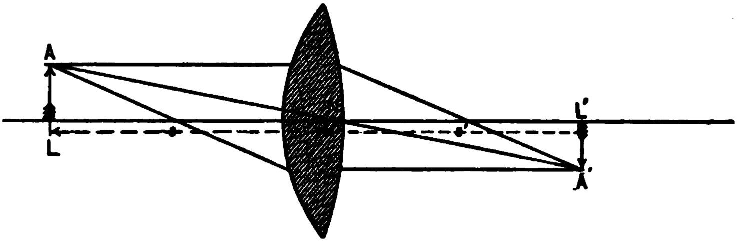

Fig. 411. — Illustrating magnifying power of lens.

Spherical and Chromatic Aberration

499. It would be interesting if time permitted to consider a more general case in which the incident ray makes a larger angle with the axis of the lens, and in which the luminous point is at some distance from the axis. The lens would then begin to show an imperfection known as "spherical aberration." The image of a point would no longer be a point.

μ	Red Light C	Yellow Light D	Green Light F
Flint glass . .	1.630	1.635	1.648
Crown glass .	1.527	1.530	1.536

So far we have assumed that μ is a constant, but if we use light of more than one color we find that μ varies from color to color and that the image of a luminous point on the axis is drawn out into a string of images each of a different color. The table illustrates this variation of refractive index.

Indeed, it is at once evident from § 495 that the focal length of a lens is shorter in proportion as $\mu - 1$ is greater. This failure of a lens to bring all colors to the same focus is called "chromatic aberration." The manner in which these errors are corrected by combining several lenses of different refractive indices and different radii of curvature forms a chapter of great interest to the advanced student, who should consult Edser's *Light* and Czapski's already classical *Theory of Optical Instruments*. The general plan is, in a word, to combine lenses into a system in such a way that the error of one element is equal to the error of another element, but is of opposite sign. Such a system is said to be "achromatic."

Case IX. *The Prism*

500. An interesting and important special case of refraction at two surfaces is that of the prism.

Definitions. (i) **The angle of the prism** is the dihedral angle

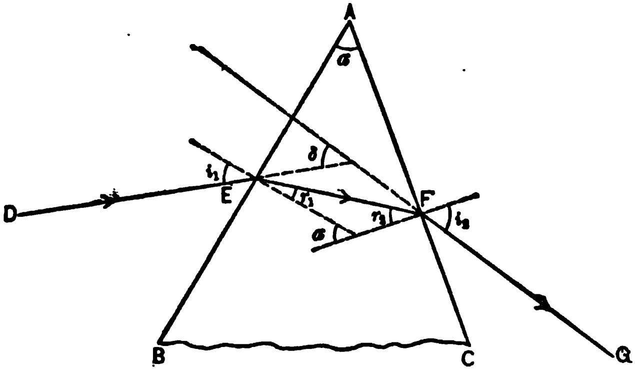

Fig. 412. — Refraction through a prism.

between the inclined planes which bound the refracting medium. This angle α is represented by $B\widehat{A}C$ in Fig. 412. The line in which these inclined planes meet is called the **refracting edge** of the prism.

(ii) Consider an incident ray DE falling upon one surface of the prism and, after two refractions as indicated in Fig. 412, emerging from the second surface as the ray *FG*. The angle between the incident and emergent rays, which we here denote by δ, is called the **angle of deviation.**

If we denote angles of incidence by i and angles of refraction by r, and distinguish between the first and second surfaces by subscripts as before, it is evident from the figure that the

$$\text{angle of the prism} = \alpha = r_1 + r_2, \qquad \text{Eq. 196}$$

and that the

$$\text{angle of deviation} = \delta = (i_1 - r_1) + (i_2 - r_2). \qquad \text{Eq. 197}$$

To derive these two equations we have merely to recall that the exterior angle of any triangle is equal to the sum of the opposite interior angles. These two equations enable us to determine the deviation of a ray as soon as we know i_1, μ, and α.

Angle of Minimum Deviation

501. As has already been pointed out, it is a consequence of the law of refraction that when any emergent ray is replaced by an incident ray which has the same direction and the same position, but an opposite sense, the path of the reversed ray through the prism is identical with that of the direct ray. In other words, if we consider, in Fig. 412, *GF* as the incident ray, then *ED* will be the emergent ray.

By applying this principle of reversibility to a prism it may be shown that **the angle of deviation is a minimum when the path of the ray inside the prism is symmetrical with respect to the two refracting surfaces.**

For since the ray is reversible, it follows that corresponding to any one angle of deviation there are two possible angles of incidence, namely i_1 and i_2. See Fig. 412. But it is an experimental fact that there is only one angle of incidence at which the deviation is a minimum; and the only way in which this single value for the angle of incidence can be obtained is by making $i_1 = i_2$; from which it follows that $r_1 = r_2$ and hence that the ray inside the prism is symmetrical with respect to the two refracting surfaces.

The position of minimum deviation is one of great usefulness in work with the spectroscope, as well as in the accurate measurement of refractive indices.

SPECIAL CASE X. *Prism of Small Refracting Angle*

502. We have already seen (§ 500) that in general

$$\delta = (i_1 - r_1) + (i_2 - r_2)$$

and $$\alpha = r_1 + r_2.$$

By definition, $$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin i_2}{\sin r_2}.$$

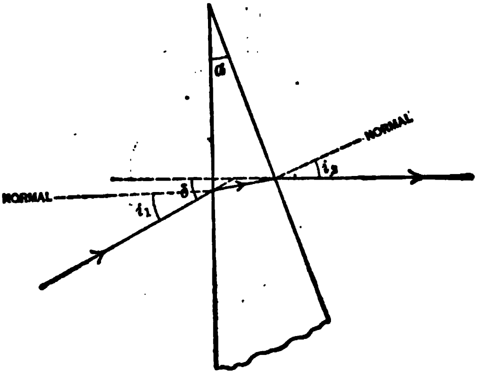

FIG. 413. — Refraction through a prism of small angle.

But when i_1 and i_2 are small, as indicated in Fig. 413, we may, approximately, write

$$\mu = \frac{i_1}{r_1} = \frac{i_2}{r_2}. \qquad \text{Eq. 198}$$

Eliminating i_1 and i_2 from these equations, we have

$$\delta = (\mu - 1)(r_1 + r_2) = (\mu - 1)\alpha, \qquad \text{Eq. 199}$$

a very convenient expression for computing the effect of thin prisms such as are used in spectacles and stereoscopes.

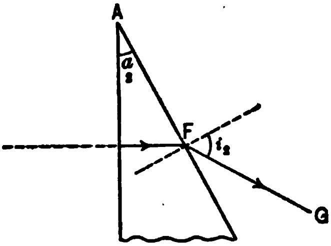

FIG. 414. — The measurement of refractive indices.

Determination of Refractive Indices

503. When any transparent material can be obtained in the form of a prism, the principle of minimum deviation enables one to measure the refractive index of the material with considerable ease. For since the path of the ray inside the prism is symmetrical, we need consider only one half of it, as shown in Fig. 414. The conditions described are as follows: —

$$i_1 = i_2 \text{ and } r_1 = r_2.$$

Hence Eqs. 196 and 197 become

$$\alpha = 2\,r$$

and

$$\delta = 2\,(i - r).$$

Hence

$$r = \frac{\alpha}{2}$$

and

$$i = \frac{\alpha + \delta}{2}.$$

Accordingly,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{\alpha + \delta}{2}\right)}{\sin\frac{\alpha}{2}}, \qquad \text{Eq. 200}$$

which is the laboratory equation for the measurement of μ. The problem then reduces itself to the measurement of the two angles α and δ. These measures can be made by use of a protractor and a few pins; but they can be carried out with a very high degree of accuracy by means of a divided circle and a pair of telescopes.

Dispersion. Color

504. Observe that the color which is produced by a prism is due, not to the refraction, but to the difference of the refraction for various rays. **This transformation of a beam of parallel light into a divergent pencil is called dispersion.** The measure of dispersion for any substance is generally defined as the ratio of the difference of deviation for red and blue rays to the mean deviation for these two rays.

This leads, through Eq. 199, to the following definition: —

$$\textbf{Dispersive Power} = \frac{(\mu_r - 1)\alpha - (\mu_b - 1)\alpha}{(\mu - 1)\alpha} = \frac{\mu_r - \mu_b}{\mu - 1}, \qquad \text{Eq. 201}$$

where μ_r, μ_b, μ are the refractive indices for the red, blue, and mean rays, respectively.

Composition of White Light

505. The fact that different rays travel at different rates enabled Newton to separate white light by means of a prism into its various constituents. For on transmitting a small beam of sunlight through a prism, he observed that the deflected light did not form a white spot on the wall beyond but was drawn out into a brilliant strip, red at one end and violet at the other. Between the red and the violet are distributed an infinite variety of colors which are usually grouped under the following four names: orange, yellow, green, blue. **But Newton's capital discovery in this domain is that any one of these colors after dispersion in one prism undergoes no further change of this kind on passage through a second prism.**

Table of Refractive Indices for Different Colors

Substance	Index of Refraction for		
	Red (C)	Yellow (D)	Blue (F)
Flint glass	1.5826	1.5867	1.5967
Crown glass	1.5311	1.5339	1.5404
Canada balsam	—	1.53	—
Oil, cinnamon	—	1.601	—
Oil, cloves	—	1.532	—
Fluor spar	1.4325	1.4340	1.4372
Water	1.3318	1.3336	1.3377
Carbon bisulphide	1.6336	1.6433	1.6688
Alcohol, ethyl	1.3596	1.3614	1.3657
Diamond	—	2.4699	—

Table of Dispersive Powers

Substance	$\frac{\mu_C-\mu_F}{\mu_D-1}$	Substance	$\frac{\mu_C-\mu_F}{\mu_D-1}$
Solids		*Liquids*	
Crown glass	0.0168	Carbon bisulphide	0.0545
Flint glass	0.0271	Alcohol	0.0171
Heavy flint glass	0.0461	Turpentine	0.0206
Fused silica	0.0147	Water	0.0180
Rock salt	0.0233		

From Kaye and Laby's *Tables.*

Newton succeeded also in compounding or adding together these various colors which he had found in sunlight, thus obtaining again white light; in this manner he demonstrated in a synthetic way the composition of ordinary daylight, a fact which he had already proved by analysis.

The majestic phenomenon of the rainbow he showed to be merely a case of refraction and dispersion in raindrops.

Problems

437. Find the focal length of a thin double convex lens made of glass whose mean refractive index is 1.50, the radius of curvature of the first surface being + 25 cm. and that of the second being − 30 cm.

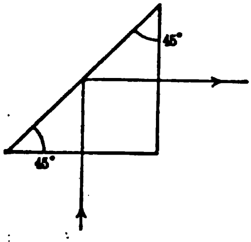

Fig. 415.

438. An object is placed on the axis of a double convex lens whose focal length is + 20 in. The object is placed 50 in. to the left of the lens. Find the position and magnification of the image.

439. The projection lens of a lantern (Fig. 427)has a focal length of 1 ft. and is used to throw a picture upon a screen which is 24 ft. away. How far back of the lens must one place the glass slide?

440. A 45-degree prism is used as a totally reflecting mirror in the manner indicated in Fig. 415. What is the least value of refractive index which the glass can have?

441. A gas flame placed in front of a thick glass mirror silvered on the back gives a series of images. Explain the relative intensity and position of these.

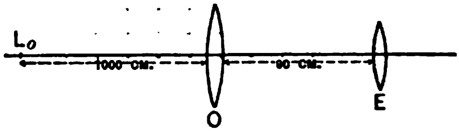

Fig. 416.

442. How far in front of a double convex lens must an object be placed in order to give an inverted image of the same size as the object?

443. A simple pocket magnifying glass has a focal length of 2 in. Find the size and position of the image when the object is placed between the lens and the focus at a distance of ½ in. from the focus.

444. Locate and describe the image produced by two thin lenses situated as shown in Fig. 416,

$$f_o = +80 \text{ cm.},$$
$$f_e = +4 \text{ cm.},$$
$$s_0 = -1000 \text{ cm.},$$

where the subscripts indicate the lenses referred to.

445. Locate and describe the image produced by the combination of thin lenses shown in Fig. 417.

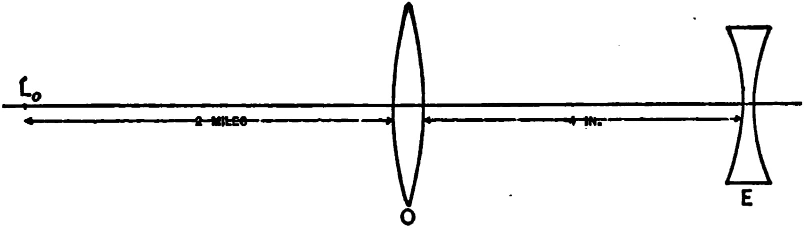

FIG. 417.

$$f_o = +6 \text{ in.}$$
$$f_e = -2 \text{ in.}$$
$$s_0 = -2 \text{ mi.}$$

446. What must be the refracting angle of a prism of ordinary glass ($\mu = 1.52$) in order that its minimum deviation should be 15°?

447. Prove that if $\mu = 1.333$ for water and 1.5 for glass, the focal length of a glass lens is increased four times by immersing it in water.

448. Prove that in the case of the ordinary pocket lens — double convex magnifying glass — the magnification is D/f, where D is the distance of distinct vision.

449. Two parallel walls are 20 ft. apart. It is desired to project upon one wall, with a magnification of 6, an object situated on the other wall. Where must the lens be placed and what must be its focal length?

VII. INTERFERENCE AND DIFFRACTION

506. Up to the present we have been considering the consequences of only two fundamental facts, namely, (i) the rectilinear propagation, and (ii) the finite, but variable, speed of light.

But there is a large group of important phenomena which find their explanation in the fact that the light waves producing them are compelled to pass through apertures which are small when measured in terms of a wave length of light. These beautiful phenomena were first cleared up by the brilliant young French physicist Fresnel (1788–1827), whose particular achievements in optics are: —

(i) The introduction of the idea that light consists in transverse vibrations, and

(ii) The combination of the principle of Huygens with that of interference; in other words, the union of the principles of Huygens and Young. Just how this second idea is employed to explain what happens when light passes through small apertures will be clear from the following illustrations.

CASE I. *Passage of Plane Waves through a Single Narrow Slit: Effect upon a Distant Screen*

507. When the glower of a Nernst lamp or any other nearly linear source of light is viewed through a narrow slit, held parallel to the source and close to the eye, a series of colored bands separated by dark bands something like those shown in Fig. 418 are seen. If monochromatic light is used, the spaces in the figure represented as white are colored. The central space is always broader and brighter than any of those on either side.

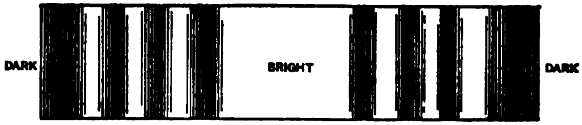

FIG. 418. — Image of a linear source produced by a narrow slit.

For this experiment a single ruling having a breadth of from $\frac{1}{20}$ to $\frac{1}{10}$ millimeter, made upon the back of a piece of silvered mirror, is well adapted.

It is often convenient also to observe this phenomenon by viewing a small brilliant source through the narrow opening between two fingers of one's hand. These bands were called by Fraunhofer, who was one of the earliest to study them, "spectra of the first class."

Fresnel's explanation is as follows: Let *AB* (Fig. 419) be a horizontal section across a vertical slit. Let *w* indicate the successive incident wave fronts, and *MM'* the screen illuminated by the light passing through the slit.

The width of the slit is indicated by s; the distance of the slit from the screen by b; the point on the screen which is nearest the slit by P_0. It is evident that, if the width of the slit is very small compared with the distance of the screen, the light *from each point* of the wave front AB, passing through the slit, will reach the point P_0 in the same phase; therefore P_0 will always be brightly illuminated. Next choose a point P, distant x from P_0, and such that its distance from the edge B exceeds its distance from the edge A by just one wave length of light; or, what is the same thing, let AC be drawn perpendicular to BP, and choose P so that BC is exactly one wave length. From the center O of the slit draw OP. Consider the two halves of the wave front just passing through the slit, AO and OB. Light from the point O will reach P just half a wave length behind that from A; and so for any other point in the region AO, there will be a corresponding point in the region OB which will send light to P just 180° out of phase with the ray from AO.

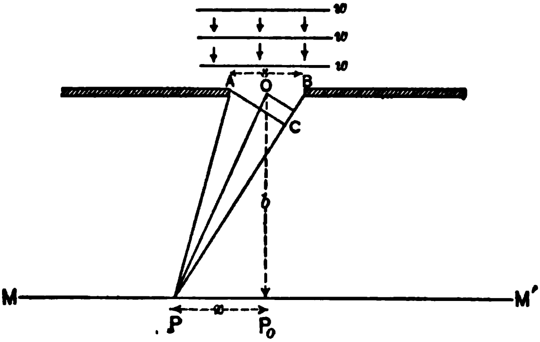

Fig. 419. — Diffraction through a slit.

The result is that these two "zones," AO and OB, interfere at P in such a way that one just annuls the effect of the other. The result is that P remains dark. This explains the first of the dark bands shown in Fig. 418.

Referring again to Fig. 419, imagine P to move still farther away from P_0 until the distance BC — generally called the *retardation* — is equal to three halves of a wave length — usually written $\frac{3}{2}\lambda$.

The wave front AB can now be divided into three "zones," such that the left hand edge of one will send light to P, just half a wave length earlier than the left hand edge of the *following* zone.

The result is that two of these neighboring zones will mutually annul each other, and the only illumination received at the point P will come from the third zone; such a point P is represented by the first *bright* band on the left of Fig. 418.

If P be moved again, still farther away from P_o, so that BC amounts to $\frac{4}{2}\lambda$, the slit width can then be divided into *four* zones, such as those described above. Since there will now be just two pairs of these zones, and since each pair is mutually destructive, P will now be dark. This explains the second *dark* band in Fig. 418.

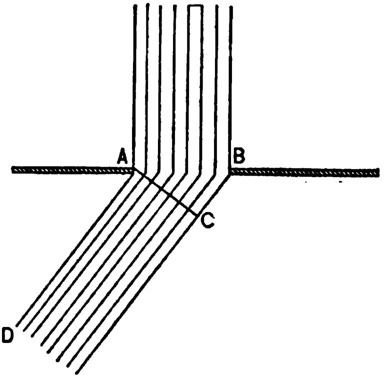

FIG. 420. — Fresnel zones.

It is now clear that the key to the situation lies in the division of the incident wave front into zones such that any point in one zone differs from the corresponding point in the neighboring zone, on either side, by exactly half a wave length. This clever idea belongs to Fresnel and these regions are called *Fresnel zones*. Figure 420 shows an aperture AB, divided into 7 Fresnel zones. A point on the screen lying in the direction AD will be illuminated in this case by only *one* zone, that is, by one seventh of the area represented by AB. The other six zones will destroy each other in pairs.

Referring again to Fig. 419, one may easily describe the position of any bright or dark band in a quantitative way as follows: since the triangles ABC and OPP_o are similar, it follows that

$$\frac{x}{b} = \frac{BC}{AC} = \frac{BC}{s}, \text{ approximately,}$$

where s is the width of the slit; and since there are n zones in the slit, it is evident from the definition of the zones that

$$BC = n\frac{\lambda}{2},$$

where λ is the wave length of the incident light. Hence

$$x = \frac{bn}{s} \cdot \frac{\lambda}{2}. \qquad \text{Eq. 202}$$

This equation tells us just how n varies as the point P moves along the screen, while b, s, and λ remain constant.

For any given position of P, either one of two things may happen, according as n is an odd or even number.

(*a*) *When n is even*, it is clear that all the zones of the slit will just annul each other in pairs. The condition for darkness at any point P is then that the slit, when divided into Fresnel zones with reference to P, shall comprise an *even* number of these zones. The value of x in Eq. 202 is the distance from the center at which any dark band, produced by n zones, appears.

(*b*) *When n is odd*, the zones of the slit will interfere in pairs as before; but *there will always be one left over* which will produce a disturbance in the ether at P. And hence the condition that P shall lie on a bright band is that the slit width be an *odd* number of Fresnel zones, *i.e.* n in Eq. 202 shall be odd.

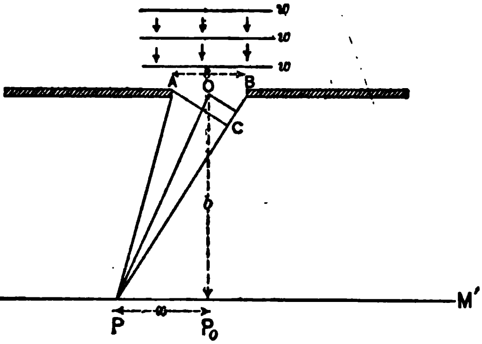

FIG. 419. — Diffraction through a slit.

Equation 202 is also competent to tell us what happens at any one point on the screen when the slit width is varied, other conditions remaining constant. As one jaw of the slit is made to approach the other it must cut off one Fresnel zone after another. As the zones diminish in number, n in Eq. 202 will become alternately odd and even; which, in turn, means that the fixed point on the screen becomes alternately bright and dark. This prediction is easily verified by viewing the filament of an incandescent lamp through a cut in a visiting card which can be bent so as to make the slit wider or narrower.

CASE II. *Diffraction of a Plane Wave at a Pair of Narrow and Equal Slits. Spectra of the Second Class*

508. Imagine two narrow slits, such as the one which we have just been studying, ruled side by side about $\frac{1}{5}$ millimeter apart. On viewing, through this pair of apertures, any linear source such as a Nernst filament or even an ordinary gas jet turned low, a series of alternately bright and dark bands are seen to occupy the center of the field, and to fall on the region covered by the broad central bright band which would be produced by either slit acting alone. We assume that the source is sufficiently distant from the apertures to make the incident waves practically plane, and that the screen, which may be the retina of the eye, is sufficiently distant to permit us to consider the arc *AC*, described about *P* as a center, as a straight line. To each element of the incident wave surface in the aperture *A* will correspond an element in the aperture *B*. The effect at any point *P* on the screen will be obtained when we add up the effects of all the elements in these two slits. Use red glass so as to make the incident light monochromatic. It is at once seen by the method of Case I that, if *P* is so chosen as to make $BC = \frac{\lambda}{2}$, we shall have darkness at *P*; for *each* element in the aperture *A* will reach *P* with a phase which is just 180° ahead of the corresponding element in *B*; the resultant amplitude at *P* will therefore be zero. The same thing will happen if *P* be moved along the screen to any position, say *x*, where *BC* is any odd number of half wave lengths,

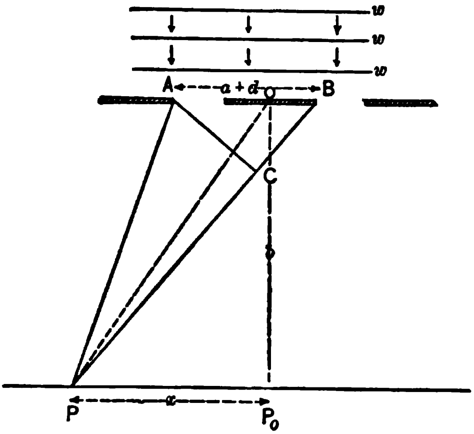

FIG. 421. — Diffraction through a pair of slits.

i.e. when $BC = (2n+1)\frac{\lambda}{2}$, n being the series of natural numbers, beginning with zero. The triangles ABC and OPP_0 are similar; therefore if we denote the distance between the slits by $a + d$, and the distance of the screen by b, we have

$$BC = (a+d)\frac{x}{OP} = (a+d)\sin\theta,$$

where θ is the angle of diffraction $P_0\widehat{O}P$. The condition that P shall lie on a *dark* band is therefore

$$(a+d)\sin\theta = (2n+1)\frac{\lambda}{2}. \quad \text{Condition for dark band.} \quad \text{Eq. 203}$$

The condition for a *bright* band is obtained in the same way, only now the difference of path between two corresponding elements — one from each slit — must be an *even* number of half wave lengths. For two rays of light which are any whole number of wave lengths apart will reënforce each other, and, if their amplitudes are equal, will produce an intensity four times as great as that of either one alone. The position x of a bright band on the illuminated screen is accordingly expressed as follows in terms of algebra: —

$$(a+d)\sin\theta = 2n\frac{\lambda}{2}. \quad \text{Condition for bright band.} \quad \text{Eq. 204}$$

Let us now remove the red glass and view these bands in white light. It is then seen that the bright bands are colored, bluish green on the side toward the center of the field and red on the side away from the center of the field. Each of these bands is in fact a spectrum of the source. Fraunhofer called these "spectra of the second class" to distinguish them from those which are seen through a *single* slit.

The distribution of intensity among these various spectra is an enticing problem for later study.

CASE III. *Diffraction through a Large Number of Equidistant Parallel Slits. The Plane Diffraction Grating*

509. Here again we shall consider the incident and diffracted wave fronts as plane, since this is the case principally used in spectroscopy.

Let *AE* (Fig. 422) represent an opaque screen in which have been cut a large number of equidistant parallel apertures. Let *a* be the width of each aperture and *d* the width of the opaque portion between any two consecutive apertures. Then $a + d$ will be what is called "the grating space," that is, the distance between any two corresponding elements of the wave surface in two consecutive apertures.

If the source of light is placed in the principal focus of the collimator lens, the emergent wave fronts w_1, w_2, etc., will be plane. In like manner, the diffracted wave *AD* will reach the principal focus of the view telescope without any change of phase in its various parts; for the "optical distance" to the principal focus *P* is the same from all parts of *AD* — it being supposed that the axis of the telescope is perpendicular to *AD*. Hence, the phase difference with which rays from the various apertures reach the plane *AD* is precisely the phase difference with which they will reach the focus *P*. We need, therefore, to consider only the phase in which the light reaches the plane *AD*.

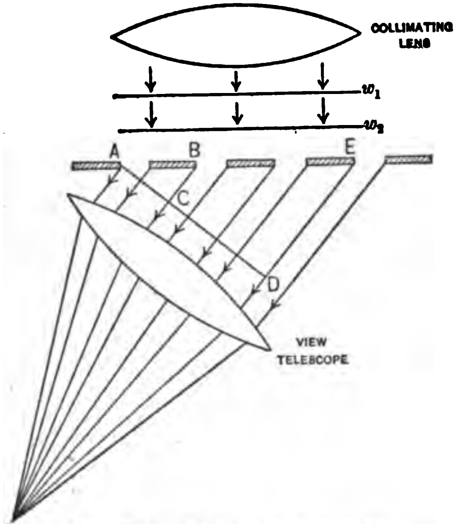

Fig. 422. — The diffraction grating.

510. Two principal questions now arise: (i) what angle must the axis of the view telescope make with the normal to the grating in order that, for any one wave length λ, the point *P* may be a maximum of intensity? and (ii) what is the intensity of light at any point *P*, in terms of the intensity of the light incident upon any one of the *n* apertures? The first only of these queries will be here considered; the answer to the second is a longer story.

It is evident from previous cases that when the telescope is directed normally upon the grating, the light from each aperture will reach P in the same phase and we shall have the greatest possible intensity. This bright band is called the "central image." As P moves to the left, say, the field will remain more or less bright until the distance DE (which is the retardation of the ray at one end of the grating over that at the other end) becomes equal to one wave length. When this occurs, the light from the aperture A will be just 180° out of phase with the light from the middle of the grating; in like manner the light from B will be just 180° out of phase with that from the ruling next to the middle, and so on, — the effect being that the light from one half the grating is annulled by the light from the other half. When $DE = 2\,\lambda$, the point P will again be dark; for then the grating will interfere by quarters, the light from each quarter being 180° out of phase with the light from the adjoining quarter, giving absolute darkness at P. At points between these there will be slight illumination; for instance, at the point where $DE = 3\frac{\lambda}{2}$; since now the first third of the grating will interfere destructively with the second third, leaving the last third to illuminate the point P. A slight maximum, due to light from one fifth of the grating, will also occur when $DE = 5\frac{\lambda}{2}$; and so on.

But the first intense illumination on either side of the central image will occur when DE is equal to as many wave lengths as there are spaces on the grating; for then CB will be equal to one wave length, and the retardation for each aperture will be a whole number of wave lengths so that the light from each aperture will reach the surface AD in precisely the same phase. The light which is thus concentrated at P is called the "spectrum of the first order." If the difference of phase between corresponding elements in two successive apertures is $n\lambda$, the light which then reaches P in maximum intensity is said to be the "spectrum of the nth order." So long, therefore, as the incident wave front is parallel to the grating, the condition that P shall be bright is that CB shall be equal to a whole number of wave lengths, or, in terms of algebra,

$$(a + d) \sin \theta = 2n\frac{\lambda}{2}, \qquad \text{Eq. 205}$$

where θ is the angle of diffraction, DAE.

If, however, the angle of *incidence* i is not zero (as in Fig. 422) but is an angle HAE such as indicated in Fig. 423, then the retardation of the extreme ray, between the two wave fronts AH and AD, is $HE + ED$. Hence, the general condition that P shall be bright becomes

$$(a + d)(\sin \theta + \sin i) = 2n\frac{\lambda}{2}. \qquad \text{Eq. 206}$$

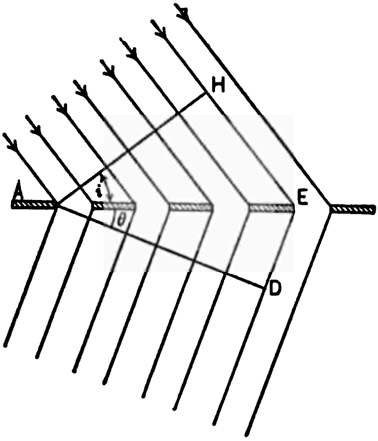

Fig. 423.

A grating of the kind just described, in which the diffracted light passes through parallel apertures, is called a **transmission grating.**

If the light falling upon such a grating were all of one wave length (*i.e.* of one color) then the spectrum of any order, on either side, would consist of a single bright line; but if the incident light is white, then the spectrum will consist of a band made up of many colors. For, in Eq. 205, each different value of λ will give a different value for θ.

Reflection Gratings

511. The opaque regions of a transmission grating are generally produced by ruling glass surfaces with a diamond point. But the diamond point deteriorates very rapidly during this work and hence it is customary to employ highly polished speculum metal for the ruled surface. The speculum metal is softer than the glass and will permit the diamond point to cut many more lines upon it than upon glass. A grating ruled upon a polished opaque surface behaves exactly as a transmission grating in which the incident wave is *the image of the actual wave* falling upon the opaque grating. Thus, in Fig. 424, where the grating is supposed to be one of speculum metal, the effect is precisely the same as if the incident wave AH were replaced by its image AH' and the grating were at the same time made transparent. In either case the retardation of the extreme ray is $HE + DE$, and the position of the spectrum of the nth order is given by

$$(a + d)(\sin i + \sin \theta) = 2\,n\frac{\lambda}{2}, \qquad \text{Eq. 206}$$

where, as above, i = angle EAH, and θ = angle EAD. Ruled surfaces at which the reflected wave front is diffracted are called **reflection gratings.**

The best gratings of the world are ruled on Rowland's engine at Baltimore, and on Michelson's at Chicago. In most cases they contain somewhere between five and twenty thousand lines to the inch; so that in many cases the polished surface between two adjacent rulings becomes vanishingly small. Notwithstanding this fineness Wallace, Thorpe, and Ives have shown that it is possible, by means of transparent celluloid, to make a "cast" of a reflection grating and to mount this cast upon a piece of plane glass in such a way as to give a very perfect spectrum. A transmission grating produced in this way, by using an opaque grating as a mold, is called a **replica.**

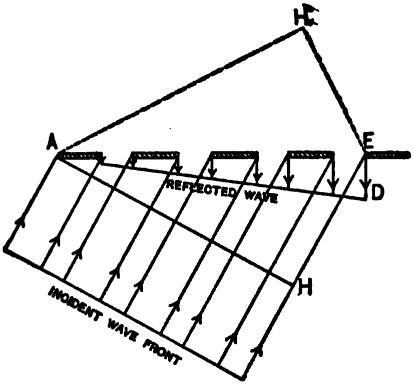

Fig. 424. — The reflection grating.

The Angstrom Unit

512. The numerical value of a wave length of light is usually expressed, not in terms of centimeters, but in terms of a much smaller unit introduced by the Swedish physicist Ångström, and hitherto called the "Ångström unit." Its value is very approximately the hundredth-millionth of a centimeter. But by action of the International Solar Union at Paris in 1907 this unit is now defined as $\frac{1}{6438.4696}$ of the wave length of the red cadmium ray in air at normal pressure and 15° C.; it is called the " angstrom," and is denoted by *A*, sometimes by *I.A*.

CASE IV. *Rectilinear Propagation of Light*

513. When, on a clear night, one observes a star with the naked eye, why is it that the plane wave front which strikes him in the face, so to speak, gives the impression of light coming from a single direction? How is it possible to screen off all of this wave front except a small portion immediately bounding the direction of the star without appreciably affecting the light which reaches the eye from the star? This is the great problem of the rectilinear propagation of light which even one hundred years ago was still demanding explanation.

The simple and direct answer which Fresnel gave, and which was perfected by Stokes near the middle of the nineteenth century, is based upon the principles of diffraction and interference which we have just been studying. The time at our disposal will not permit of the reproduction of their argument here; but the interested student will find himself easily able to follow the discussion of this subject in some of the larger compendia of physics. The same remark might be made concerning diffraction in the case of light passing a straight edge, a circular aperture, or a narrow wire.

Problems

450. A linear aperture whose width is $\frac{1}{500}$ cm. is placed parallel to and 60 cm. in front of a screen. If this aperture be illuminated with a beam of parallel rays whose angle of incidence is zero, and whose wave length is $\frac{1}{20000}$ cm., what will be the distance between the central image and the first dark band of the diffraction image on the screen?
Ans. 1.5 + cm.

451. A glass grating is ruled with 4250 lines to the centimeter. When a plane wave of yellow light strikes this grating at perpendicular incidence, it is observed that the spectrum of the second order is deviated (diffracted) through an angle of 30°. What is the wave length of the yellow light? *Ans.* $\lambda = 0.00005880$ cm.

452. A source of light emits two colors. When examined through a grating, it is found that the spectrum of the 4th order for one color is seen in the same direction as that of the 5th order for the other color. What is the ratio of the wave lengths for these two colors?

453. A certain spectrum of the 2d order is produced by a grating which is ruled with 400 lines to the millimeter. Another spectrum of the same source but of the 4th order is produced by a grating which contains 300 lines to the cm. Compare the lengths of these two spectra.

454. Two point sources of light, each in the same phase, are situated at a distance of $\frac{1}{20}$ mm. apart. If the wave length of the light which they emit is 6500 A, how far apart will be the spectra of the second class on a screen one meter away from the sources and parallel to the line joining the sources?

455. How much will the phase of a ray of light be retarded by inserting in its path a cover glass one millimeter thick having a refractive index of 1.6? Assume the light to be that of the yellow sodium flame for which $\lambda = 5893\ A$.

456. Prove that, if the opaque spaces are exactly equal to transparent spaces in a transmission grating, spectra of the fourth order will be missing.

"Nothing is too wonderful to be true."
FARADAY.

CHAPTER XIII

OPTICAL INSTRUMENTS

I. *The Photographic Camera*

514. A photographic camera, which is perhaps the simplest and best known of all optical instruments, is merely a box fitted at one end with a converging lens and at the other end with a device for holding a plate which is sensitive to light. Etymology of *camera?* The sides of this box are usually made of folded leather so that the distance of the plate *P* (Fig. 425) from the lens *L* may be varied at will.

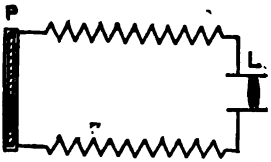

FIG. 425.—Section of photographic camera.

The rear end of the camera is furnished also with a ground glass which may be placed in the same position that the sensitive plate is to occupy later.

The *first* step in taking a photograph consists in placing this ground glass in such a position that the lens will produce upon it a sharp image of the object whose photograph is desired. This is called "focusing."

The *second* step is to cover the lens so that no light can enter the camera; then put the sensitive plate in the position formerly occupied by the ground glass and "expose" the plate by again uncovering the lens for the proper length of time.

The *third* step is to "develop" and "fix" the plate. The lights and shadows are now just reversed from what they are in the object. The plate in its present condition is therefore called a "negative."

The *fourth* step, called "printing," again reverses the lights and shadows and gives the finished picture, which is called the "positive."

A good photographic lens is one: —

(*a*) Which has the same focal length for yellow as for violet light; yellow rays being those which are principally used in focusing on the ground glass, violet rays being those which most affect the sensitive plate. In other words, the lens must be corrected for chromatic aberration (§ 499).

(*b*) Which has the same focal length for the edges as for the center of the lens. When this is so, the whole of the lens can be used, and the photograph taken with a short exposure. Such lenses are said to be " quick "; they are corrected for spherical aberration.

(*c*) Which produces an image distinct, not only at the center, but clear out to the edges of the sensitive plate for which it is intended. Such a lens is said to have a " flat field."

(*d*) Which gives the same magnifying power at the center and at the edge of the photographic plate. If the lens is not carefully corrected so as to give this result, buildings and geometrical figures will be distorted. A lens so corrected is said to be " rectilinear."

For special purposes, such as portrait or landscape photography, it is obviously unnecessary that the lens should possess all of these four qualities.

II. *The Human Eye*

515. The eye, from an optical point of view, is a camera in which the sensitive photographic plate is replaced by a sensitive membrane, called the **retina.** This retina is connected with a nerve called the **optic nerve,** which conveys the impression to the brain.

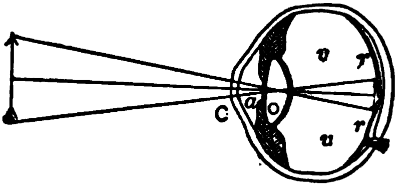

Fig. 426. — Longitudinal section of the human eye.

In the front part of the eye is placed a converging lens *O*; this is called the **crystalline lens.** This lens projects upon the retina an image of external objects. The distance between this lens and the retina remains practically constant. How, then, can the eye see objects which lie at different distances from the eye? For it is evident from

Fig. 409 that the distance of the image from the lens ordinarily varies with the distance of the object from the lens. The answer to this question lies in the fact that the muscles of the eye are able to change the focal length of the crystalline lens at will. The power of this lens is thus varied automatically to suit the distance of the object. This process of changing the focal length of the eye to suit the distance of the object is called **accommodation.**

Immediately in front of the crystalline lens hangs a dark-colored diaphragm, in the center of which is an opening called the *pupil*. This aperture varies with the amount of light falling upon the eye. As the light grows brighter, the pupil contracts; as the light grows weaker, the pupil enlarges. Examine the pupil of your own eye in a mirror after remaining for a few minutes in a dark room; in a moment after coming into the light the pupil contracts again to its natural size.

Every student should examine carefully one of these little pasteboard models, called "skiascopic eyes"; nothing except perhaps an actual dissection will make the structure of the eye so plain. The model is provided with a lens which enables one to easily examine its retina with the ophthalmoscope of Helmholtz, and to try the effect of various spectacle lenses. It is interesting to note that the only kind of light which the eye can use for vision is a convergent or at most a parallel pencil.

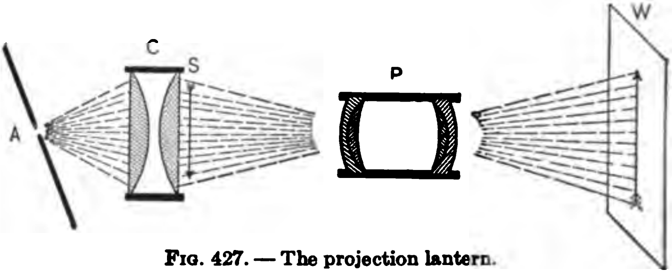

FIG. 427. — The projection lantern.

III. *The Projection Lantern*

516. This instrument consists essentially of three parts, namely: —

(i) A brilliant source of light A (Fig. 427); preferably a direct-current arc or a nitrogen-filled incandescent lamp with concentrated filament.

(ii) A pair of large short focus lenses *C*, of large aperture, which shall distribute this strong light uniformly over the object *S* which is to be projected, and

(iii) A fairly well-corrected lens *P*, which throws upon the screen or wall *W* an enlarged image of the picture on the glass slide. This second lens is generally of the rectilinear type and is called the "projection" or "focusing" lens.

All that here needs special mention is the function of the condensing lens *C*, which has nothing to do with producing the picture on the screen, except to deflect towards the axis those rays which would otherwise pass through the edge of the slide and never enter the projection lens. If the condenser is removed, the effect is to diminish the area of the slide which is visible on the screen. It is absolutely essential that the convex surfaces of these lenses should be placed facing in, as shown in Fig. 427. Evidently the slide and the screen must be in conjugate foci of the projection lens. If the object to be projected is opaque, the light *A* and the lens *P* must of course both be on the same side of the object.

IV. *The Simple Microscope*

517 This instrument consists usually of a single converging lens of short focal length. The distinguishing feature of the simple microscope is that the lens is always placed at a distance from the object which is less than its focal length. Now, if the object *O* (Fig. 428) were placed exactly at the focus, the power of the lens would be just sufficient to flatten out the curved wave front wf_1, so that it would emerge as a plane wave front. But if the object be placed between the focus and the lens, as in this case, wf_1 will be too convex for the lens to over-

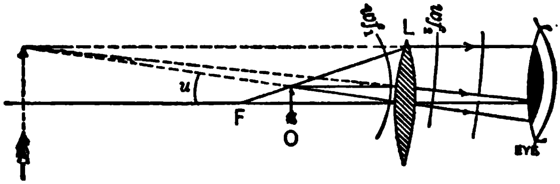

FIG. 428. — The simple microscope.

come its curvature entirely; the emergent wave front wf_2 will therefore still be convex on its advancing side. This image I will therefore lie on the same side of the lens as the object O. The image will, therefore (§ 475), be virtual and erect. Since the image and the object subtend the same angle θ at the center of the lens, it is evident that the image will be larger than the object in the following ratio: —

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{Distance of image from lens}}{\text{Distance of object from lens}}.$$

But since the lens is always held very near the eye, the distance of the image is practically the distance of distinct vision, say 10 inches; and since the object is always held very near the principal focus of the lens, its distance from the lens is practically the focal length. Denote this focal length by f, and the distance of distinct vision by v; then the magnifying power of the simple microscope may be described in terms of algebra as follows: —

$$\frac{I}{O} = \frac{v}{f}, \quad \text{Equation of simple microscope.} \qquad \text{Eq. 207}$$

where O is a linear dimension of the object, and I the corresponding linear dimension of the image.

We might, of course, bring an object up close to the eye so that it would subtend an angle as large as that of the image seen through the microscope, but unfortunately the human eye cannot accommodate for these short distances. The necessity for the microscope arises then from our limited power of accommodation.

Distinguish carefully between the "power" of a lens (§ 495), which is the reciprocal of its focal length, measuring its ability to change the curvature of a wave surface, and its "magnifying power," which is the ratio of the size of the image to the size of the object.

V. *The Compound Microscope*

518. The compound microscope is an instrument which is rendered necessary by the fact that the magnifying power of the simple microscope cannot be pushed very far. As indicated in Fig. 429, the essential features of this instrument are: —

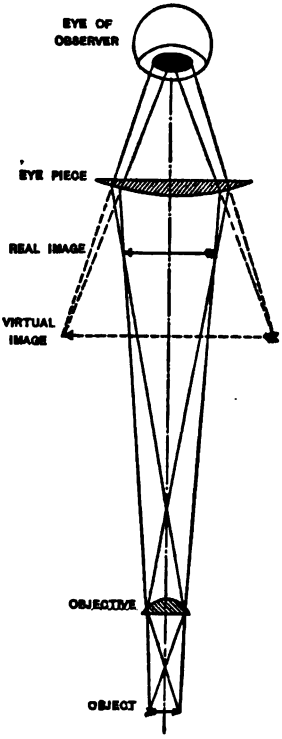

FIG. 429. — The compound microscope.

(i) A very perfect, but highly complex, lens placed close to the object and giving a magnified real and inverted image of the object. From its proximity to the object, this lens is called the **objective**.

(ii) The other essential of the compound microscope is a simple microscope with which to examine the real image produced by the objective. From its proximity to the eye, this lens is called the **eyepiece**.

If we denote by F the focal length of the objective and by T the length of the tube of the microscope, the magnification which the objective produces is practically T/F; for T and F are the respective distances of image and object from the objective. As we have seen above (§ 517), the magnification which the simple microscope — here the eyepiece—produces is the distance of distinct vision divided by its focal length v/f. Consequently the magnifying power of the compound microscope may be described as follows: —

$$\frac{I}{O} = \frac{Tv}{Ff}. \qquad \text{Eq. 208}$$

Equation of compound microscope.

The perfection of the modern compound microscope is due very largely to the late Professor Abbe of the University of Jena, working in conjunction with the optician Carl Zeiss. Its complete theory forms several of the most beautiful chapters in geometrical and physical optics.

VI. *The Astronomical Telescope*

519. The astronomical telescope is an instrument used for viewing very distant objects. The rays of light which fall upon the telescope are, therefore, generally parallel.

The purpose of the telescope is twofold: —

(1) To gather a large amount of light and condense it into a small bundle of rays, so that these can all enter the pupil of the eye, and thus make any luminous *point* appear much brighter than it would to the naked eye.

(2) To magnify the angle subtended by two luminous points, and thus make them appear farther apart than when seen by the naked eye.

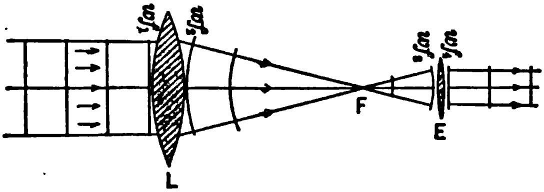

FIG. 430. — The astronomical telescope.

These purposes are accomplished most simply by two converging lenses, *L* and *E*. The lens *L* is called the object lens, or simply the **objective**; the lens *E* is called the eye lens, or, more frequently the **eyepiece**.

The wave front wf_1 sent out by a star is practically plane when it strikes the objective, as is indicated in Fig. 430. The effect of the objective is to turn this plane wave front into a spherical one wf_2. Let *F* denote the center of curvature of the wave front wf_2. This is called the principal focus of the telescope. The eye lens is so adjusted that its principal focus also lies at *F*. **Therefore the rays which emerge from the eye lens are again parallel.**

The effect, therefore, of a large telescope, such as that at the Lick Observatory or at the Yerkes Observatory, is to take a bundle of parallel rays forming a cylinder a yard or so in diameter and condense it into a small cylinder of parallel rays just large enough to enter the pupil of the eye. So that a star (which is practically a luminous point) appears immensely brighter when viewed through a telescope than it does to the naked eye.

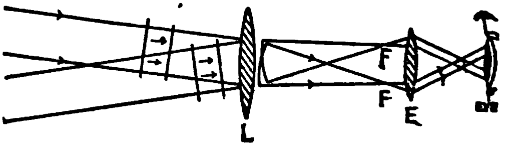

FIG. 431. — The telescope as used to magnify an angle.

520. If two stars close together, say a double star, be viewed through a telescope, *each* will send a beam of parallel rays through the instrument, as indicated in Fig. 431. **But the two parallel beams which emerge from the eye lens make with each other a much larger angle than do the two parallel beams which enter the objective. The telescope thus magnifies the angular distance between two stars.** It thus enables us to recognize certain stars as double when to the naked eye they appear single. For we can only distinguish two luminous points as separate when they subtend at the eye an angle which is greater than about two minutes of arc.

VII. *The Opera Glass*

521. The earliest telescopes did not employ a converging eyepiece such as those represented in Figs. 430 and 431; but instead they used a double concave eye lens, the power of which was equal to and opposite in sign to that of the human eye. The telescope with which Galileo discovered four satellites of Jupiter is shown in Fig. 432. On account of its compactness and because it gives an upright image, this same principle is employed in the modern opera glass.

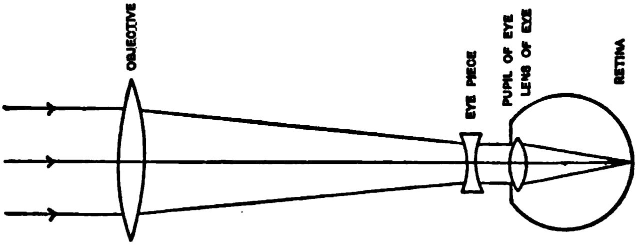

FIG. 432. — The opera glass.

If we assume what is nearly true, namely, that the eyepiece produces a curvature of the incident wave front which is exactly equal and opposite that produced by the human eye, then by placing the objective of the opera glass at such a distance that its principal focus lies on the retina, we shall see the object distinctly; and since the image is much larger than that produced by the naked eye, we shall see the object magnified.

In short, the eyepiece and the eye act together, as if they constituted a piece of plane glass.

The ordinary opera glass here described has been immensely improved in recent years by the introduction of a pair of prisms between the eyepiece and the objective. Just how the introduction of the prisms allows one to put into the small compass of an ordinary opera glass an instrument of nearly three times its focal length will be clear from an inspection of Fig. 433. This increased focal length permits of greater magnifying power. The two totally reflecting prisms which double the ray back upon itself serve also to make the image erect. Accordingly a converging lens is used in the eyepiece.

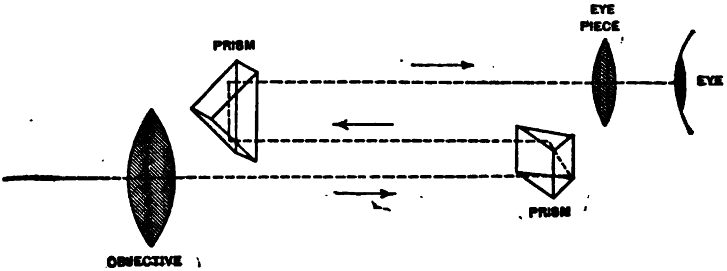

FIG. 433. — The prism binocular.

This instrument, which we owe largely to J. Porro and E. Abbe, is called a *prism-binocular*. It has the marked advantage of giving a field which is uniformly sharp clear out to the edge.

VIII. *The Interferometer*

522. The interferometer is an instrument devised by Michelson (1881) for the study of aberration; it is now employed for a great variety of purposes, but its fundamental use is perhaps the accurate measurement of small distances. As the name would imply, the principle involved is that of the interference of light waves. But, as we shall see presently, the interference is not between two single point sources or two single linear sources, as in § 463, but between pairs of corresponding points which, taken together, constitute *two extended wave surfaces*.

523. Just what is meant by this will be clearer after the student has repeated the following experiment: Take two small strips of ¼-inch glass plate and clamp them together at one end, as shown in section in Fig. 434. At the other end *DB* place a single thickness of tissue paper between the plates. Now examine these plates in the light reflected from some monochromatic source, such as a sodium flame or mercury arc. The region between the two plates will be seen filled with a large number of alternately bright and dark lines. The explanation of these "interference bands," as they are called, is perhaps most simply given in terms of Fig. 435, which is taken from an article by Dr. Henry G. Gale in the *American Machinist*, July 11, 1901.

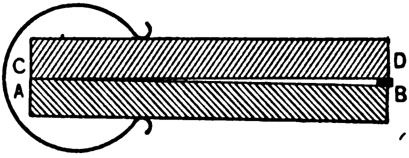

Fig. 434.—A simple interference experiment.

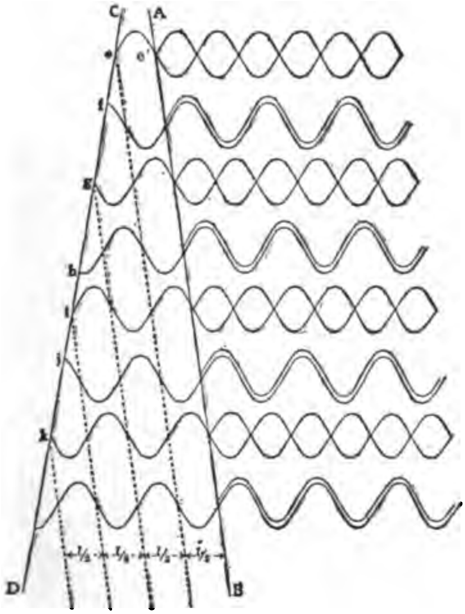

Fig. 435. — Illustrating interference between two large wave fronts.

Imagine *AB* and *CD* to be the two glass surfaces inclosing the wedge-shaped air film in Fig. 434. The fine parallel dotted lines in Fig. 435 are drawn ½ wave length apart so that one can estimate the thickness of the film *at any point* to within a small fraction of a wave length, by merely looking at the diagram. Next imagine a ray of light coming from the right and incident upon the surface *AB* (the inner surface of the glass), at the point *e'*; part of the light will be reflected from *AB* and part of it will be refracted into the air film, and there reflected from the surface *CD*. These two rays are now in a condition to interfere; for they each have their origin in the same luminous particle.

Let us choose the point *e* such that the thickness of the film at that point is just one half wave length, and then ask what the retardation of the second train of waves is when compared with the first. It follows that the total retardation due to the air film is λ, where λ is the wave length of the light employed. The same thing happens, of course, at points such as *g*, *i*, *k*, etc., where the thickness of the air film is 2, 3, 4, etc., any whole number of half wave lengths. So far, therefore, as the difference in path between the two rays is concerned, they would be in the same phase, and hence reënforce each other, whenever the air was any whole number of half wave lengths in thickness. But consider what happens at *f*, where the air film has a thickness of $\frac{3}{4}\lambda$. Here the retardation due to one round trip across the film is $\frac{3}{2}\lambda$; in the same manner it will be seen that at *h* and *j* the difference in phase is $\frac{5}{2}\lambda$ and $\frac{7}{2}\lambda$, respectively; in other words, the two trains here differ by an odd number of half wave lengths, and ought therefore to interfere destructively, giving rise to a dark band. We are thus led to expect exactly such a series of alternate bright and dark bands as those observed with the two pieces of plate glass in Fig. 434.

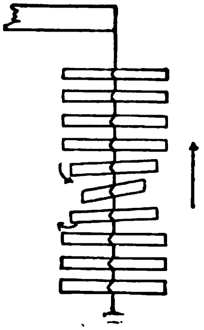

FIG. 128. — Kelvin's wave model.

524. But oddly enough, experiment shows that the black band occurs at exactly the position where the above view would lead us to expect a bright band. The one consideration which is here needed to complete the theory, is very clearly set forth by Professor Michelson in terms of Kelvin's wave model, as follows: "This discrepancy is due to the assumption that both reflections took place under like conditions, and that the phase of the two trains of waves would be equally affected by the act of reflection. This assumption is wrong, for the first reflection takes place from the *inner* surface of the first glass, while the second occurs at the *outer* surface of the second glass. The first reflection is from a rarer medium — the air; while the second is from a denser medium — the glass. A simple experiment with the Kelvin wave apparatus (§ 186) will illustrate the difference between the two kinds of reflection. The upper end of this apparatus is fixed, while the lower end is free; the fixed end, therefore, represents the surface of a denser medium, the free end that of a rarer medium. If now a wave be started at the lower end, by twisting the lowest element to the *right*, the twist travels upward till it reaches the ceiling, whence it returns with a twist to the *left*, *i.e.* in the *opposite* phase. When, however, this *left* twist reaches the lowest element, it is reflected, and returns as a twist to the *left* — so that the reflection is in the *same* phase.

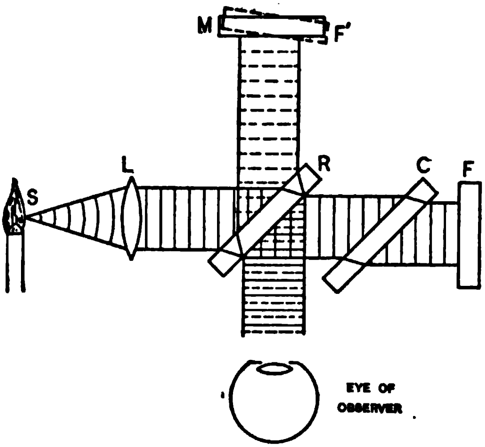

FIG. 436. — Diagram of Michelson's interferometer.

"There is thus a difference of phase of one half a period between the two reflections, and when this is taken into account, experiment and theory fully agree." — *Light Waves and Their Uses*, p. 16.

Accordingly we see that destructive interference occurs when the thickness of the film is any even number of quarter wave lengths; reinforcement occurs when the thickness of the film is any odd number of quarter wave lengths.

525. Now, instead of thinking of these two surfaces *AB* and *CD* (Fig. 435) as made of glass, let us imagine them produced as follows: A Bunsen flame containing sodium is placed in the principal focus of a converging lens *L*. A certain luminous particle *S* in this flame emits a spherical wave. The lens *L* transforms this spherical wave into a plane wave, as indicated in Fig. 436.

This plane wave strikes a fixed plane parallel plate of glass *R* at an angle of 45°. The rear surface of this plate is thinly silvered. Part of the light, indicated by solid lines, is therefore transmitted; part of it, indicated by dotted lines, is reflected. The *transmitted* wave front proceeds next through a plane parallel plate *C* and is reflected from a plane silvered mirror *F*. It now retraces its path to the silvered surface of the mirror *R*, where it is reflected to the eye as shown in Fig. 436. The reflected wave front proceeds to a movable mirror *M*, which is carried on the nut of a screw whose axis lies in the direction joining the mirrors *R* and *M*. From the mirror *M* the wave front is reflected normally and is transmitted by *R* so as to enter the eye from the same direction as the other wave front. In brief the incident wave front from the source *S* is split up by the mirror *R* into two wave fronts which enter the eye in a condition for interference. The wave front from the fixed mirror *F* behaves exactly as if it came from its image in the mirror *R*, *i.e.* from a position indicated by the dotted line *F'*. Any wave front leaving *M* and the corresponding wave front from *F'* may therefore be regarded as a pair of wave fronts replacing the two which were reflected from *AB* and *CD* in Fig. 435; only these two surfaces are not fixed as were *AB* and *CD*. Here the mirror *M* can be moved with the screw so as to make these two surfaces coincide, or so as to put either one in advance of the other. Not only so, but one observes that as the mirror *M* advances, the fringes (interference bands) move

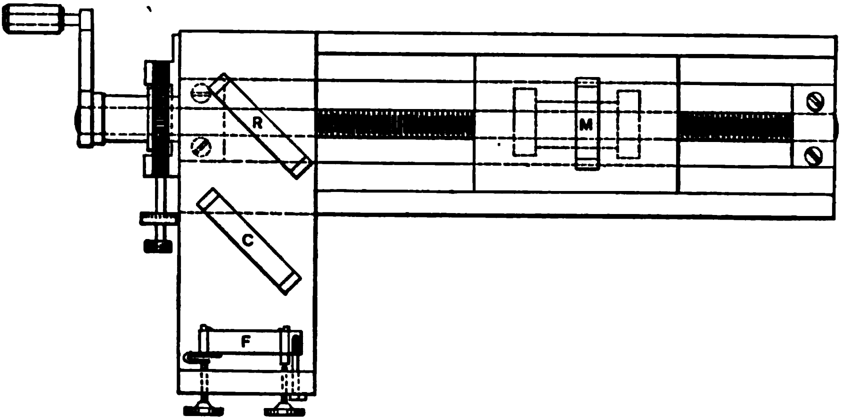

FIG. 437.—Michelson's interferometer.

across the field at the rate of one fringe for each half wave length of advance in the position of the mirror. Note that the lettering in Fig. 437 is precisely the same as that used in the immediately preceding figure.

In this manner Michelson has succeeded in using the wave length of light as a new unit of length for the accurate measurement of very minute distances. In 1892 he, in conjunction with Benoit, measured the standard meter at Paris in terms of the wave length of the red cadmium ray and found, with an error probably less than one part in a million, that the meter is equal to 1553163.5 such wave lengths. The moral effect of Michelson's determination was to render the red cadmium wave length our standard of length.

The extraordinary accuracy of these measures is firmly established by the recent work (1907) of Benoit, Fabry, and Perot, three eminent Frenchmen who, using slight improvements, find the meter to contain 1553164.13 red cadmium waves; or

Wave length of red cadmium ray = $6438.4696 \times 10^{-10}$ *meter.*

This is the value adopted by the International Solar Union at Paris in 1907.

IX. *The Spectroscope*

526. If one observes a Bunsen flame through an ordinary glass prism held close to the eye, he sees a succession of colored images of the flame. This series of colored images is called the **spectrum** of the flame.

An instrument such as the prism which will separate the various radiations emitted by any source of light is called a **spectroscope.** Accordingly we may define **spectroscopy** as that branch of physics which has for its object **the determination and description of the various radiations which bodies emit, reflect, and absorb.**

Sources of Radiation

527. In practice we are limited to two artificial sources upon which we can *experiment* — flames and electric currents. In addition to these there are a number of natural sources whose occurrence is so far beyond our control that we can only *observe* them. The following classification may be used to summarize these facts: —

- SOURCES
  - Natural
    - Lightning
    - Stars, including the Sun
    - Comets
    - Auroras
    - Meteors
    - Glowworms
  - Artificial
    - Flames
      - Candle
      - Alcohol
      - Bunsen
      - Oxyhydrogen
    - Electric Currents
      - Arc
      - Spark
      - Brush discharge
      - Electric furnace

The **Bunsen flame**, devised by the German chemist of that name in 1856, has three marked advantages as a spectroscopic source, namely: —

(i) It is simple in construction and operation.

(ii) Its outer mantle is almost totally devoid of any characteristic spectrum of its own.

(iii) It possesses a high temperature, not less probably than 1788° C.

The **electric arc**, with a temperature lying probably between 3500° and 3900° C., is exceedingly efficient as a source because it easily volatilizes any substance placed in it.

The **electric spark**, obtained by the use of the induction coil, is the source of radiation which is most generally convenient and is most widely employed. In general, the electric spark produces two spectra, namely, the spectrum of the electrodes and the spectrum of the air or other gas between the two electrodes. But it was discovered by the German mathematician Plücker, in 1858, that the spectrum of the electrodes may be avoided by inclosing them in a partial vacuum. Accordingly, the universal method of studying the spectra of gases is now to seal platinum or aluminum electrodes into a glass tube and then fill the tube under low pressure with the gas to be studied. The converse problem of getting the spectrum of the electrodes without that of the air between them has recently been solved by Schuster and Hemsalech, who have proved that the air ceases to become luminous when a sufficient amount of inductance (electrical inertia) has been placed in the circuit through which the spark passes.

Separation of Radiations

528. Let us suppose now that we have before us one or more of the above-mentioned sources which we wish to examine. The next step will be to separate the rays of various wave lengths so that we may examine them individually. For the student may here assume, what is the experimental fact, that each different chemical element introduced into the flame or arc gives a different spectrum, and one which is peculiar to itself.

The Prism Spectroscope

529. If the source of light be small or linear, and if it is necessary to make only a hasty visual examination, the most convenient plan is to view the source through a prism placed immediately in front of the eye, as indicated in Fig. 438. The chief difficulty in the naked prism is that the source, however small, is still so large that the successive colored images formed by the prism will overlap each other, leaving the separation incomplete. To avoid this difficulty a small astronomical telescope — called a collimator — is placed between the prism and the source, as shown in Fig. 439. In the principal focus of its objective is placed a narrow straight slit with movable metal jaws. The source is now placed immediately back of this slit, or a real image of the source is focused upon the slit by means of a lens called the image

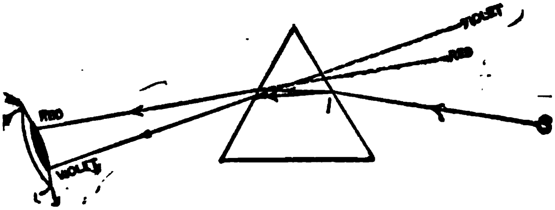

FIG. 438.—A simple form of spectroscope.

lens. When the slit is properly placed and illuminated, each point of it will emit through the lens of the collimator a beam of nearly parallel light, whose cross section is the **effective aperture** of the objective. If now this emergent beam be examined through a prism by the naked eye as before, a series of colored images of the slit will be seen, each appearing to be at an infinite distance from the observer; and if the slit is narrow, these images will be sharp and fine and easily separated. But they will not in general appear very bright, because the human eye is too small to admit more than a fraction of the emergent beam. Accordingly, these colored images, each at an infinite distance, are viewed through another small telescope called the **view telescope.** This serves to condense the entire beam which emerges from the prism and to reduce it to another parallel beam just large enough to fill the aperture of the average human eye.

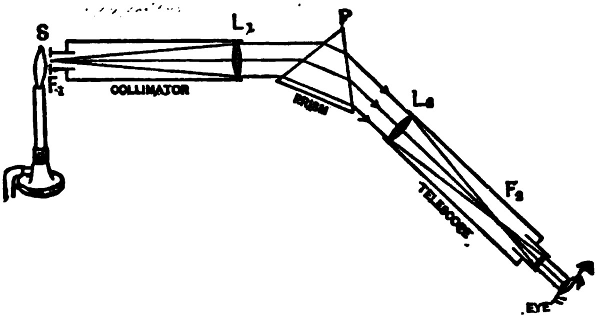

FIG. 439. — A prism spectroscope adjusted to view one particular color.

The Spectrograph

530. When it is desired to photograph a spectrum, the view telescope is removed and is replaced by a camera provided with a photographic objective. Such an instrument is known as a **spectrograph,** a convenient form of which is shown in Fig. 440.

The Spectrometer

531. When the typical spectroscope represented in Fig. 439 is provided with a graduated circle and so arranged that the view telescope and the prism can rotate about the axis of the circle, the instrument is known as a **spectrometer**, and can be employed for the measurement of prism angles (§ 503) and wave lengths (§ 510) as well as for the examination of spectra.

In the prism spectroscope it is nearly always necessary for the sake of good definition to place the prism in such a position that it will produce a minimum deviation for the pencil of rays under observation. For it may be shown by geometrical optics that, if the prism be placed in any other position, the image of a point source will not be a point, but a line.

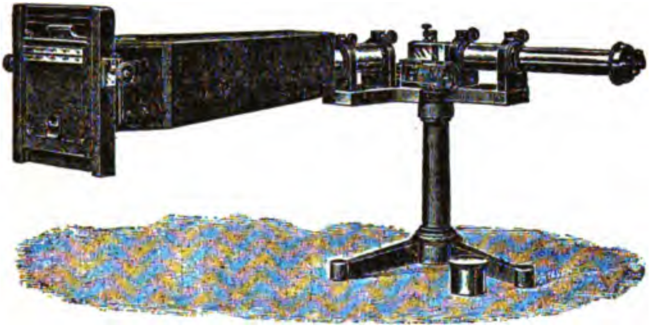

FIG. 440. — Hilger's spectrograph.

The Grating Spectroscope

532. So far as auxiliary parts are concerned, this piece of apparatus is very like the prism spectroscope; but as regards its essential feature, the dispersing piece, the two instruments are radically different. For the dispersion which is produced by a prism rests ultimately upon the fact that the waves of longer length travel through glass with a higher speed than those of shorter length; while the dispersing effect of a grating is due to the fact that when waves of different lengths are forced to pass through a narrow opening, they are all spread out like a fan, — diffracted, — the longer waves being more spread out, however, than the shorter ones.

If a gas flame, turned low, or the filament of an incandescent lamp be viewed through a single narrow slit in a visiting card, a a number of spectra are seen on each side of the bright central image. These have been called by Fraunhofer "spectra of the first class." They are not used in spectroscopy. If, however, the same source be viewed through two or more fine parallel equidistant slits placed close together, say less than $\frac{1}{64}$ inch apart, another series of spectra will be observed. These are the combined effect of light passing through several openings; they are known as "spectra of the second class" and are practically the only ones used in the grating spectroscope.

These narrow equidistant parallel slits were first prepared by the German optician Fraunhofer, at Munich, in 1821. He made them by wrapping fine wire over the threads of two parallel screws, then soldering the wire to the screws, and afterwards cutting away one entire side of the little cage thus formed. Such a device he called a **grating**. The modern grating is made by ruling straight lines with a diamond point on a flat plate of highly polished speculum metal. Here the light is reflected from the unruled surface between two furrows made by the diamond; and diffraction occurs exactly as if the light had passed through a narrow aperture of the same width. Such a device is called a **reflection grating**, and when placed where the prism stands in an ordinary prism spectroscope, we have what is known as **a grating spectroscope**. The student should now review those sections of this book which deal with the passage of light through narrow apertures.

When a beam of parallel light emerges from the collimator lens and falls upon a grating placed with its rulings parallel to the slit of the collimator, part of the light is reflected according to the laws of ordinary reflection, and forms in the focal plane of the view telescope what is known as the **central image**. The remainder of the light, however, is spread out on either side of this central image, the blue light being deviated least and the red light (longer waves) being deviated most. Unlike the prism, the grating yields several spectra, distributed on each side of the central image.

If monochromatic light be viewed with such an instrument, a series of bright lines — sharp maxima — each an image of the slit, will be seen on each side of the central image. The image nearest the central, on either side, is said to be the "spectrum of the first order," the next is called the "second order spectrum," etc.

Position of Spectra

533. Three questions may now be fairly asked about these spectral lines. The first is as follows: —

Given light of wave length λ, what will be the position of its line in a spectrum of, say, the nth order? The answer to this question is completely given in the following equation

$$(a + d)(\sin i + \sin \theta) = n\lambda, \qquad \text{Eq. 206}$$

which has already been derived in § 510. This equation, solved for θ, tells one in exactly what direction (angular position) he must look for the spectrum of the nth order when the incident light, of wave length λ, makes an angle of incidence i, upon a grating whose rulings are $a + d$ centimeters apart.

Resolving Power of a Grating

534. A second question is this. What is the smallest difference of wave length which two lines may have and yet be recognized as double by use of any particular grating?

It is not difficult to show that the sharpness of the image of any linear source of light, such as the illuminated slit of a spectroscope (§ 529), viewed through a grating, varies directly as the number of lines on the grating. The same source presents very different appearances according to the number of apertures through which it is viewed. If ordinates represent intensities and abscissas wave lengths, (a) in Fig. 441 will illustrate the broad diffuse appearance of a line when seen through a grating of few rulings: (b) and (c) show the same line through gratings which contain more lines. The reason for this is as follows: suppose the spectrum of the nth order to be seen in any direction θ; then if the incident ray be

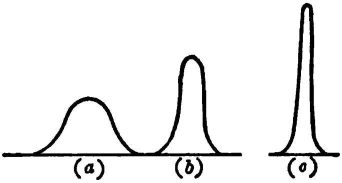

FIG. 441. — Illustrating the effect of small and large resolving power.

normal to the grating, we have

$$(a + d) \sin \theta = n\lambda. \qquad \text{Eq. 205}$$

If N be the total number of lines in the grating, we have $N(a + d) \sin \theta = Nn\lambda =$ retardation of last ray over the first ray. This retardation would amount to ED in Fig. 422. Suppose now that the direction of view be changed from θ to θ', a very small amount, so that

$$N(a + d) \sin \theta' = Nn\lambda + \lambda = Nn\lambda',$$

in other words, so that the retardation of the last ray becomes one wave length greater than before. We shall then have destructive interference between the light from the two halves of the grating. If it contain one thousand lines, the light from the first aperture will be just half a wave length ahead of that from the 501st aperture: in like manner the second aperture interferes with the 502d; and so on.

From the two preceding equations, it follows that

$$(a + d)(\sin \theta' - \sin \theta) = n(\lambda' - \lambda) = \frac{\lambda}{N}, \qquad \text{Eq. 209}$$

which means that for any given wave length the angular distance from a point of maximum intensity θ to the next minimum θ' is inversely proportional to the total number of lines N on the grating and to the spacing of the lines.

The ability of a grating to separate the images of a single source, emitting two nearly equal wave lengths λ and λ', is called its *resolving power*. It follows from the preceding equation that the numerical value of the resolving power, which we shall denote by R, is measured by the following expression: —

$$R = \frac{\lambda}{\lambda - \lambda'} = nN, \qquad \text{Eq. 210}$$

where $\lambda - \lambda'$ is the difference of wave length between the closest images which the grating will show as double. Evidently this resolving power depends (*if the grating be perfect in other respects*) upon the product of the total number of rulings on the grating N by the order of the spectrum n.

Brightness of Spectra

535. The third question is, How bright will any particular spectrum be compared with the intensity which one would obtain by use of a single aperture of the same width as those on the grating? The complete answer is too long a story for this volume, but the result is as follows: —

If I is the intensity, in any given direction, of the spectrum of a monochromatic source of wave length λ, produced by a grating which has N lines ruled upon it at the uniform distance of $a + d$ cm. apart, then

$$I = I' \frac{\sin^2 \frac{N\mu(a+d)}{2}}{\sin^2 \frac{\mu(a+d)}{2}}, \qquad \text{Eq. 211}$$

where I' is the intensity which a single aperture would give in the same direction. The "given direction" is here defined by μ, whose value is

$$\mu = \frac{2\pi}{\lambda}(\sin i + \sin \theta),$$

where the algebraic symbols have the same meaning as in Eq. 206.

536. In 1883 Rowland invented a **concave grating spectroscope,** the distinguishing feature of which is that it requires neither collimator nor view telescope, consisting as it does of a grating ruled upon a concave spherical mirror of speculum metal. If one has regard both to convenience and resolving power this instrument, for general spectroscopic purposes, is probably unexcelled.

The Echelon Spectroscope

537. Measured by resolving power the echelon spectroscope devised by Michelson in 1898 is a still more effective instrument. This is essentially a grating with only a few rulings, in which, however, the form of the ruling or groove is given a perfectly definite form. The result is obtained by using a pile of from ten to thirty plane parallel glass plates of equal thickness, the edge of each plate being slightly displaced over that of its neighbor. The high resolving power is here obtained by the use of spectra of high orders, even as high as twenty or thirty thousand.

PRINCIPLES OF SPECTROSCOPY

538. The science of spectroscopy is a large and independent subject which must be left for later study. But it may be here stated that the entire structure is built upon a foundation consisting of something like the following principles: —

(i) *Spectra of Gases.* — The emission spectrum of a gas, unless strongly compressed, is always a spectrum of bright lines.

(ii) *Kirchhoff's Laws.* — (1) The ratio of the thermal emission to the absorption, in a solid or liquid body, depends upon the temperature only; and (2) this ratio is numerically equal to the emission of an absolutely black body at the same temperature.

(iii) *Doppler's Principle.* — The effect of motion in the line of sight is to apparently shorten the wave length when source and observer approach each other; and to lengthen the wave length when source and observer recede from each other. See § 207. It is by use of this principle that the rate of approach of the "fixed" stars is measured.

(iv) *Principle of Humphreys and Mohler.* — Effect of increased pressure about the source is, in general, to slightly increase the wave length.

(v) *The Zeeman Effect.* — When a source of light is placed in a strong magnetic field, each individual line in its spectrum will, in general, become a triplet, or a still more complex line.

(vi) *The Stark-LoSurdo Effect.* — When a source of light is placed in a strong *electric* field, each individual line in its spectrum will, in general, become a more complex line.

(vii) *Temperature Effects.* — (1) The maximum radiation of a solid body varies as the fifth power of its absolute temperature. (2) The wave length of the maximum radiation from a solid body varies inversely as the absolute temperature of that body. See § 274.

(viii) *Law of Spectral Series.* — It has been shown by Kayser and Runge and Rydberg and others that, in the case of many chemical elements nearly all their spectral lines are arranged in series, and that their respective wave lengths λ may be computed from the following formula: —

$$\frac{1}{\lambda} = A - \frac{B}{(m + C)^2},$$

in which A, B, and C are constants for each series and m denotes the natural numbers beginning with 3. **Each chemical element has its own different characteristic values for the constants A and C.** This fact is the foundation of spectrum analysis, a great science which has made a "triple alliance of astronomy, chemistry, and physics."

539. In conclusion the student is warned not to think of the preceding outline of optics as anything but a very incomplete sketch. For instance, the entire subject of color vision and color mixture has been omitted; likewise the phenomena of polarized light. The subject of astrophysics has not been touched upon. These and many other interesting topics are discussed in the following books: —

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